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THE PHASE AMBIGUITY RESOLUTION BY THE EXHAUSTION METHOD IN A SINGLE-BASE INTERFEROMETER

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ABSTRACT

The paper considers the phase methods for measuring an object spatial orientation by means of satellite navigation equipment. Methods for the resolution of phase ambiguity are analyzed. Effectiveness and applicability of the one-step methods are discussed in more detail. It is proved that for the realization of the exhaustion method the minimal group of navigation space crafts should include 5–6 observed ones. When measuring signals of 8 space crafts with base length of 1 m, an unambiguous solution is achieved practically in all cases.

Keywords: interferometer, the phase ambiguity, single-step exhaustion methods, satellite navigation systems.

INTRODUCTION

Development of professional radio-navigation equipment to improve accuracy was ensured by the application of phase methods for the measurement of navigation signals parameters. Their application in the geodesic class equipment allows determining relative coordinates of the objects with a centimeter and millimeter accuracy. Phase methods can also be applied to determine space orientation of the objects by GLONASS/GPS signals.

Angular location of the object in the space by the signals of satellite radio-navigation systems may be determined based on the measurement of difference in the speed of navigation spacecrafts signals between the antennas located at the ends of base vectors. Interferometers with the distance between the antennas (base length) up to several meters are applied to improve the accuracy of space orientation determination. The main problem in phase changes is the phase ambiguity resulting from the fact that the wavelength of the measured signals is relatively short (about 19 cm) which is much lower than the length without the interferometer [16].

Currently, LAMBDA-method is applied to the majority of goniometric and geodetic equipment to resolve phase ambiguity. To solve the problem by LAMBDA-method at least two dimensions are required, on condition that the phase ambiguity of the measurements does not vary. Therefore, single-stage methods of resolving ambiguities are of interest. Single-stage method based on maximum likelihood use the redundancy of the system of equations that can be obtained by using excessive constellation of navigation spacecrafts [13], [18].

Exhaustion search method is used to resolve phase ambiguity in a single-base interferometer. The solution is chosen based on the maximum likelihood criteria. The minimum constellation of navigation spacecrafts for the implementation of exhaustion search method is 5-6. When measuring signals of 8 navigation spacecrafts with the base length of 1 m the solution in almost all cases is unique.

Satellite radio-navigation systems (SRNS) are widely used in almost all areas of science and technology. Modern navigation receivers are able to determine the current position of the object with an accuracy of 3-5 meters and the velocity vector of up to 0.1 m/s. In addition, the frequency and time-based equipment is used based on GLONASS/GPS radio-navigation systems allowing synchronizing on-board time scale with the UTC scale with an accuracy of up to 100 ns. Scientists and engineers have applied significant efforts to further improve the accuracy of navigation parameters. Work on the integration of receivers for satellite navigation systems and inertial navigation systems are ongoing [8]. The application of a joint digital filtering of navigation data and stand-alone systems data with Kalman filter may also significantly improve the accuracy [2], [11], [12], [14]. The multipath reception error compensation [15] allows increasing the accuracy of determining the coordinates and velocity vector of the object.

Further development of the radio-navigation equipment is represented by the application of phase methods for measurement of navigation signals parameters. Application of phase methods in geodetic equipment allows determining the relative coordinates of objects with a centimeter and millimeter accuracy.

Phase methods can also be applied to determine space orientation of objects by GLONASS/GPS signals with high accuracy. In contrast to magnetic sensors and inertial systems measuring the space orientation of objects, the satellite goniometric equipment determines the angle of head, pitch and bank to the true meridian and has no drift of measured parameters. Furthermore, compared to inertial sensors, the goniometric satellite equipment is characterized by short initialization time and low cost. However, the implementation of this method requires measurement of the coordinates of the phase centers of the antennas with millimeter accuracy [19].

The angular position of an object in space by SRNS signals can be determined by measuring the path difference of the navigation spacecrafts (NS) signals between the antennas located on the ends of base vectors [3]. To determine the space orientation, two non-collinear base vectors are sufficient, i.e. three antennas.

The phase shift of the NS signal received at two spaced antennas, and the cosine of angle between the base vector and the direction vector to the NS is associated with the following formula (Figure-1):

$$\cos\alpha = \frac{\lambda \cdot \varphi}{2\pi B}, \tag{1}$$

where λ – wavelength of the NS signal; φ - phase shift; B - base length; α - angle between base vector and direction vector to NS.

Formula (1) is an equation of a single-base interferometer and is widely applied in the theory of phase direction finders and antenna arrays.

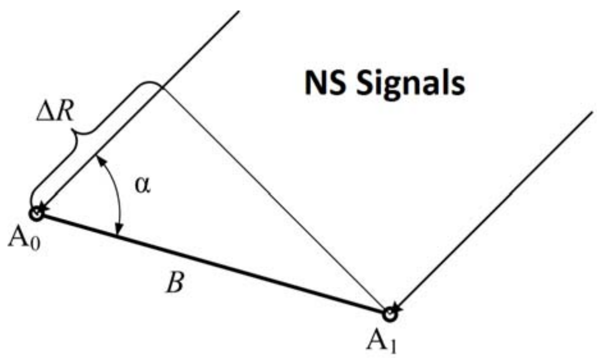

Figure-1. Single-base interferometer.

The base vector coordinates may be determined by the equation based on scalar product of vectors:

$$k_x x + k_y y + k_z z = \Delta R = \lambda \frac{\Phi}{2\pi}, \tag{2}$$

where k_x, k_y, k_z – directional cosines of direction vector at NS; x, y, z – coordinates of the base vector; ΔR – path difference; Φ – signal phase shift; λ – wavelength.

Not less than three equations are required for determination of all unknowns. Taking into account the interrelation of base vector coordinates, knowing the base length B the system of equations may be as follows:

$$\begin{cases} k_{xi}x + k_{yi}y + k_{zi}z = \Phi_i, \\ x^2 + y^2 + z^2 = B^2, \end{cases} \tag{3}$$

where i is the satellite number.

The main problem in phase measurements is phase ambiguity. In order to improve the accuracy of determining the space orientation, the interferometers with the distance between the antennas (base length) up to several meters are applied. The ambiguity of the measurement of the phase shift is due to the fact that the wavelength of the measured signal is short enough (about 19 cm), which is much shorter than the interferometer base lines [11], [12]. Phase ambiguity resolution methods may be divided into two classes: single-stage operation based on the results of each measurement [8], [14] and methods based on filtration, requiring the measurement of phase shifts within a certain time interval [2], [15], [19].

$$k_x x + k_y y + k_z z = \Phi_i + n_i \lambda_i \tag{4}$$

where n – integer-valued phase ambiguity; i = 1, 2, … N – order number of the NS under consideration.

Currently, the majority of goniometric and geodetic equipment to resolve phase ambiguities applies LAMBDA-method [4], [7], [10].

According to this method, at the first stage the integer ambiguity n is represented as an additional unknown quantity without regard to its integer. Then each measurement gives one unknown value n_i. As a result, each measurement by N of the NS, the system of equations will have N + 3 unknowns; therefore, the equations prove to be insufficient. Hence, to solve the problem by LAMBDA-method, at least two measurements are required, considering that the phase ambiguity of the measurements does not vary. However, due to the fact that the position of NS is changing slowly, with a stationary object in each new measurement, equations (2) are strongly correlated with the previous ones, and the system of equations, despite the redundancy, is close to degeneration. To solve the obtained improperly justified system of equations, decorrelation is used; herewith, a more or less reliable estimation of the unknowns n_i is ensured. At the second stage the obtained values of phase ambiguity are reduced to an integer (mainly by rounding), and then the original system of equations is solved.

THE EXHAUSTION METHOD

Single-stage methods applied to resolve phase ambiguity [1], [5], [6] are of special interest. Single-stage methods based on maximum likelihood use the redundancy of the equation system that can be obtained by using an excessive constellation of NS.

To resolve phase ambiguity in a single-base interferometer, and exhaustion method is applied in a single-stage method. The solution is selected by the criteria of maximum likelihood. The likelihood formula (LF) with the system of signals N of the NS may be presented as follows:

$$W\left(\Phi_1, \Phi_2, \ldots \Phi_n \mid x, y, z\right) = \prod_{i=1}^{N} \left[\frac{1}{\sigma_i \sqrt{2\pi}}\right] \cdot \exp\left[-\sum_{i=1}^{N} \frac{\left[\Phi_i + n_i \lambda_i - \left(k_{xi}x + k_{yi}y + k_{zi}z\right)\right]^2}{2\sigma_i^2}\right] \tag{5}$$

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$$x^2 + y^2 + z^2 = B^2. \quad (6)$$

With an additional condition

Formula (5) may have a local minimum for each combination of ambiguities n_i. The task to minimize the likelihood formula by all possible values n_i is solved by their sorting. The main disadvantage of the minimization method is a large amount of ambiguity combinations n_i. The number of ambiguity combinations upon receipt of signals N of the NS will be $n_{\max}^N$, where $n_{\max}$ = int($2B/\lambda+1$). For instance, with a base length B = 1 m the ambiguity n for each NS may take 11 values (from –5 to 5). The general amount of ambiguity combinations in case of measurements at three NS will be 11^3=1331, at four NS - 11^4 = 14641, at eight NS - $\approx 2\cdot10^8$. A local minimum of formula (5) corresponds to each combination n_i. In case of large amount of combinations n_i, a situation may occur when the values of local minimums are close to the global minimum resulting in the likelihood of a wrong solution. To reduce the volume of calculations in resolving ambiguities of phase dimensions, the base length may be reduced; however, in this case the accuracy of angle measurements deteriorates.

The amount of calculations may be significantly reduced if we choose an initial constellation with a minimum number of NS (non-excessive constellation of NS). Based on possible combinations of phase ambiguities and solving the problem at these values of phase ambiguity, an initial set of solutions is developed. Then every solution of the initial set of solutions is checked by the solution with regard to a complete constellation. Screening of the spurious solutions by the maximum likelihood criteria, or similarly, by a total discrepancy between the minimum least squares solutions (MLS).

Potential exhaustion search methods may be explored through the analysis of the likelihood function. The angular position of the base vector at a known length may be set by two parameters – angle of head K and angle of pitch Ψ; herewith, the likelihood function will be two-dimensional. Angles of head and pitch are connected with rectangular coordinates with the help of the following formula:

$$X = B\cdot\cos\mathrm{K}\cdot\cos\Psi\ ;\ Y = B\cdot\sin\mathrm{K}\cdot\cos\Psi\ ;\ Z = B\cdot\sin\Psi\ . \quad (7)$$

When resolving phase ambiguities, special interest is paid to the likelihood of gross errors, i.e. the cases in which the phase ambiguity is determined incorrectly. Gross errors occur when LF has side lobe values comparable with the main maximum value corresponding to the right solution. This situation is illustrated in Figure-2, which shows the likelihood function for one NS. The figure shows that the resolution of phase ambiguities in the measurement of one base for each NS alone is impossible, because the likelihood function takes the extreme values in the whole areas and spurious solutions are indistinguishable from true solutions.

By increasing the number of observed NS the total residual error represents the sum of sinuous functions obtained by measuring each NS, and is the result of the interference of these functions. Figure 3 shows LF at four NS. There are clearly distinguished main and side lobes.

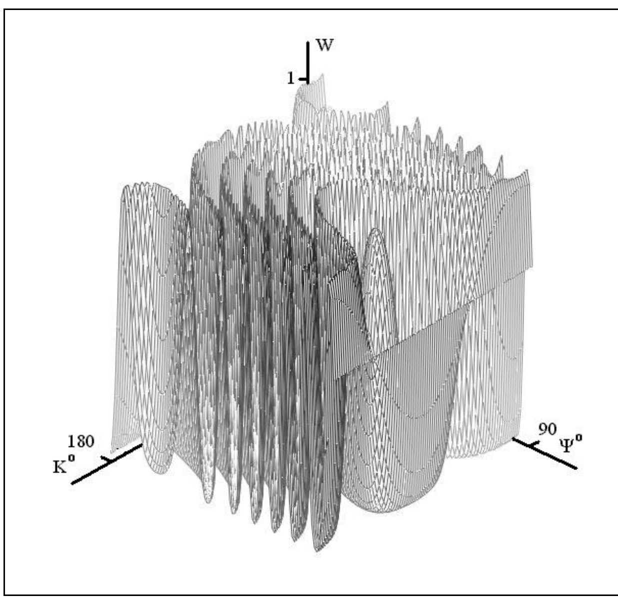

Figure-2. LF in measurements by one NS.

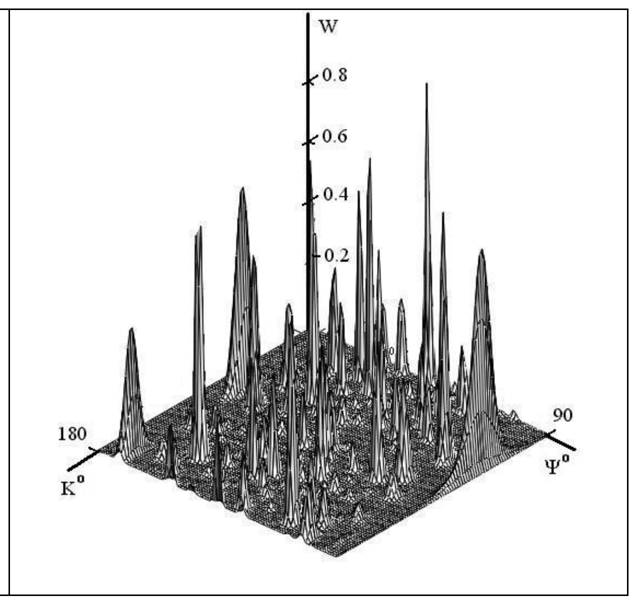

Figure-3. LF in measurements by four NS.

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LF is quite complicated to analyze; therefore, it is necessary to introduce one parameter enabling to assess the likelihood of missing the correct solution and the probability of gross errors, i.e. spurious solution. LF indicator may be used as such parameter representing a total residual error MLS of solution, equal to the sum of squared residual errors for all NS, or the square root of this value.

Residual errors are composed of two elements: one of them is due to discrepancies in the side lobes resulting from spurious solutions adopted, and the other is due to the dispersion of the measured phase shifts. The side lobe of LF provides spurious solution; herewith in case of redundancy, the system of equations becomes inconsistent even at zero values of error of phase shifts measurement. The value of residual errors in side lobes of LF in case of no measurement noise depends on the configuration of NS and values of phase ambiguity; hence, this value can be considered as the mathematical expectation of residual errors. Noise measurement error of the phase shift is characterized by normal distribution. Thus, the residual error for each NS in solving the system of equations (3) in the main and side LF, maximums LF are distributed in accordance with the normal law with the mathematical expectation, equal residual errors that arise in the absence of noise measurements.

Let us consider the function of the likelihood of closing error distribution. If mathematical expectations of values x_i equal to zero and their dispersions are equal, than the value $z = x_1^2 + x_2^2 +\ldots+ x_n^2$ is distributed in accordance with the law χ^2 with n degrees of freedom [9]. This refers to the main maximum of LF at equally accurate measurements of phase shifts. In side lobes mathematical expectations are not equal to zero and the application of the distribution law χ^2 is not allowed.

The formula for the distribution of total residual error may be obtained in accordance with the following procedure. Firstly, we need to obtain the likelihood density of the square for one random value; then by applying the law of random variables addition, we can obtain the desired likelihood density. To calculate the distribution function of the total residual error square value, the characteristic formulas should be used [17].

The characteristic formula for the calculation of the random variable square value with a non-zero mathematical expectation is as follows:

$$\Theta(v) = \frac{1}{\sqrt{1-2i\sigma^2 v}} \cdot \mathrm{e}^{\frac{im^2 v}{1-2i\sigma^2 v}}. \tag{8}$$

The characteristic formula for the calculation of the total square values of the independent normal random values with a non-zero average equals to the multiplication of characteristic formulas of additives:

$$\Theta_n(v) = \left(1-2i\sigma^2 v\right)^{-\frac{n}{2}} \cdot e^{i\frac{v\cdot\sum_{k=1}^{n} m_k^2}{1-2i\sigma^2 v}} \tag{9}$$

Formula (9) shows that one of the features of the characteristic formula is as follows: it depends not on mathematical expectations of the initial random values but on the total of their square values $m^2 = \sum_k m_k^2$. The formula for distribution of the total square value of residual errors should also have the same characteristic.

The likelihood density may be obtained by Fourier reverse transformation of the characteristic formula (9).

$$P_n(x) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} \Theta_n(v) \cdot e^{-ivx} dv = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} \left(1-2i\sigma^2 v\right)^{-\frac{n}{2}} \cdot e^{i\frac{v\cdot m^2}{1-2i\sigma^2 v}} \cdot e^{-ivx} dv \tag{10}$$

The likelihood density diagrams at different values m in case of five NS under consideration are shown in Figure-4. The diagrams show principal possibility of spurious solutions screening at $m > 5\sigma$.

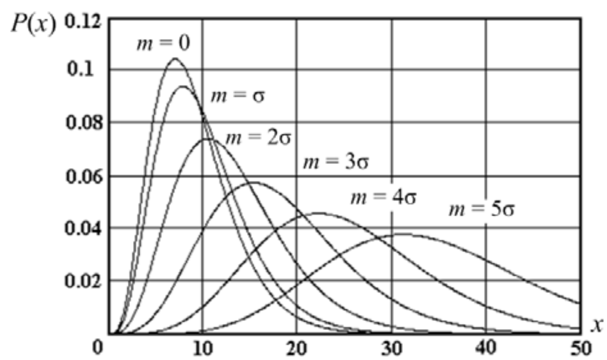

Figure-4. Likelihood density of total residual error at different parameter values m

The likelihood density (10) is not represented via elementary and tabulated formulas that complicate the task to calculate likelihood of finding a specific value in a certain area. For this purpose, a cumulative distribution formula may be applied:

$$W_n(x) = \frac{1}{2\pi} \int_{-\infty}^{x} \int_{-\infty}^{\infty} \Theta(v) \cdot e^{-ivx} dv dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\Theta(v)}{iv} \cdot \left(1-e^{-ivx}\right) dv = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\left(1-2i\sigma^2 v\right)^{-\frac{n}{2}}}{iv} \cdot e^{\frac{im^2 v}{1-2i\sigma^2 v}} \left(1-e^{-ivx}\right) dv \tag{11}$$

To determine the likelihood of a spurious solution with the specified likelihood of missing the correct solution it is necessary to determine the threshold value at which the correct solution gets to the list of possible solutions with the specified probability. The threshold value can be selected by using the cumulative distribution function (11), assuming $m = 0$.

Figure-5 shows the likelihood of a spurious solution versus the ratio between total expected residual errors and their mean-square deviation (MSD) at different number of the observed NS. The likelihood of producing a spurious solution is largely characterized by a minimum expectation of the total residual error in the side lobes. One can see from Figure 5 that the effectiveness of the rejection of spurious solution is achieved in case $m > (5…6)\cdot\sigma$.

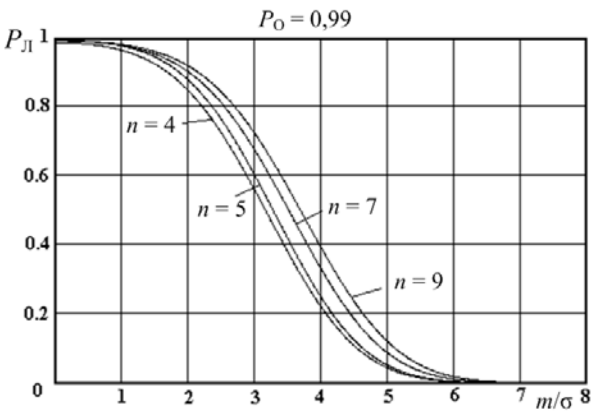

Figure-5. Likelihood of a spurious solution versus the ratio between total expected residual errors and their MSD.

Exhaustion of possible solutions gives a set of residual error components by means of incorrect phase ambiguity resolution, which are a deterministic magnitude and represent residual error expectations m. They can be calculated a priori for each combination of phase ambiguities. The minimum value of this magnitude is of interest, since precisely such options have the greatest likelihood of a spurious solution, while increasing the value of m the likelihood of a spurious solution decreases rapidly. However, the calculation of residual errors for each specific case runs into considerable difficulties, primarily because of the large number of phase ambiguity combinations, which occur during exhaustion of all options.

When analyzing, the residual error expectations in side lobes (with zero phase shift measurement error) can be considered as a random variable. According to the obtained data, distribution of the residual error expectations is practically independent of the configuration of the constellation of NS, the spatial arrangement and the base vector length; however the minimum value of the total residual error decreases with the increasing base length. This dependence is explained by the quadratic increase in the number of possible positions of the base vector with an increase in its length. The square root of the sum of squared residual errors (total residual error) is described rather accurately by a normal distribution, and in this case the standard deviation does not depend on the number of NS in the constellation (the number of NS was considered in the constellation of 4 to 13, at different positions, and the base vector length) and makes 28 mm. The only exception is the case when measuring on 4 NS. Distribution histograms for $n = 4$ and $n = 9$ are shown in Figure-6. The normal distribution with a large number of NS can be explained as a conclusion of the central limit theorem. The mean value of the total residual error with the number of NS exceeding 5 is linearly dependent on the number of NS in the constellation.

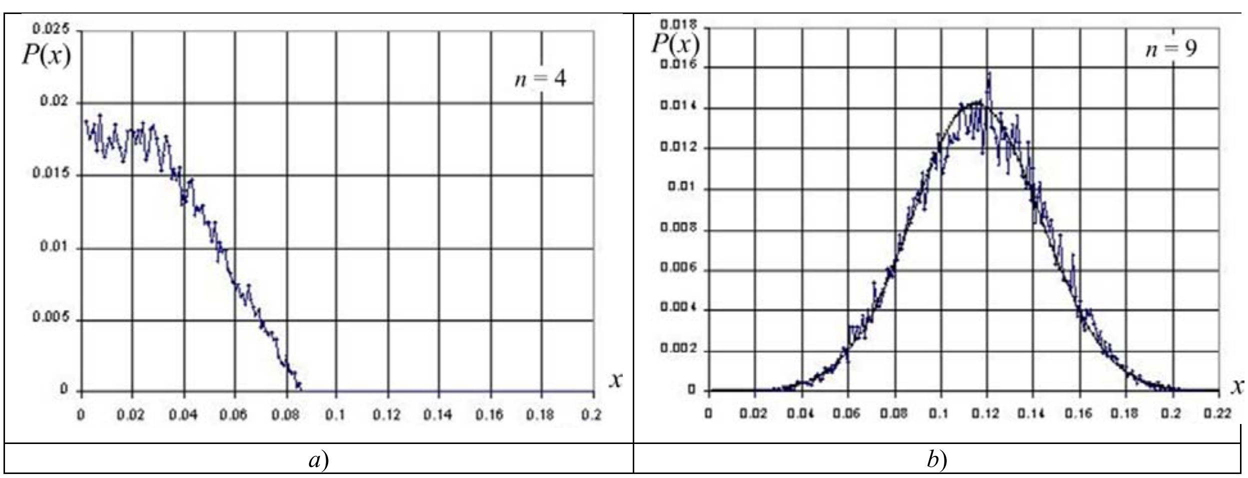

Figure-6. Distribution histograms of total residual error with four (*a*) and six (*b*) observed NS.

The likelihood of producing spurious solution is largely characterized by the minimum residual error in side lobes. Making use of expression (11) for cumulative distribution function, it is possible to determine likelihood of producing spurious solution. Figures 7 and 8 demonstrate gross error likelihood with the minimum residual error and the base length of 1 and 10 m versus phase shift measurement error.

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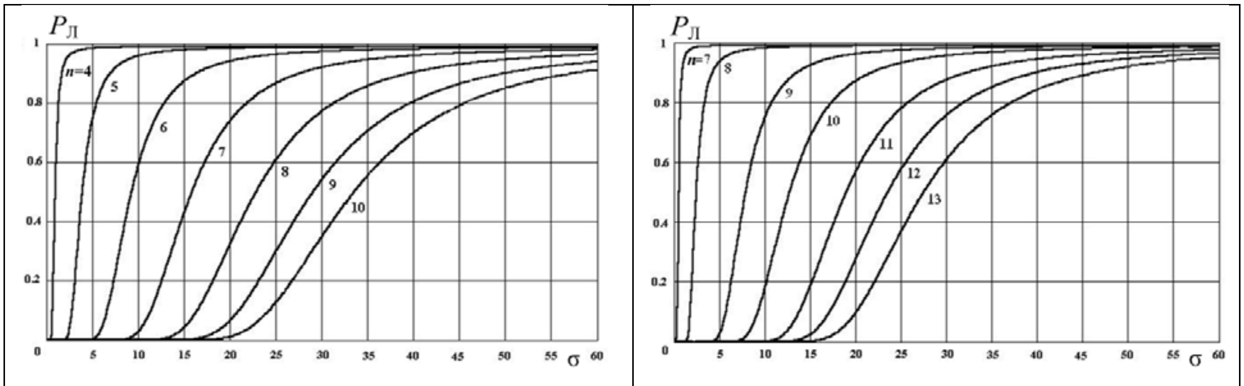

a) b)

Figure-7. Gross error likelihood with the base length of 1 m (*a*) and 10 m (*b*).

CONCLUSIONS

Based on the research findings, the following conclusions may be made:

1) The efficiency of the exhaustion method for the phase ambiguity resolution depends on the number of the observed NS and the base length. With the base length of 1 m the exhaustion method can operate even at 5 observed NS, noise error of the phase shift measurement being 5°, whereas the base length of 10 m and the same phase shift measurement error requires observation of 7–8 NS.

2) Exhaustion method of phase ambiguity resolution may be applied with interferometer base length up to 3 m and limiting MSD of phase shift measurement error ranging from 15…20°.

3) Minimum constellation of NS for the exhaustion method implementation makes 5–6 observed NS. Measurement of signals of 8 NS and use of the base length of 1 m produces an unambiguous solution in almost all cases.

It should be noted that the single-step exhaustion method in a single based interferometer is used in practice to compile an initial set of solutions; therefore the most important characteristic is the likelihood of missing the correct solution, which is determined by a threshold value of the likelihood function. The presence of spurious solutions in the initial set does not mean a gross error if the correct solution is also present in the initial set of solutions. Further rejection of spurious solutions can be carried out by filtration of solutions from the initial set of solutions and by using a multi-base antenna system.

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