INVERSE KINEMATICS ANALYSIS OF A 5-AXIS RV-2AJ ROBOT MANIPULATOR

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ABSTRACT
This paper presents the inverse kinematics analysis of the five degree of freedom (DOF) Mitsubishi Melfa RV-2AJ industrial robot. The proposed method is used specifically for controlling the z-axis Cartesian position. The kinematics problem is defined as the transformation from the robot’s end-effector Cartesian space to the joint angles of the robotic arms. An analytical solution using trigonometry illustration is presented to describe the relation between the position of the robot end-effector to each of the robot joints. Several lab experiments to validate the established kinematics equations have been conducted. In this study, the developed kinematics solutions were found to be accurate to approximately 99.83% compared to the real robot. These findings have significant implication for developing a kinematic simulation model that can be used to evaluate position and force control algorithm.

Keywords: inverse kinematics, mitsubishi melfa RV-2AJ, pythagoras’s theorem.

INTRODUCTION
Robotics technology is at the convergence of diverse branch of knowledge that one must realize to successfully express the motion of complex robotic systems. At present, incorporation of human in the process is fundamental for a rapid setup, programming and robot system maintenance. Therefore, before any robotic system is associated to a specific workplace, a simulation approach is often needed to provide deep understanding upon the control framework and its behavior. By simulating the robot and its environment, the human can consistently improve the overall system, reduce the build cost, and eliminate the risks the robot might exerts on the user.

The Mitsubishi RV-2AJ robot as shown in Figure 1 has been chosen as a case study as it technically offers the best performance as a small, compact and powerful articulated-arm robot in its class. The robot operates using AC servomotors that can produce a maximum speed of 2100 mm/s with a repeatability of ±0.02 mm. The high precision motors with integrated absolute position encoders consistently ensure reliable and maintenance-free operation (Esa, Ibrahim, Mustaffa, & Majid, 2011).

The maximum payload of the robot is rated at 2 kg and thus is exceptionally sufficient for low payload handling, placing and separating small parts. Another prominent applications that are worth mentioning include quality control and handling samples in medical and other laboratories. Since the robot is able to cover horizontal motion of up to 410 mm (with the gripper pointed downwards), it is also ideal for applications where a small and compact robot needs to be installed directly next to or even in any automated system.

Figure-1. RV-2AJ at work in cramped quarters. (Mitsubishi Electric, 2009).

Taking into consideration the way the five jointed robot arm of the RV-2AJ is engineered, the mechanical structure is designated as anthropomorphic articulate (having human-like characteristics). Every single joint has one freedom of rotation around its own axis. The motion axes for the model are assigned with their own nomenclature as follows: base rotation for Joint 1, shoulder rotation for Joint 2, elbow rotation for Joint 3, wrist pitch for Joint 4, and wrist roll for Joint 5. Figure-2 shows the assigned naming convention.
Table-1 lists the maximum working envelope for each joint of the robotic arm while Table-2 lists the link distance between those joint. Each link has its own range limitation allocated by the manufacturer so that the arm will not move beyond the range and thus damaging the servo motor. The complete dimensions of the link frames, the distance between the joints and the robot work envelope are given in Figure-3.

Table-1. Rotation range for RV-2AJ. (Mitsubishi Electric, 2002).

<table>
<thead>
<tr>
<th>Joint</th>
<th>Angle Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Waist rotation</td>
<td>-150° to +150°</td>
</tr>
<tr>
<td>2: Shoulder rotation</td>
<td>-60° to +120°</td>
</tr>
<tr>
<td>3: Elbow rotation</td>
<td>-110° to +120°</td>
</tr>
<tr>
<td>4: Wrist pitch</td>
<td>-90° to +90°</td>
</tr>
<tr>
<td>5: Wrist roll</td>
<td>-200° to +200°</td>
</tr>
</tbody>
</table>

Table-2. Link distance for RV-2AJ. (Mitsubishi Electric, 2002).

<table>
<thead>
<tr>
<th>Link</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Waist to shoulder</td>
<td>300 mm</td>
</tr>
<tr>
<td>2: Shoulder to elbow</td>
<td>250 mm</td>
</tr>
<tr>
<td>3: Elbow to wrist</td>
<td>160 mm</td>
</tr>
<tr>
<td>4: Wrist to end</td>
<td>72 mm</td>
</tr>
</tbody>
</table>

Inverse kinematics of RV-2AJ robot

The inverse kinematics study consists of determining the joint angles of a robot from its specified end-effector Cartesian position. The analysis of this inverse kinematics for the robot mechanical structures is vital to realize the mechanical system, allowing the development of further studies and applications.

In the past, previous researchers have established different method for developing inverse kinematics of the RV-2AJ robot than the one being discussed in this paper. Such is by implementing an iterative algorithm based on Jacobian transpose matrix (Haklidir & Tasdelen, 2009). However, for no reasonable justification, the kinematic model was developed only up to the first three joints (instead of five) and no experiments result were showed at all to verify the accuracy of the model. As a result of the limited DOF and the unknown accuracy, the position of the end-effector will always be ambiguous. Several other researches have also developed the inverse kinematics solution for the RV-2AJ, but again the accuracy of the models were not proven by any approach (Coman, Balan, Donca, & Verdes, 2011; Coman, Stan, Manic, & Balan, 2009; Šljivo, 2013). On that account, this paper addresses this matter thoroughly with comparison to experimental results in order to validate the accuracy of the developed model. In addition, considering that the proposed method in this paper is limited to just controlling the z-axis Cartesian position of the robot, the process is much simpler and straightforward to develop compared to other conventional method from previous researchers.

Pythagoras’s theorem in inverse kinematics problem

In order to solve the inverse kinematics problem for the 5-axis RV-2AJ robot using Pythagoras’s theorem, a conclusion has been made in which that the solution is not applicable for the wrist roll of the robot. This is because the rotation of the wrist roll does not have any influence on the end-effector Cartesian position. Therefore, the joint angle of the wrist roll is not considered.

Referring to the top view of the robot as shown in Figure-4, the waist joint angle of $\theta_1$ can be easily resolved. Additionally, it can be seen that wherever the robot moves in any Cartesian position that is permissible.
for the robot, the waist joint angle could always be calculated by using the same technique that will be discussed in this section.

**Figure-4.** Determining waist angle at joint 1.

Since the robot posture resulted in Figure-4 produces a right-angled triangle that involves the angle at first joint, the following Pythagoras's theorem applies:

\[
tan \theta_1 = \frac{P_y}{P_x}
\]

Hence, \( \theta_1 \) can be calculated as follows:

\[
\theta_1 = tan^{-1} \left( \frac{P_y}{P_x} \right)
\]

Before the same theorem is applied to identify the subsequent joint angles of the robot, the initial condition of the wrist pitch has to be configured as shown in Figure-5 so that the wrist pitch is always perpendicular to the ground level at all times during the experiments. To achieve the required setting, both the elbow and the wrist pitch were inclined at +90° while the shoulder is at 0° respectively.

**Figure-5.** RV-2AJ robot with wrist pitch at vertical position.

Figure-6 shows an example position to acquire the shoulder joint angle of \( \theta_2 \) to \( \theta_4 \) of the robot. The law of sine will be used in this section as the following equation:

\[
\frac{a}{sin \theta_A} = \frac{b}{sin \theta_B} = \frac{c}{sin \theta_C}
\]

(3)

\[
c = \sqrt{P_x^2 + [300 - (P_z + 72)]^2}
\]

(4)

\[
c^2 = a^2 + b^2 - 2ab \cos \theta_C
\]

(5)

\[
\theta_C = \cos^{-1} \left( \frac{a^2 + b^2 - c^2}{2ab} \right)
\]

(6)

Since the joint angle of each link of the robot is based on the previous link inclination, hence:

\[
\theta_A = 180 - \theta_C
\]

(7)

Substitute the calculated \( \theta_C \) to equation Error! Reference source not found. to acquire \( \theta_4 \) and \( \theta_6 \):

\[
\theta_4 = sin^{-1} \left( \frac{a}{c} \sin \theta_C \right)
\]

(8)

\[
\theta_6 = sin^{-1} \left( \frac{b}{c} \sin \theta_C \right)
\]

(9)

Next, to solve the following triangle, we get:

\[
\theta_{\theta} = tan^{-1} \left( \frac{300}{P_x} \right)
\]

(10)

Since both \( \theta_D \) and \( \theta_6 \) are within their right-angled triangle, this yields to:

\[
\theta_D = \theta_6 = 90 - \theta_{\theta}
\]

(11)
For the next section of the triangle, we get:

\[
\frac{c}{\sin \theta_G} = \frac{h}{\sin \theta_H} = \frac{Pz + 72}{\sin \theta_F}
\]  \hspace{1cm} (12)

with:

\[
h = \sqrt{300^2 + Px^2}
\]  \hspace{1cm} (13)

Calculating \( \theta_F \) from (12), we have:

\[
\theta_F = \sin^{-1}\left(\frac{Pz + 72}{c} \sin \theta_G\right)
\]  \hspace{1cm} (14)

Consequently, the remaining joint angles of \( \theta_2 \) and \( \theta_4 \) can be obtained as follows:

\[
\theta_2 = 180 - \theta_A - \theta_D - \theta_F
\]  \hspace{1cm} (15)

\[
\theta_H = 180 - \theta_G - \theta_F
\]  \hspace{1cm} (16)

\[
\theta_4 = 180 - \theta_B - \theta_H
\]  \hspace{1cm} (17)

RESULTS AND ANALYSIS

The inverse kinematics of the RV-2AJ robot has been implemented and simulation study has been performed using the MATLAB program. To find out the angle at joint 1, several random positions of the x-axis position, \( Px \) and the y-axis position, \( Py \) were taken. For experimental results of finding the joint angles at joint 2 to joint 4, the Cartesian coordinates for the end-effector of the robot were set in a way that \( Px \) is fixed at 302.45 mm while \( Py \) is fixed at 0 mm at all time. However, it should be noted that the applied inverse kinematics method is still applicable when there are any movements anywhere in these axes. The experiment variable, which is the z-axis position, \( Pz \) of the robot end-effector changes from 397.35 mm down to 207.36 mm with 10 mm reduction (±1-3 mm) for each sample taken.

The desired Cartesian coordinates of the robot were manually controlled via the teach pendant and the joint angles of the robot were directly obtained from its display panel. Comparison between simulation and experiment results are analyzed in this section. In order to fully validate the workability of the developed inverse kinematics method, data taken from the experiments were categorized into three possible configurations.

Determining \( \theta_1 \)

To determine the first joint angle, \( \theta_1 \) of the RV-2AJ robot, the end-effector was placed in different Cartesian positions such as in Figure-4. Comparison for both simulation and experimental results were recorded in

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{No. of test} & \text{\( Px \) (mm)} & \text{\( Py \) (mm)} & \text{Simulation (degree)} & \text{Experiment (degree)} & \text{Difference (degree)} & \text{Error (degree)} \\
\hline
1 & 0.02 & 0 & \theta_1 & \theta_1 & 0 & 0 & 0 \%
\hline
2 & 224.52 & 156.22 & \theta_1 & 34.83 & \theta_1 & 34.83 & 0 & 0 \%
\hline
3 & 63.37 & 14.74 & \theta_1 & 13.0943 & \theta_1 & 13.09 & -0.0043 & -0.43 \%
\hline
4 & 50.73 & 58.3 & \theta_1 & 48.0801 & \theta_1 & 48.08 & -0.0001 & -0.01 \%
\hline
5 & 247.56 & -93.02 & \theta_1 & -20.594 & \theta_1 & -20.59 & 0.004 & 0.4 \%
\hline
6 & 7.04 & 140.77 & \theta_1 & -87.137 & \theta_1 & -87.14 & -0.003 & -0.3 \%
\hline
7 & 24.77 & -25.69 & \theta_1 & -46.0045 & \theta_1 & -46.05 & 0.0055 & 0.35 \%
\hline
8 & -41.65 & 65.76 & \theta_1 & -57.6513 & \theta_1 & -57.65 & 0.0013 & 0.13 \%
\hline
9 & -60.48 & -49.01 & \theta_1 & 39.0198 & \theta_1 & 39.02 & 0.0004 & 0.04 \%
\hline
10 & -37.99 & -67.95 & \theta_1 & 60.791 & \theta_1 & 60.79 & -0.001 & -0.1 \%
\hline
\end{array}
\]

Table-3. Comparison between simulation and experimental results for determining \( \theta_1 \).

First configuration for determining \( \theta_2 \), \( \theta_3 \), and \( \theta_4 \)

The first configuration for the conducted experiments to find the joint angles at joint 2 to joint 4 took place when the wrist joint of the robot was located above the shoulder joint as featured in Figure-7. By implementing the developed inverse kinematics method, identical end-effector positions from the robot have been used in MATLAB simulation and the results were recorded.

Figure-7. RV-2AJ robot at first configuration.
The comparison between the simulation and experimental results are presented in Table-4. The average error for \( \theta_2 \), \( \theta_3 \), and \( \theta_4 \) over the 10 data readings are 0.34%, 0.82%, and 2.28%, respectively.

**Table-4.** Comparison between simulation and experimental results for first configuration.

<table>
<thead>
<tr>
<th>No. of test</th>
<th>( P_r ) (mm)</th>
<th>Simulation (degree)</th>
<th>Experiment (degree)</th>
<th>Difference (degree)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>397.35</td>
<td>( \theta_2 ) 35.7388</td>
<td>( \theta_2 ) 35.73</td>
<td>-0.0088</td>
<td>-0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 66.3738</td>
<td>( \theta_3 ) 66.39</td>
<td>0.0162</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 77.8877</td>
<td>( \theta_4 ) 77.85</td>
<td>-0.0277</td>
<td>-0.36</td>
</tr>
<tr>
<td>2</td>
<td>387.34</td>
<td>( \theta_2 ) 36.3252</td>
<td>( \theta_2 ) 36.32</td>
<td>-0.0052</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 68.9218</td>
<td>( \theta_3 ) 68.93</td>
<td>0.0082</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 74.5373</td>
<td>( \theta_4 ) 74.73</td>
<td>-0.023</td>
<td>-0.31</td>
</tr>
<tr>
<td>3</td>
<td>377.35</td>
<td>( \theta_2 ) 37.0279</td>
<td>( \theta_2 ) 37.02</td>
<td>-0.0079</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 71.2079</td>
<td>( \theta_3 ) 71.28</td>
<td>0.0091</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 71.7012</td>
<td>( \theta_4 ) 71.67</td>
<td>-0.0003</td>
<td>-0.01</td>
</tr>
<tr>
<td>4</td>
<td>367.25</td>
<td>( \theta_2 ) 37.84</td>
<td>( \theta_2 ) 37.84</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 73.4453</td>
<td>( \theta_3 ) 73.45</td>
<td>0.0047</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 68.7147</td>
<td>( \theta_4 ) 68.69</td>
<td>-0.0247</td>
<td>-0.34</td>
</tr>
<tr>
<td>5</td>
<td>357.35</td>
<td>( \theta_2 ) 38.7597</td>
<td>( \theta_2 ) 38.76</td>
<td>0.0003</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 75.4433</td>
<td>( \theta_3 ) 75.45</td>
<td>0.007</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 65.7973</td>
<td>( \theta_4 ) 65.78</td>
<td>-0.0173</td>
<td>-0.27</td>
</tr>
<tr>
<td>6</td>
<td>347.36</td>
<td>( \theta_2 ) 39.7787</td>
<td>( \theta_2 ) 39.78</td>
<td>-0.0014</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 77.2742</td>
<td>( \theta_3 ) 77.28</td>
<td>0.0058</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 62.947</td>
<td>( \theta_4 ) 62.95</td>
<td>-0.0216</td>
<td>-0.32</td>
</tr>
<tr>
<td>7</td>
<td>337.34</td>
<td>( \theta_2 ) 40.8968</td>
<td>( \theta_2 ) 40.84</td>
<td>-0.0068</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 78.9515</td>
<td>( \theta_3 ) 78.94</td>
<td>0.0075</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 60.1518</td>
<td>( \theta_4 ) 60.13</td>
<td>-0.0218</td>
<td>-0.35</td>
</tr>
<tr>
<td>8</td>
<td>327.36</td>
<td>( \theta_2 ) 42.1013</td>
<td>( \theta_2 ) 42.1</td>
<td>-0.0013</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 80.4576</td>
<td>( \theta_3 ) 80.48</td>
<td>0.0014</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 57.4311</td>
<td>( \theta_4 ) 57.41</td>
<td>-0.0211</td>
<td>-0.37</td>
</tr>
<tr>
<td>9</td>
<td>317.27</td>
<td>( \theta_2 ) 43.3936</td>
<td>( \theta_2 ) 43.39</td>
<td>-0.0036</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 81.8342</td>
<td>( \theta_3 ) 81.84</td>
<td>0.0028</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 54.7721</td>
<td>( \theta_4 ) 54.75</td>
<td>-0.0025</td>
<td>-0.05</td>
</tr>
<tr>
<td>10</td>
<td>307.35</td>
<td>( \theta_2 ) 44.7725</td>
<td>( \theta_2 ) 44.77</td>
<td>-0.0025</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_3 ) 83.0556</td>
<td>( \theta_3 ) 83.06</td>
<td>0.0044</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_4 ) 52.1719</td>
<td>( \theta_4 ) 52.15</td>
<td>-0.0219</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

Second configuration for determining \( \theta_2 \), \( \theta_3 \), and \( \theta_4 \)

For the second configuration experiment, the wrist joint of the robot was placed horizontally equal to the shoulder joint. By realizing this Cartesian position of the robot, the robot configuration in Figure-8 was achieved.

![Figure-8. RV-2AJ robot at second configuration.](image1)

The simulation and experiment results from this particular position are presented in Table-5. The error produced for \( \theta_2 \), \( \theta_3 \), and \( \theta_4 \) are 0.40%, 1.24%, and 1.84%, respectively.

**Table-5.** Comparison between simulation and experimental results for second configuration.

<table>
<thead>
<tr>
<th>( P_r ) (mm)</th>
<th>Simulation (degree)</th>
<th>Experiment (degree)</th>
<th>Difference (degree)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>( \theta_2 ) 45.8340</td>
<td>( \theta_2 ) 45.83</td>
<td>-0.0040</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>( \theta_3 ) 83.8576</td>
<td>( \theta_3 ) 83.87</td>
<td>0.0124</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>( \theta_4 ) 50.3084</td>
<td>( \theta_4 ) 50.29</td>
<td>-0.0184</td>
<td>-1.84</td>
</tr>
</tbody>
</table>

Third configuration for determining \( \theta_2 \), \( \theta_3 \), and \( \theta_4 \)

The wrist joint of the robot was assigned to be lower than the shoulder joint for the third configuration. Detail posture of the robot is given in Figure-9.

![Figure-9. RV-2AJ robot at third configuration.](image2)
The results for the third region data readings between the MATLAB simulation and experiment are compared in Table-6. The average error for $\theta_2$, $\theta_3$, and $\theta_4$ readings are 0.17%, 0.45%, and 1.49% respectively.

Table-6. Comparison between simulation and experimental results for third configuration.

<table>
<thead>
<tr>
<th>No. of test</th>
<th>$P_t$ (mm)</th>
<th>Simulation (degree)</th>
<th>Experiment (degree)</th>
<th>Difference (degree)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>297.35</td>
<td>$\theta_2$ 46.228</td>
<td>$\theta_2$ 46.22</td>
<td>-0.0068</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_3$ 84.127</td>
<td>$\theta_3$ 84.14</td>
<td>0.0127</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_4$ 40.549</td>
<td>$\theta_4$ 40.63</td>
<td>-0.0159</td>
<td>-1.29</td>
</tr>
<tr>
<td>2</td>
<td>287.36</td>
<td>$\theta_2$ 47.754</td>
<td>$\theta_2$ 47.75</td>
<td>-0.0048</td>
<td>-0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_3$ 83.048</td>
<td>$\theta_3$ 83.05</td>
<td>0.0002</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_4$ 47.197</td>
<td>$\theta_4$ 47.18</td>
<td>-0.0072</td>
<td>-1.72</td>
</tr>
<tr>
<td>3</td>
<td>277.36</td>
<td>$\theta_2$ 49.357</td>
<td>$\theta_2$ 49.35</td>
<td>-0.0017</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_3$ 85.829</td>
<td>$\theta_3$ 85.83</td>
<td>0.0009</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_4$ 44.819</td>
<td>$\theta_4$ 44.8</td>
<td>-0.0012</td>
<td>-0.27</td>
</tr>
<tr>
<td>4</td>
<td>267.38</td>
<td>$\theta_2$ 51.009</td>
<td>$\theta_2$ 51.01</td>
<td>0.0001</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_3$ 86.468</td>
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</table>

Comparison with other method

To compare the accuracy of the developed inverse kinematic model, another conventional method of using inverse matrix multiplication (Niku, 2011) has been established and the results are shown in Table. For this comparison, $\theta_{234}$ represents the total summation of $\theta_2$, $\theta_3$, and $\theta_4$ which should theoretically produced 180°.

Based on these findings, it is clear that the method discussed in this paper is more accurate.

Table-7. Comparison between Pythagoras’s simulation and inverse matrix simulation results.

<table>
<thead>
<tr>
<th>No. of test</th>
<th>$P_t$ (mm)</th>
<th>Pythagoras’s Simulation (degree)</th>
<th>Inverse Matrix Simulation (degree)</th>
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</table>

CONCLUSIONS

In this paper, the inverse kinematic equations for the joint angle of the RV-2AJ industrial robot arm with regards to the end-effector position have been derived. It can be seen that the developed inverse kinematics solution provides approximately 97.72% to 99.83% accuracy identical to the robot itself. On the other hand, the errors produced from the conducted experiments (when compared to the simulations) were possibly because of the robot calibration issue and mechanical properties that contributes to slightly false data readings whenever the robot arms were moved.

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REFERENCES


(STUDENT), pp. 138–143. IEEE. doi:10.1109/STUDENT.2011.6089341


