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A REVIEW OF ACOUSTIC FDTD SIMULATION TECHNIQUE AND ITS APPLICATION TO UNDERGROUND CAVITY DETECTION

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ABSTRACT

The feasibility of the Finite Difference Time Domain (FDTD) technique for acoustic wave created an opportunity for underground cavity detection that poses strong potential for acoustic wave spectrum application. The numerous significant advances have been attained to date, and more technological challenge awaits the optimization of an acoustic system to fill the gap between expectations and practical performance. Despite this tremendous progress, challenging issues related to the FDTD technique for acoustic wave simulation within the underground cavity detection yet to conclude. This review presents the development of the numerical approach in the acoustic wave simulation to excite pulse from the source, with particular emphasis being placed on the recent progress, a portion of travelling waves interact with the underground cavity structure and the cavity properties measurement of the cavity through different approaches. .

Keywords: FDTD, acoustic wave, cavity defect detection.

INTRODUCTION

Finite Difference Time Domain technique for solving acoustic wave problems is one of the fastest growing branches of simulation method for the underground detection technology. FDTD is a numerical analysis which acquires computational simulation for modelling, in this case acoustic wave. Acoustic waves is a longitudinal wave consist of particle velocity and sound pressure that have same direction of vibration when travels. The direction of wave propagation (eg. reflection and refraction) affected by propagation medium such as in the air, soil, concrete or cavities. Acoustic wave has received a lot of attention from both, industry and academic field. This is due to some beliefs that acoustic wave process may offer high analysis efficiency, simple operation, low cost and energy efficiency compared with the conventional underground detection processes in some other applications.

Most of the acoustic experts have been investigated standard finite difference that is able to exhibit good overview of both stability and accuracy methods for the past two decades (Bording and Lines, 1997; Carcione *et al.*, 2002). Presently, the accuracy, stability and grid dispersion of the finite difference have become the focus of the studies among researchers. In addition, they are aiming for reducing the computational cost and more accurate characterization data density on selective soil. A simple option is to introduce a more symmetric operator for the Laplacian in the scalar wave equation or its equivalent in the acoustic wave equation (Cole, 1994; Fomel and Claerbout, 1997). Even higher accuracy can be obtained with non-standard finite difference schemes (Mickens, 1984; Cole, 2000). Significant progress has been made in the isotropic and anisotropic elastic media of finite difference modelling (Bale, 2002; Manning and Magrave, 1999, 2002).

Those aspects are related to the acoustic wave parameters as well as have been well recognized as the important parameters influencing the modelling simulation for pressure and velocity. Recently geophysics engineer has recognized that the boundary condition in the acoustic wave simulation is another aspect, which is also playing an important role in determining the soil density characterization (Bording and Lines, 1997).

Since numerical modelling of acoustic wave is very complex, involving various sound dynamic condition and density parameters, it is therefore, FDTD has become one of popular numerical techniques to be used for the study of acoustic wave propagation. This article reviews some previous acoustic wave propagation studies for underground detections, especially in homogenous medium. Some progress in the development of our in-house acoustic FDTD tool in UTHM is discussed and some preliminary results are presented.

NUMERICAL EXPERIMENT

### Introduction to FDTD discrete model

FDTD method has been develop a wide variety of problems (Bottledoren, 1994). FDTD solves acoustical wave field in solid medium, by employing finite differential equations to approximate the derivatives of stress and the particle evaluation points, both temporally and spatially, (Teramoto and Tsuruta, 2003) as shown in Figure-1.

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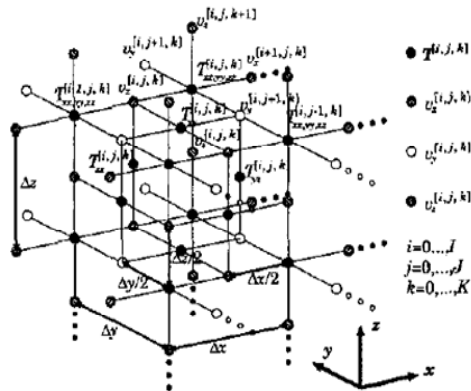
Figure-1. Geometry of nodes for stress and particle velocity is adopted in the FDTD simulation (Teramoto and Tsuruta, 2003).

The 2D standard acoutic wave FDTD method was derived empirically by (Geiger and Daley, 2003). It requires spatial discretization of the cell size lower than λ /10 to avoid dispersion and the curved boundaries that are modeled in a staircase way (Navarro *et al.*, 2008). Traditionally, proper used of space and time sampling is required (depending on applications) to avoid grid dispersion and numerical instability (Kelly and Marfurt, 1990).

Acoustic FDTD has been examined based on Navier-stokes equation to analyze frequency by considering the air viscosity (Kawai *et al.*, 2011). Furthermore the acoustic FDTD based on Navier Stokes equation can be used to analyze longitudinal force (Tanada and Kajikawa, 2014). Acoustic propagation can be described in terms of scalar pressure field and vector velocity based on Figure-2 by considering only the acoustic propagation without elastic media.
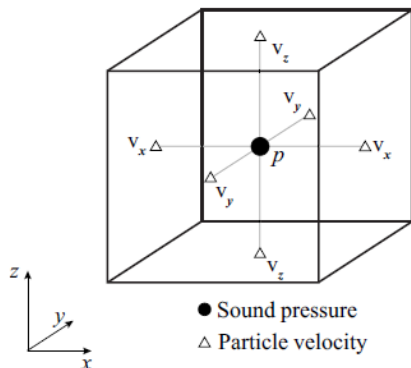
Figure-2. Acoustic FDTD methods based on the motion equation and Navier-Stokes equation (Tanada and Kajikawa, 2014).

Fundamental acoustic FDTD method based on motion equation and the equation of continuity is given by (Tanada and Kajikawa, 2014);

$$\frac{\partial \mathbf{V}}{\partial t} = -\frac{1}{\rho}\nabla p\mathbf{I}, \tag{1}$$

$$\frac{\partial p}{\partial t} + \kappa \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0, \tag{2}$$

where

$$\mathbf{V} = [v_x, v_y, v_z]^T,$$

where, t is the time, p is the sound pressure, u,v and w are particle velocities in the x,y, and z directions, p is the air density, k is the modulus of volume elasticity, and I is the unit matrix.

Navier-stokes is used to describe the motion of viscous subtances similar to the wave equations of the sound wave. Navier-stokes equation and equation of continuity are discretized by using central difference scheme where;

$$\frac{D\mathbf{V}}{Dt} = -\frac{1}{\rho}\nabla p\mathbf{I} + \nu\nabla^2\mathbf{V} + \frac{\nu}{3}\nabla^T(\nabla \cdot \mathbf{V}), \tag{3}$$

$$\frac{\partial p}{\partial x} + \kappa \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0, \tag{4}$$

where

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}, \tag{5}$$

There are many method to discretized the equation such as central difference scheme to solve the partial difference acoustic equation. Staggerd gridding method has been adopted in (Ehrlich, 2008) paper to discretized the temporal derivatives between the spatial step size.

Spatial derivatives

A spatial derivative is a measurement of a quantity of changing in space contrast with temporal derivative which would be a measure of how a quantity is changing in time. In this case, the scheme is using the staggered grid of Yee (Yee, 1966) with the evaluation points for pressure and velocity separated by half a step shown in Figure 3 by (Ehrlich, 2008).

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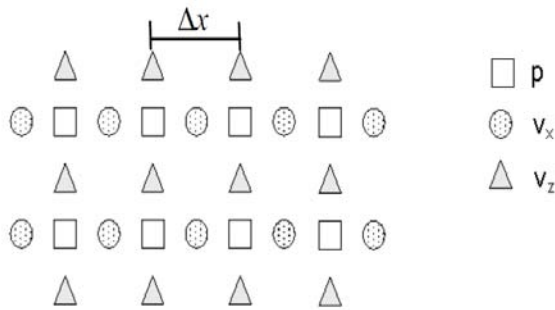
Figure-3. Calculation grid for 2 dimensions with grid
points for the P, Vox and V_z separated by half
the spatial step size (Ehrlich, 2008).

On a staggered grid the scalar variables (pressure, velocity) are stored in the cell centers of the control volumes, whereas the velocity or momentum variables are located at the cell faces. A staggered storage is mainly used on structured grids for compressible or incompressible flow simulations. Using a staggered grid is a simple way to avoid odd-even decoupling between the pressure and velocity (Harlow *et al*., 1965).

Absorbing boundary conditions

Absorbing Boundary Condition (ABC) is used to ensure no energy is transmitted or reflected from the boundary of the computation domain back into interior. For computational reason, it is necessary to limit the computational cost by using boundary condition in FDTD numerical modeling. According to (Nataf , 2013), to reduce undesired numerical reflections, there is a need for special treatment at these boundaries such as Absorbing Boundary Condition(ABC) and perfectly matched layer (PML) formulations. Therefore, in acoustic FDTD modeling, there is very essential boundary condition formulation need to be taken into account for numerical modeling instead of propagation of the wave itself.

ABC technique was introduced by (Engquist, 1977) while the PML technique was introduced by (Berenger, 1994). Each ABC approach has it own characteristic, advantages, and disadvantages. For example, as reported in Clayton and Engquist (1977), Higdon (1986) and Higdon (1987), there have an advantage of requiring relatively little memory but only work well within a limited range of angles incidence. PML method has some limitation due to the discretization and numerical reflections. Based on Johnson *et al.* (2002) and Oskooi *et al*. (2008), the boundary between the PML and the regular medium is no longer reflection less, but the reflection are small because the discretization is well approximated with the exact wave equation.

However, reflections supressed due to adiabatic theorem. With a non PML absorber, it might need a very

thick absorbing layer to get acceptable reflections. However, the PML solved the interface problem between the physical zone and the layers that led reflection. So designing of the PML is to distinguishes it from ordinary absorbing material so that waves incident upon the PML from a non-PML medium without reflecting back into the interior (Nataf, 2013). Based on (Schroeder, 1999), Figure 4 illustrates the two dimensional finite difference model with perfectly match layer (PML) as absorbing boundary condition. The model shows the normal source point is located at free surface and the space discretized by using staggered finite difference grid.
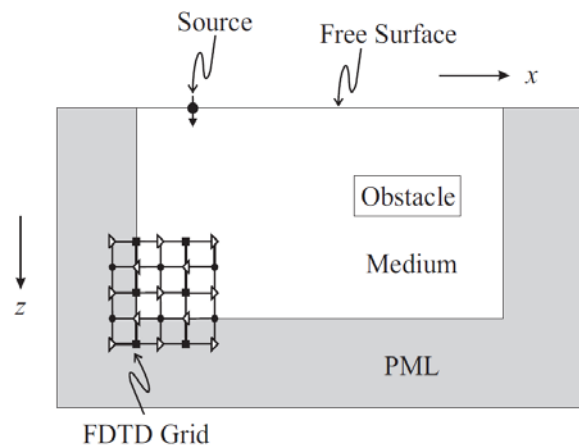
Figure-4. Two-Dimensional finite difference model
(Schroeder, 1999).

Acoustical source

The FDTD method requires discretization of time and space. These important factors will dictate the behaviour of the fields in a simulation such as the courant number and the points per wavelength for any given frequency (source). The source will be considered as the transmission coefficient for a planar interface from a FDTD simulation such as Gaussian pulse as used in Nesvijski, (2008), sinusoidal source in Ehrlich (2008), and impulse function source used in Edip *et al.* (2010). In order to calculate the spatial derivatives properly, it is essential that the source has to be modeled as a small area rather than a single point source. Figure 5 below shows the Gaussian pulse source with an absorbing boundary condition.
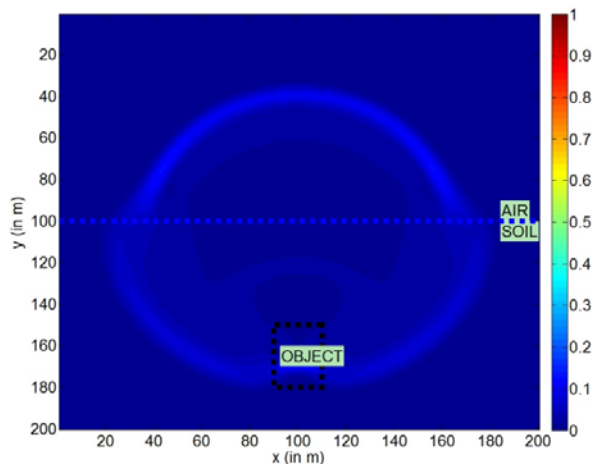
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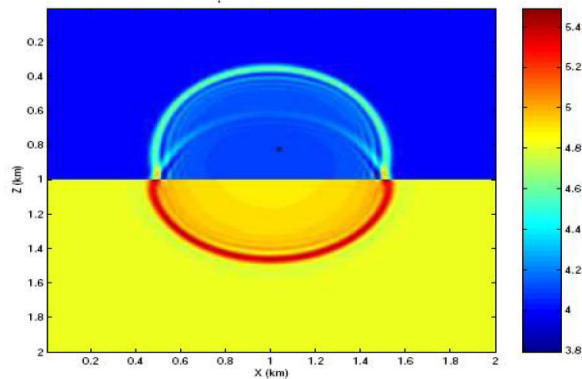
Figure-5. Snapshot of an acoustic simulation in a 2-layer medium. The star denotes the location of the point source (Nesvijski, 2008).
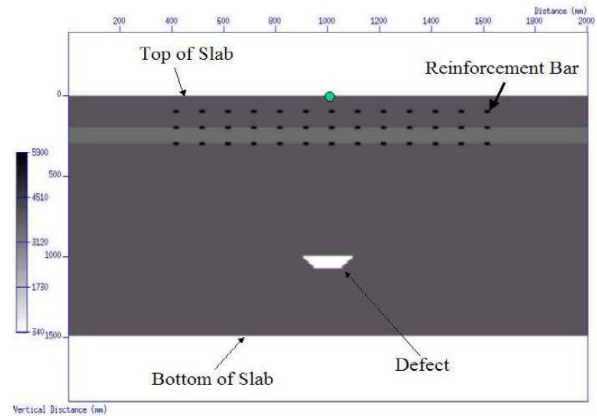

Figure-7. An acoustic velocity model used to numerically generate acoustic wave (Alkhalifah, 2005).

SIMULATION FOR DETECTION

Underground detection

Figure-6 illustrates the acoustic wave model for underground detection by using FDTD acoustic wave equation. The simulation tool is developed by EMCenter research group in UTHM. This model has well-defined thickness of void or cavity for underground detection with invisible defect supported on a homogenous half space layer or called as sub-layer. These were consistent with our expectations and appeared to be a very similar to those found in (Alkhalifah, 2005). The simulation was constructed cavity as an anomaly to record the reflected wave before reach the object and after the wave absorption inside the cavity. There is a model of acoustic velocities used to generate acoustic wave propagation using FDTD applied to the acoustic wave equation as Figure-7.

The effect of cavity and void

Acoustic wave propagation are far more sensitive to air-filled anomaly, which are the most sought in geotechnical (Alkhalifah, 2005). The effect of the void or cavities has been studied by Njihovih *et al.* (2004). Table-1 shows the parametric study for each of properties of the void or cavity. As a result, Figure 8 illustrated the wave reflected from the cavities boundaries where it is observed that the reflected waves have a relatively narrow frequency bandwidth. However, this effect actually give an advantage to underground void or cavity detection studied (Nasseri-Moghaddam, 2005; Chai *et al.*, 2014; Nasseri-Moghaddam *et al.*, 2007) in order to determine the location and embedment depth of void.

Table-1. Parametric study cases (Alkhalifah, 2005).

Case	$V_{s,obj}$	Distance	Width	Height	Depth
0	N/A (half space)	N/A	N/A	N/A	N/A
1	0 (cavity)	10 m	1 m	1 m	1 m
2	20 m/s	10 m	1 m	1 m	1 m
3	1000 m/s	10 m	1 m	1 m	1 m
4	2000 m/s	10 m	1 m	1 m	1 m
5	0 (cavity)	10 m	2 m	1 m	1 m
6	0 (cavity)	10 m	3 m	1 m	1 m
7	0 (cavity)	10 m	1 m	2 m	1 m
8	0 (cavity)	10 m	2 m	2 m	1 m
9	0 (cavity)	10 m	1 m	1 m	2 m
10	0 (cavity)	10 m	1 m	1 m	3 m
11	0 (cavity)	10 m	2 m	1 m	2 m
12	0 (cavity)	10 m	1 m	2 m	2 m
13	0 (cavity)	10 m	2 m	2 m	2 m

Figure-6. Snaphot of an acoustic simulation with Gaussian pulse. This simulation tool is developed in UTHM.

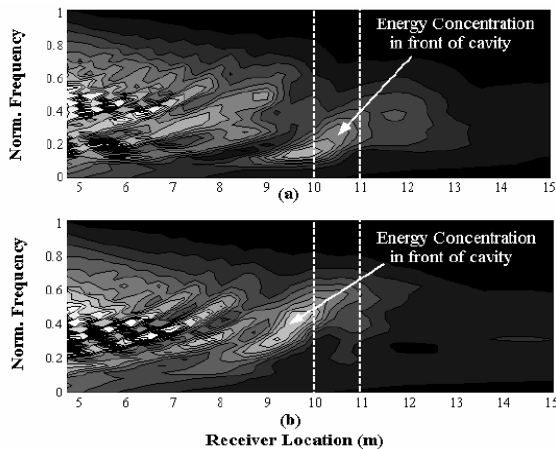ARPN Journal of Engineering and Applied Sciences

**Figure-8.** Power spectral for void model at (a) velocity=0 ms^{-1} and (b) velocity,Vs=2000 ms^{-1} (Njihovih *et al.*, 2004).

From Norville and Scott (2003), the structure that buried deeper underground have less visibility to higher frequency waves. It is much difficult to interpret the various anomalies because of the limitation of resolutions. Therefore selection of operating frequency must be done carefully for best results.

Effect of frequency response based on acoustic FDTD

Tanada and Kajikawam (2014) demonstrated the effectiveness of each of the acoustic FDTD method through the comparisons with measured characteristics. Figure 9 shows the measured characteristic of the no viscosity method and Navier-Stokes equation method.
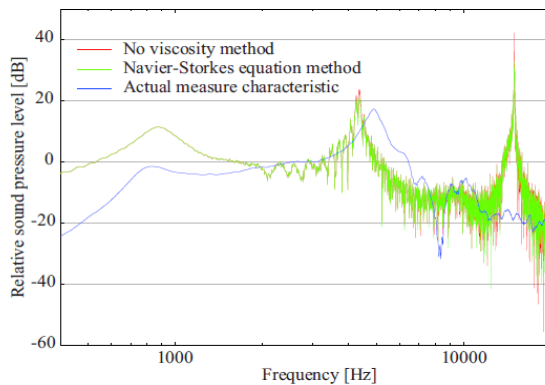

Figure-9. Comparison between measured response and simulated frequency response using no viscosity method and Navier-Stokes equation (Tanada and Kajikawam 2014).

Navier- Stokes method shows the low sensitivity happens at higher resonance frequency due to the air viscosity effect compared to the no viscosity method.

Some visible reflections may be due to imperfection in the boundary condition at the FDTD model. The acoustic source mode should be explored further with appropriate acoustic analysis method and boundary conditions. One of the recent approaches was by combining the FDTD method with Multi-Region as in Fu *et al.* (2013).

SUMMARY

Figure-10 summarized the numerical FDTD Acoustic Wave methods of conventional method based on motion equation and Navier-Stokes equation. In both methods the source is longitudinal wave (Pressure wave) and best applied in homogeneous medium. Navier-Stokes has an additional parameter of air viscosity which can be adapted on the fluid or liquid medium in sound system applications. Motion equation on the other hands is only good for solid medium applications. Another available approach is based on the elastic wave equation. One advantage of this technique is, it can be applied for heterogeneous medium. However in term of implementations, the technique is much complex, as both longitudinal (pressure wave) and transverse wave (stress wave) have to be used (Zhou *et al.*, 2007). Additionally the elastic equation requires additional parameter for each medium such as Young Modulus and the shear modulus. By minimizing the required parameters, it helps the FDTD method to numerically simulate the wave propagation without interfering with the computational cost factor. Therefore an improvement based on a sensitivity measurement by using motion equation second Newton Law will be proposed in our future work. An improvement can be made via the sensitivity measurement by understanding the reflection wave behaviour for detection of cavity in any conditions. Thus, the longitudinal wave excitation (Pressure Wave) gives a great potential for cavity detection on the homogeneous area.
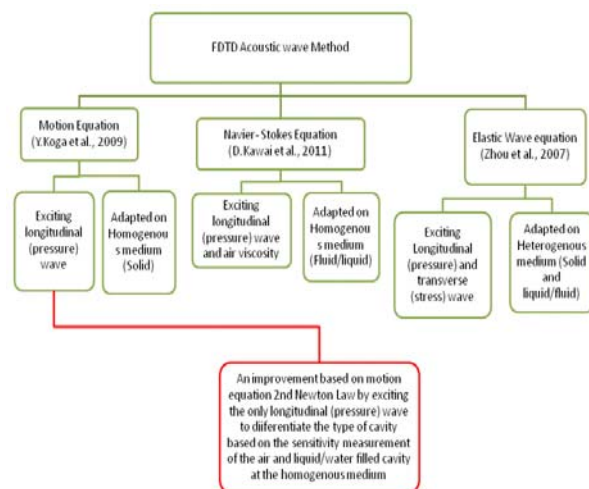
Figure-10. Summarizing the FDTD acoustic wave method.

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CONCLUSIONS

Numerical FDTD simulation for underground cavity detection is suitable for acoustic wave propagation in the homogeneous media. The FDTD model significantly influenced the propagation of the reflection waves of the acoustic wave modelling. In addition, from the review of cavity detection simulation, the pattern shows the underground cavity is strongly depend on the dimension and condition of the cavity i.e. air-filled or water-filled. These cavity properties are essential due to the energy absorption and reflection when the acoustic waves propagated through the cavity. In addition, other parameter influences the detection is the depth of cavity. Thus, further improvement by creating the sensitivity measurement based on motion equation is useful for cavity detection in homogenous medium.

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