COMPUTATION OF ELASTOMERS PROPERTIES USING FORTU-FEM CAD SYSTEM

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ABSTRACT

This paper presents developed by authors FORTU-FEM Computer Aided Design (CAD) system, which implements the moment scheme of the Finite Element Method (FEM) for computation of the properties of complex mechanical systems. Examples of application of FORTU-FEM CAD for calculation of the tensional and the deformed states of constructions from elastomers are considered.

Keywords: finite element method, moment scheme, CAD, FORTU-FEM, elastomers.

INTRODUCTION

The design of constructions, made from elastomer materials, is an actual problem of the stress and the strain states mechanics. In solid mechanics, there is big class of problems for which we cannot accept the hypothesis that elastomer materials are incompressible. To solve them, we need explore properties of elastomers; investigate the laws of deformation of the elements, typically made of rubber. Here, the compressibility of a material plays an important role. Traditional approaches, used in solid mechanics, do not take into account these features. Other problem is excluding a contact between firm elements, which have elastic layers of rubber, both for small and large deformations. Use of existing mathematical apparatus to solve these problems may lead to considerable difficulties. For example, analyses of movements of Finite Elements (FE) with Poisson's ratio about 0.5 will lead to significant computational errors. This is caused by the fact that existing CAD systems typically implement a scheme of computation using the traditional method of variational calculus - the Finite Element Method (FEM).

Let’s note that the modern CAD systems, developed for studying properties of materials being in the stress and the strain states, contain a number of methods for approximate computations. They support solution of large systems of algebraic and transcendental equations, numerical integration and differentiation, etc. Implementation of iterative and incremental computation schemes is associated with multiple repetitions of the basic operations. It needs specific knowledge of a user and considerable efforts to develop a method.

To overcome these problems it is necessary to allow the CAD users –specialists in a specific domain – to develop methods and define a schema of computation in the terms, which are close to the users’ domain of expertise. This task lies inside the general problem of development of Domain Specific Languages (DSL) for system engineering [1]. In our previous works, we propose the methodology for domain specific mathematical modelling [2], allowing development of DSLs for modelling domains, having different mathematical properties and structure [3].

In this paper, we consider a DSL FORTU, based on energetic variational principle [4]. It is used in FORTU-FEM CAD system, intended for the design of non-standard mechanical constructions, being in the tensional and the deformed conditions [5; 6].

The proposed FORTU-FEM system by the given minimum of input information and the large-scale scheme of computation, which corresponds to the minimization of energetic functional, interactively realizes the process of analysis and numerical solution of non-standard problems of firm bodies, being in the stress and the deformed states. Interactive flexibility of FORTU-FEM allows its users change both formulation of the problem and its computation scheme.

In this paper, we consider an additional module for FORTU-FEM, which implements the moment scheme of FEM. It allows empower FORTU-FEM and increase the accuracy of computations.

This paper is organized as follows. First, we analyse existing approaches and present the general structure and the principles of the CAD system FORTU-FEM. We give the inference of variation ratios, used in FORTU-FEM. Proving the concept samples of computation of properties of elastomers are considered. Conclusions finalize this paper.

LITERATURE REVIEW

The modern mechanics uses the methods of analysis, which should take into account different characteristics of spatial deformations. As an important numerical method for analysis in mechanical engineering,

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FEM becomes very popular among professionals, engaged in applied problems of mechanics of deformed solid bodies. Finite element method is widely used here for solving complex problems in construction of buildings, machinery, aircraft and rocket design and other areas. Although this method already used for a long time, many aspects are not yet sufficiently covered or still not well developed.

For example, there are considerable difficulties with traditional FEM in the form of the method of displacements, built on the basis of the Lagrangian variational principle for the solution of problems with singularities (such as taking into account weak compressibility, calculation using three-dimensional FE, and others) [7]. To overcome these problems other variational principles - Castigliano (method of forces), Hellinger-Reissner, Hu-Vashitsu (mixed method) are developed.

Note, that FEM in the form of the method of forces does not receive significant development because of the complexity in the approximation of the stress state. Greater use has the mixed FEM scheme.

With positive features [8], these methods have a number of drawbacks, as an increasing order of the system of solutions' equations (comparatively with the FEM in the form of displacement), violation of the positive definiteness of the matrix of equations. Therefore, the development of the hybrid schemes in the form of the FEM method of displacement based on the variational Lagrange principle is needed.

Standard FEM in the form of displacement requires that the field of displacements of points within a finite element is approximated by the polynomial functions and the contact of the elements at the borders is carried out under the conditions of continuity. However, this option of FEM has slow convergence due to polynomiality of functions.

In addition, approximating displacement fields do not include the term that describes the rigid displacement of FE. This effect is significant when using curvilinear FE, and taking into account the rigid displacements of FE should not only be seen as a necessary condition for convergence, but also as an important method for increasing efficiency of FEM in the calculation of curved bodies.

Use of the standard scheme in the form of the FEM method of displacement along with appearance of rigid movements of FE, shows another negative property called "false shear effect" [9]. At a bend of thin plates and shells, based on three-dimensional FE, the errors associated with fictitious shear strains are significantly increased.

To address these shortcomings the Moment Scheme of Finite Elements was developed [10], which allows us to take into account the basic properties of rigid displacement for curved FE of isotropic elastic bodies. Its idea is rejection of terms of the expansion of deformations, which are responsive for rigid displacements and for emerging fictitious shear deformations. In this regard, the exact equations of deformations and displacement are replaced with approximations.

Let us consider the use of moment scheme of the FEM for solving problems of elastomers in more details.

MATERIALS AND METHODS

Inference of variation ratios in static

The general formula for the strain energy of a system, based on Lagrange variational principle [4], can be written as

$$\Pi = \iiint_V |J| \left(\begin{pmatrix} \varepsilon_{11}(\frac{1}{2}D_{11}\varepsilon_{11} + D_{12}\varepsilon_{22} + D_{13}\varepsilon_{33}) + \\ \varepsilon_{22}(\frac{1}{2}D_{22}\varepsilon_{22} + D_{23}\varepsilon_{33}) + \varepsilon_{33}(\frac{1}{2}D_{33}\varepsilon_{33}) + \end{pmatrix} + \varepsilon_{12}(\frac{1}{2}D_{44}\varepsilon_{12}) + \varepsilon_{13}(\frac{1}{2}D_{55}\varepsilon_{13}) + \varepsilon_{23}(\frac{1}{2}D_{66}\varepsilon_{23}) \right) dV, \quad (1)$$

where D_{mn} are coefficients of the matrix of elastic moduli of a finite element

$$D_{m\times n} = \begin{pmatrix} 2\mu+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix} \quad (2)$$

Let us consider the moment scheme for the finite elements. Its main principle is a representation of the approximating function in a Taylor series, followed by discarding its n members. For a spatial rectangular hexagonal FE, this function provided in the form of the series will be as follows

$$u_{k'} = w_{k'}^{000} + w_{k'}^{100}\psi^{100} + w_{k'}^{010}\psi^{010} + w_{k'}^{001}\psi^{001} + w_{k'}^{110}\psi^{110} + w_{k'}^{101}\psi^{101} + w_{k'}^{011}\psi^{011} + w_{k'}^{111}\psi^{111} \quad (3)$$

where $w_{k'}^{pqr}$ - expansion coefficients, ψ^{pqr} - the set of power coordinate functions defined by the formula

$$\psi^{pqr} = \frac{x^p}{p!}\frac{y^q}{q!}\frac{z^r}{r!}$$

($p=0, 1; q=0, 1; r=0,1$) (4)

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The components of the strain tensor we expand in Maclaurin series in the neighborhood of the origin

$$\varepsilon_{ij} = \sum_{stg}^{ij} e^{(stg)} \psi^{(stg)} \tag{5}$$

or

$$\begin{aligned}
\varepsilon_{11} &= e_{11}^{00} + e_{11}^{001}\psi^{001} + e_{11}^{010}\psi^{010} + e_{11}^{011}\psi^{011};\\
\varepsilon_{22} &= e_{22}^{000} + e_{22}^{001}\psi^{001} + e_{22}^{100}\psi^{100} + e_{22}^{101}\psi^{101};\\
\varepsilon_{33} &= e_{33}^{000} + e_{33}^{010}\psi^{010} + e_{33}^{100}\psi^{100} + e_{33}^{110}\psi^{110}\\
\varepsilon_{12} &= e_{12}^{000} + e_{12}^{001}\psi^{001};\\
\varepsilon_{13} &= e_{13}^{000} + e_{13}^{010}\psi^{010};\\
\varepsilon_{23} &= e_{23}^{000} + e_{23}^{100}\psi^{100}.
\end{aligned} \tag{6}$$

Along with coefficients of the strain, the expansion of the strain components also contains the coefficients of rigid rotations. This fact causes the slow convergence of FEM. To discard it, we eliminate these members of the series.

After converting to a given FE, and taking into account the formulas (5), (6), (7), the strain tensors will have the following form

$$\begin{aligned}
\varepsilon_{11} &= \sum_{i=1}^{8} u_i b_{100}^{k'} (w_{k'}^{i2} + w_{k'}^{i5} y + w_{k'}^{i6} z + w_{k'}^{i8} yz);\\
\varepsilon_{22} &= \sum_{i=1}^{8} v_i b_{010}^{k'} (w_{k'}^{i3} + w_{k'}^{i5} x + w_{k'}^{i7} z + w_{k'}^{i8} xz);\\
\varepsilon_{33} &= \sum_{i=1}^{8} \omega_i b_{001}^{k'} (w_{k'}^{i4} + w_{k'}^{i6} x + w_{k'}^{i7} y + w_{k'}^{i8} xy);\\
\varepsilon_{12} &= \sum_{i=1}^{8} u_i b_{100}^{k'} (w_{k'}^{i3} + w_{k'}^{i7} z) + v_i b_{010}^{k'} (w_{k'}^{i2} + w_{k'}^{i6} z);\\
\varepsilon_{13} &= \sum_{i=1}^{8} u_i b_{100}^{k'} (w_{k'}^{i4} + w_{k'}^{i7} y) + \omega_i b_{001}^{k'} (w_{k'}^{i2} + w_{k'}^{i5} y);\\
\varepsilon_{23} &= \sum_{i=1}^{8} v_i b_{010}^{k'} (w_{k'}^{i4} + w_{k'}^{i6} x) + \omega_i b_{001}^{k'} (w_{k'}^{i3} + w_{k'}^{i5} x),
\end{aligned} \tag{7}$$

where u_i, v_i, ω_i - the displacement components of each node within FE; $w_{k'}^{ij}$ - the twiddle factors of nodes, $b_{(\mu\nu\eta)}^{k'}$ - coefficients, which relate the values of nodal displacement and power functions $\psi^{(pqr)}$. For a cubic FE, which has the unit metrics of measurement, and is represented in the natural system of coordinates, we can determine coefficients $b_{\mu\nu\eta}^{k'}$

$$\begin{aligned}
&b_{100}^{k'} = b_{010}^{k'} = b_{001}^{k'} = 2,\\
&b_{110}^{k'} = b_{101}^{k'} = b_{011}^{k'} = b_{111}^{k'} = 0.
\end{aligned} \tag{8}$$

Using (10) we obtain the formula for calculating the coefficients of the stiffness matrix [K_{ikjm}]

$$\begin{aligned}
&K_{i1j1} = \frac{\partial^2 \Pi}{\partial u_i \partial u_j},\ K_{i1j2} = \frac{\partial^2 \Pi}{\partial u_i \partial v_j},\ K_{i1j3} = \frac{\partial^2 \Pi}{\partial u_i \partial w_j};\\
&K_{i2j1} = \frac{\partial^2 \Pi}{\partial v_i \partial u_j},\ K_{i2j2} = \frac{\partial^2 \Pi}{\partial v_i \partial v_j},\ K_{i2j3} = \frac{\partial^2 \Pi}{\partial v_i \partial w_j};\\
&K_{i3j1} = \frac{\partial^2 \Pi}{\partial w_i \partial u_j},\ K_{i3j2} = \frac{\partial^2 \Pi}{\partial w_i \partial v_j},\ K_{i3j3} = \frac{\partial^2 \Pi}{\partial w_i \partial w_j}.
\end{aligned} \tag{9}$$

The design scheme in the FORTU-FEM CAD system

CAD FORTU-FEM was developed for the analysis of the tense and deformed states of complex engineering structures. The system allows the engineer to describe the design scheme to solve a problem in terms, close to his domain of expertise.

FORTU-FEM CAD system consists of three subsystems: the subsystem for preparation of input data for the subsequent computation (preprocessor), the subsystem for calculation of the data (processor) and the subsystem for analysis of the results and the generation of documentation (postprocessor).

Figure-1 shows the general scheme of the FORTU-FEM CAD system. The mathematical model allows users not only formally describe different problems of solid mechanics, but also automate the process of obtaining the equations for numerical solution of the problem. The accuracy of the calculation is depend on the user-selected type of the FE, the density of FEs' mesh, its structure and the chosen computation scheme.

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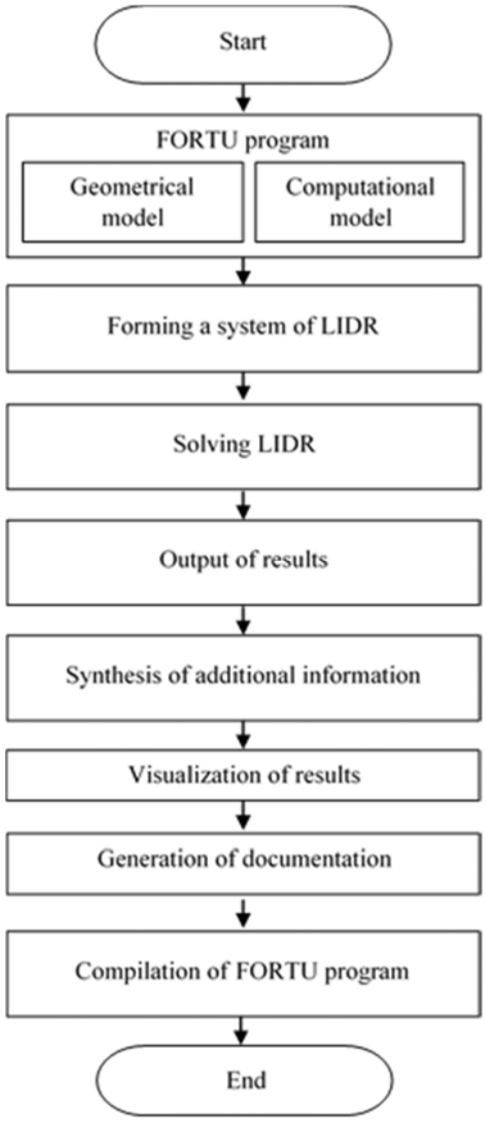

Figure-1. General scheme of a problem solution using FORTU-FEM.

Unlike many existing CAD systems that use FEM method, in FORTU-FEM we implement the moment scheme of the FEM. It allows us significantly empower design process and increase the accuracy of computations.

To prove this statement let us consider several examples.

Study of the elongation of a prismatic rod under own weight

To show possibilities of FORTU-FEM CAD system let us consider a simple problem from elasticity theory. A prismatic rod is rigidly fixed on the one side and is extended under own weight to the other side (Figure-2).

The input parameters are the following: the size of the rod is $0{,}01 \times 0{,}01$ m, its height $h = 0{,}02$ m, the elastic modulus is $E = 2$ MPa, the density of the material is $\rho = 1200\, kg/m^3$, the Poisson ratio is $\nu = 0{,}49$.

The analytical solution for this problem we may obtain from the formula $w = -\frac{\rho g}{2E}\left(h^2 - z^2\right)$, which expresses the shift of points along the axis Z. By this formula, the maximum elongation for such the construction is $w = 11{,}76 \cdot 10^{-7}\, m$.

Table-1 shows the results, made with FORTU-FEM CAD; computations are made for the case of irregular partitioning into finite elements. The best result $w = 12{,}00 \cdot 10^{-7}\, m$ is achieved by splitting a prismatic rod into 1258 FEs.

Figure-3 shows a graph, comparing results of computations of shifting nodes at optimal partitioning into FE located on the Z axis, by the FORTU-FEM CAD system and by using the analytical formula.

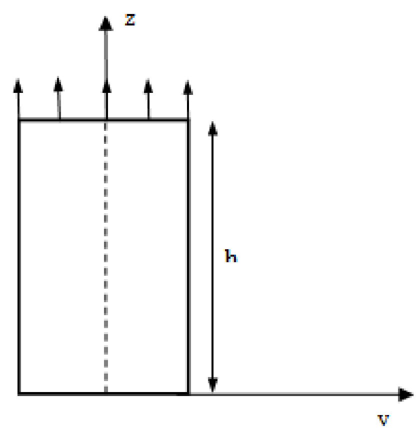

Figure-2. The schema of a strain of a prismatic rod under own weight.

Table-1. The results of the computations.

№	Number of FE	Number of nodes	w $\times 10^{-7}$ м
1	69	39	10,81
2	176	66	10,55
3	337	138	11,03
4	558	198	11,12
5	1258	371	12,00
6	4345	1239	12,51
7	9828	2247	12,93
8	11109	2618	12,97

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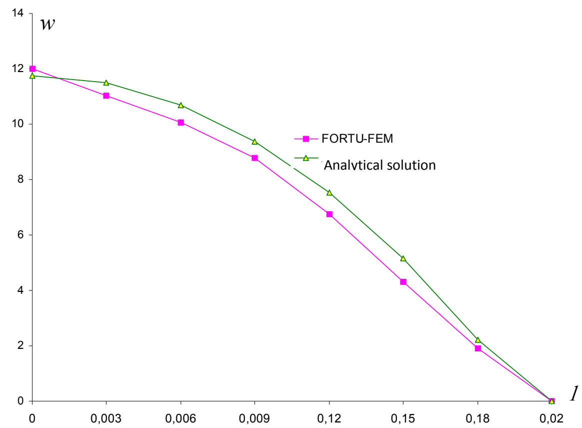

Figure-3. Comparison of the results of calculation of the prismatic rod with the of FORTU-FEM and the analytical solution.

This example shows good correspondence of the results, obtained by FORTU-FEM (by the moment scheme of FEM) and the analytical solution.

Modelling two-sided blade from heat-resistant rubber

The next method is often used in mechanics of elastomers for the definition of elastic properties of a rubber sample under tension. The essence of the method is as follows: the sample is fixed on both sides and stretched at a constant speed until the break occur. This allow us investigate a material strength, its elongation at the break and a stress at a given elongation.

Samples for testing are typically two-sided blades (see Figure-4 as example). They are cut with special knives from the vulcanized plates having thickness 2.0± 0.2. Since the investigated blade is symmetrical, then to calculate the deformed state only a half of the object can be used (see Figure-5). One end is fixed, and to the other a force is applied.

Using preprocessor of FORTU-FEM CAD system we built a regular finite element model of the blade, consisting of 1463 nodes and 4347 elements. The material used in this case is a heat resistant rubber, having following characteristics: Young's modulus $E = 6$ MPa, Poisson's ratio $\nu = 0{,}49$. A distributed load, applied to the unfastened end, is $q = 10{,}5$ MPa.

Figure-6 shows the distribution of displacement u along the sample. The relative value of elongation, computed with FORTU-FEM system, is 100%. This result corresponds to the technical documentation [11]. Table-2 shows the results of calculations of various physical and mechanical properties of the sample.

Table-2. Results of calculations of physical and mechanical properties of a rubber sample.

Type of rubber	Distributed load q, MPa	Young's modulus E, MPa	Elongation, in %, not less	Elongation, obtained by FORTU- FEM, in %
1	2	3	4	5
Acid and alkali resistant	5,6	2,75	100	105
Frost resistant	5,75	3,25	100	98
Oil and petrol resistant, brand A	11	4,75	150	143
Oil and petrol resistant, brand B	7	3,06	150	152
Oil and petrol resistant, brand C	6,85	3,00	150	148

The Poisson's ratio of the samples is $\nu = 0{,}49$. The column 4 shows values, taken from the technical documentation for the given type of the rubber. Column 5 gives results, obtained by FORTU-FEM. Their comparison shows that FORTU-FEM represent good results for the computation of properties of elastomers, being in the deformed states.

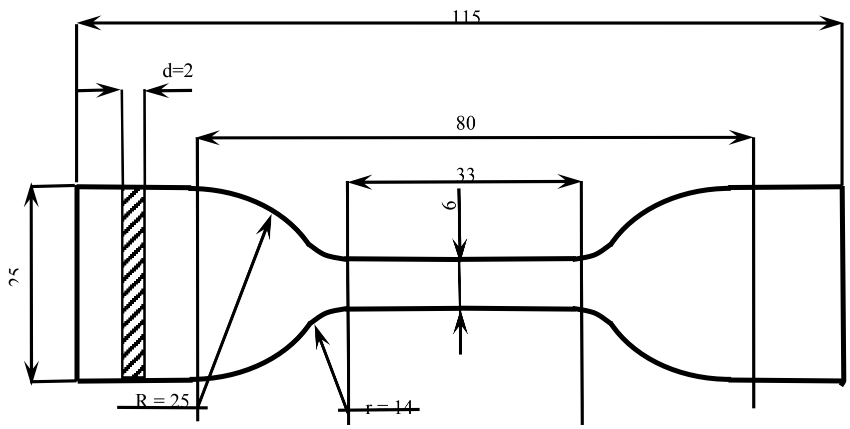

Figure-4. Test sample in the form of two-sided blade.

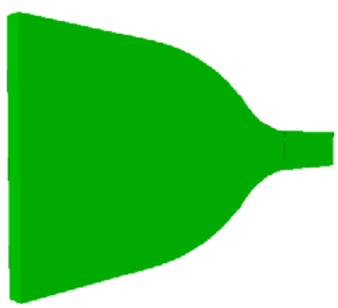

Figure-5. Geometrical model of the blade.

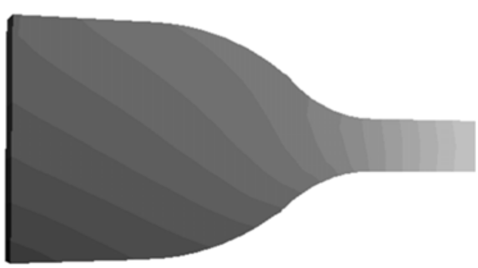

Figure-6. The distribution of displacements in the blade, made from heat-resistant rubber.

CONCLUSIONS

Methods for solving various problems of solid mechanics by finite element method show several limitations. A traditional version of the FEM often has slow convergence, especially for massive bodies and shells, made of complex curved shapes.

This paper shows results, obtained with FORTU-FEM CAD system, which implements the moment scheme of the FEM. Parameters of deformed states of the samples, made from various brands of rubber are calculated. Results allow us to draw the conclusion about good convergence of the implemented method.

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