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TRACKING WITH MULTIRATE OUTPUT FEEDBACK (MROF) BASED DISCRETE SLIDING MODE CONTROL

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ABSTRACT

In this paper, a discrete sliding mode controller with multirate output feedback is designed to control the inverted pendulum system at the upright position. Most of the SMC control strategies are based on state feedback, however not all of the state feedbacks are available. The multirate output feedback (MROF) used output feedback, therefore the state are always available at any condition. The error state variable was added to the system to achieve reference tracking. The MROF was compared with discrete Proportional Integral Derivative **(**PID) and discrete Linear Quadratic Regulator (LQR).

Keywords: multirate output feedback, discrete sliding mode control, inverted pendulum.

INTRODUCTION

The problem of controlling uncertain system such as modeling error and external disturbance become an interest topic to control researcher. Among the controllers, Sliding Mode Control (SMC) is well known as a robust controller that can deal with uncertainties. The concept of achieving desired dynamics by changing the controller structure had been proposed by (Utkin, 1977) and (Decarlo, 1988). Since then, SMC become popular and is used as a controller for much application. In order to design SMC, a stable sliding surface is selected and then a control law is designed which drives the system states to reach and stay in that sliding surface in finite time. In recent years the trend of controller has changed from continuous to discrete as well as sliding mode contol. In continuous SMC, the structure switching may be made at any time once the space trajectory across the sliding surface. However in discrete sliding mode control (DSMC) the control signal are performed only after regular intervals of time and produce a zigzag motion called quasi sliding mode band (Weibing and Yufu, 1995) due to finite sampling frequency.

The DSMC have many advantages compared to continuous SMC such as low power consumption, low cost, high accuracy and the others (Wang, 2008). Same as continuous SMC, the DSMC can be designed based on reaching law (Yan *et al.*, 2006), (Jung *et al.*, 2000) or equivalent control law approach (Dong *et al.*, 2007). By designing the controller with the equivalent control law, the selection of sliding surface will gives different performance (Nai *et al.*, 2006) , (Abdennabi *et al.*, 2012) and (Khalid *et al*., 2007).

The state feedbacks are normally used to design the SMC. However not all of the state always available especially during experiment. In this case, the Multirate output feedback (MROF) method that only used the output samples to design the controller can be considered. In MROF, the control input and output of a system are sampling at different sampling rates. The output is sampled at a faster rate; meanwhile the control input is updated at a slower rate (Bandyopadhyay and Janardhanan, 2006). The MROF also able to control the system even exists uncertainties either that are match or unmatched uncertainties (Janardhanan and Vinay, 2008), (Janardhanan and Satyanarayana, 2012), (Takao, S., Masanori, N. and Akira, I. 2009). The nonlinear plant system such as Hydraulic Manipulator (Waterman and Nanomi, 2009), Magnetic Levitation System (Vitthal and Pratik, 2010) and Inverted Pendulum System (Premand Siva, 2014) was controlled by MROF. Normally the designed MROF are without reference tracking. In this paper the reference tracking will be considered by adding the error state variable to the system.

Inverted pendulum system

Inverted pendulum system consists of a freely pivoted rod, mounted on a motor driven cart. The aim of designing the controller for this system is to maintain the pendulum at the up position by controlling the movement of the cart. The inverted pendulum is known as a nonlinear model, however by considering the small variations of angle about the equilibrium point when the pendulum is at upright position, the nonlinear plant can be transform into a linear plant. The linear plant of inverted pendulum system can be represented in state space as

$$\dot{X}(t) = AX(t) + Bu(t)$$
$$y(t) = CX(t) \tag{1}$$

where

$$A=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{(M_pl_p)^2g}{P} & -\frac{B_{eq}(I_p+M_pl_p^2)}{P} & -\frac{M_pl_gB_p}{P}\\ 0 & \frac{(M_c+M_p)M_pgl_p}{P} & -\frac{M_pl_pB_{eq}}{P} & -\frac{(M_c+M_p)B_p}{P}\end{bmatrix}$$

$$B=\begin{bmatrix}0\\ 0\\ \frac{(I_p+M_pl_p^2)}{P}\\ \frac{M_pl_p}{P}\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

$$X(t)=\begin{bmatrix}x & \theta & \dot{x} & \dot{\theta}\end{bmatrix}$$

$$P=(M_c+M_p)I_p+M_cM_pl_p^2 \tag{2}$$

x = cart position,
θ = pendulum angle,
$\dot{x}$ = cart velocity,
$\dot{\theta}$ = pendulum angle velocity,
M_p = pendulum mass,
M_c = Cart mass,
I_p = Pendulum moment of inertia,
l_p = Pendulum length,
B_p = Viscous damping coefficient,
B_{eq} = Equivalent viscous damping coefficient,
g = Gravitational constant on earth,,

The external disturbance, $d(t)$ will be added to the system to create the match uncertainties. Then, the inverted pendulum system from equation (1) can be expressed as

$$\begin{aligned}\dot{x}(t)&=Ax(t)+B(u(t)+d(t))\\ &=Ax(t)+Bu(t)+Bd(t)\end{aligned} \tag{3}$$

where $x(t)\in R^n$ is the state vector, $u(t)\in R$ is the control input, A, B and C are known matrices and n is also known.

CONTROLLER DESIGN

Multirate output feedback

The system in equation (3) also can be rewrite as below to separate the disturbance term

$$\dot{x}(t)=Ax(t)+Bu(t)+Dd(t) \tag{4}$$

To achieve reference tracking, the state variable z which is defined as $\dot{z}(t)=r(t)-y(t)$ is added to the system. r is the reference to be tracked. Using equation (4) and consider the tracking, the system can be represented as

$$\begin{aligned}\dot{x}_p(t)&=A_px_p(t)+B_pu(t)+D_pd(t)+R_pr(t)\\ y(t)&=C_p(t)\end{aligned} \tag{5}$$

where

$$x_p=\begin{bmatrix}z\\ x\end{bmatrix},\ A_p=\begin{bmatrix}0 & -C_1\\ 0_{n\times1} & A\end{bmatrix},\ B_p=\begin{bmatrix}0\\ B\end{bmatrix}$$
$$D_p=\begin{bmatrix}0\\ D\end{bmatrix},\ H_p=\begin{bmatrix}1\\ 0_{n\times1}\end{bmatrix},\ C_s=\begin{bmatrix}1 & 0_{1\times n}\\ 0 & C\end{bmatrix} \tag{6}$$

Figure-1 shows the diagram of the multirate output feedback with reference tracking. In MROF the output is sampled at a faster rate at η second and the control input is computed at a slower rate of $\tau=\eta N$, where N is an integer. The N is chosen to be greater or equal to observability index of (Φ,C). The constant control signal $u(k)$ is applied over a period during interval $k\tau\leq t(k+1)<\tau$ .The main input for the controller are based on output feedback, y_k, past value of input control $u(k-1)$ and not the state of the system.

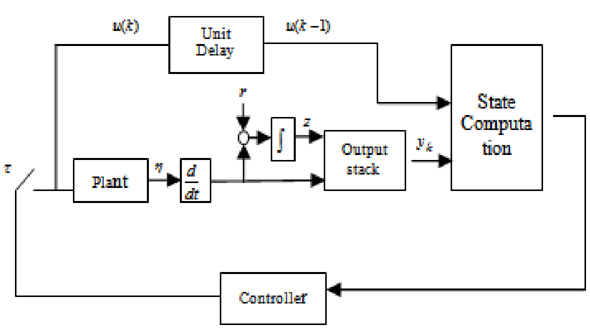

Figure-1. Diagram of multirate output feedback.

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The system in equation (5) is sampled at a slow rate as 0.004 second. The discrete time of the system is given as

$$x(k+1) = \Phi\, x(k) + \Gamma u(k) + Ff(k) + Hr(k)$$
$$y(k) = C_p x(k) \tag{7}$$

where

$$\Phi = e^{A\tau} \text{ and } \Gamma = \int_0^T e^{A\tau} d\tau B$$

$$f(k) = \int_0^T e^{A\tau} B((k+1)T - \tau) d\tau \tag{8}$$

where $u(k) \in R$, $\Phi(k) \in R^{n\times n}$ and $\Gamma(k) \in R^{n\times 1}$, assume that the (Φ,Γ) is controllable and the pair (Φ, C) is observable. The past N multirate-sampled system outputs can be represented as

$$y_k = \begin{bmatrix} y(k-1)\tau \\ y(k-1)\tau+\eta \\ y(k-1)\tau+2\eta \\ \vdots \\ y(k\tau-\eta) \end{bmatrix} \tag{9}$$

Then, system in equation (7) can be represented in faster sampling time, η as

$$x(k+1) = \Phi x_\eta(k) + \Gamma_\eta u(k) + F_\eta f(k) + H_\eta r(k)$$
$$y_{k+1} = C_0 x(k) + C_u u(k) + C_f f(k) + C_r r(k) \tag{10}$$

$$C_0 = \begin{bmatrix} C_p \\ C_p\Phi_\eta \\ C_p{\Phi_\eta}^2 \\ \vdots \\ C_p{\Phi_\eta}^{N-1} \end{bmatrix} C_u = \begin{bmatrix} 0 \\ C_p\Gamma_\eta \\ C_p(\Phi_\eta\Gamma_\eta + \Gamma_\eta) \\ \vdots \\ C_p\sum_{i=0}^{N-2}{\Phi_\eta}^i\Gamma_\eta \end{bmatrix} \tag{11}$$

where C_0, C_u, C_f and C_r can be expressed as

$$C_f = \begin{bmatrix} 0 \\ C_pF_\eta \\ C_p(\Phi_\eta F_\eta + F_\eta) \\ \vdots \\ C_p\sum_{i=0}^{N-2}{\Phi_\eta}^i F_\eta \end{bmatrix} C_r = \begin{bmatrix} 0 \\ C_pH_\eta \\ C_p(\Phi_\eta H_\eta + H_\eta) \\ \vdots \\ C_p\sum_{i=0}^{N-2}{\Phi_\eta}^i H_\eta \end{bmatrix} \tag{12}$$

Based on equation (7) and (10), the state $x(k+1)$ can be represented in term of $y_{k+1}, u(k)$ and $f(k)$ as

$$x(k+1) = L_y y_{k+1} + L_u u(k) + L_f f(k) + +L_r r(k) \tag{13}$$

where

$$L_y = \Phi\left(C_0^T C_0\right)^{-1} C_0^T$$
$$L_u = \Gamma - \Phi\left(C_0^T C_0\right)^{-1} C_0^T C_u$$
$$L_f = F - \Phi\left(C_0^T C_0\right)^{-1} C_0^T C_f \tag{14}$$
$$L_r = H - \Phi\left(C_0^T C_0\right)^{-1} C_0^T C_r$$

The state $x(k)$ can be expressed using the output y_k as

$$x(k) = L_y y_k + L_u u(k-1) + L_f f(k-1) + +L_r r(k-1) \tag{15}$$

There are several types of sliding surface in SMC such as linear sliding surface, nonlinear sliding surface and integral sliding surfcace. Different sliding surface will give different tracking result. In this paper, the linear sliding surface will be considered as

$$\sigma(k) = Gx(k) \tag{16}$$

σ = sliding surface,
G = switching surface,

The $\sigma(k+1)$ can be expressed as

$$\sigma(k+1) = G\Phi\, x(k) + G\Gamma u(k) + GFf(k) + GHr(k) \tag{17}$$

The controller can be define as

$$u(k) = -(G\Gamma)^{-1}\left[G\Phi x(k) + GFf(k) + GH(k)\ \right] + M\left|\sigma(k)\right|\operatorname{sgn}(\sigma(k))\} \tag{18}$$

Since the value of $f(k)$ is unknown, then the $f(k)$ will be replaced by estimated disturbance $\tilde{f}(k) = f(k-1)$.

$$u(k) = -(G\Gamma)^{-1}\left[G\Phi x(k) + GF\,\tilde{f}(k) + GHr(k)\ \right] + M\left|\sigma(k)\right|\operatorname{sgn}(\sigma(k))\} \tag{19}$$

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Then the controller can be expressed in multirate output feedback as

$$u(k) = -(G\Gamma)^{-1}\begin{bmatrix} G\Phi(L_y y_k + L_u u(k-1) + L_f f(k-1)) \\ + L_r r(k-1) + GF\tilde{f}(k) + GHr(k) \end{bmatrix} \quad (20)$$
$$+ M|\sigma(k)|\operatorname{sgn}(\sigma(k))\}$$

Stability of the system

To guarantee the stability of the discrete time sliding mode control, the condition in equation below must be satisfied.

$$[\sigma(k+1) - \sigma(k)]\operatorname{sgn}(\sigma(k)) < 0 \quad (21)$$

The equation (21) was decomposed into two inequalities, which are

$$[\sigma(k+1) - \sigma(k)]\operatorname{sgn}(\sigma(k)) < 0 \quad (22)$$

$$[\sigma(k+1) + \sigma(k)]\operatorname{sgn}(\sigma(k)) > 0 \quad (23)$$

Using equation (22)

$$[\sigma(k+1) - \sigma(k)]\operatorname{sgn}(\sigma(k)) < 0$$
$$= [G\Phi x(k) + G\Gamma u(k) + GFf(k) + GHr(k) - \sigma(k)]\operatorname{sgn}(\sigma(k)$$

$$= \begin{bmatrix} G\Phi[L_y y_k + L_u u(k-1) + L_f f(k-1) + + L_r r(k-1] \\ + G\Gamma\{(-G\Gamma)^{-1}[G\Phi(L_y y_k + L_u u(k-1) + L_f f(k-1)) \\ + L_r r(k-1) + GF\tilde{f}(k) + GH_r(k)] \\ + M|\sigma(k)|\operatorname{sgn}(\sigma(k)\} + GFf(k) + GHr(k) - \sigma(k) \end{bmatrix}\operatorname{sgn}(\sigma(k))$$
$$= -|\sigma(k)|M < 0 \quad (24)$$

Based on equation (23)

$$[\sigma(k+1) - \sigma(k)]\operatorname{sgn}(\sigma(k)) < 0$$
$$= |\sigma(k)|(2-M) < 0 \quad (25)$$

In order to make sure that sliding condition in equation (24) and equation (25) is achieved and the system is stable, the range of M must be between $0 < M < 2$.

Simulation results

The inverted pendulum system that discretised with $\tau = 0.004$ second as in equation (5).

$$\Phi = \begin{bmatrix} 1 & -0.004 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.0039 & 0 \\ 0 & 0 & 1.004 & -0.0040 & 0.004 \\ 0 & 0 & 0.0034 & 0.9534 & 0 \\ 0 & 0 & 0.1845 & -0.2016 & 0.9984 \end{bmatrix}.$$

$$\Gamma = \begin{bmatrix} 0 \\ 0 \\ 0.0001 \\ 0.0061 \\ 0.0265 \end{bmatrix} \quad F = \begin{bmatrix} 0 \\ 0 \\ 0.0001 \\ 0.0061 \\ 0.0265 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.004 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (26)$$

The value of N is chosen as 4 because of the observability index of the system is 4. The external disturbance is set to

$d(k) = 0.05\cos(0.1\pi k \tau)$.

The parameters of L_u, L_r and L_f are

$$L_u = \begin{bmatrix} 0 \\ 0 \\ 0.0001 \\ 0.0046 \\ 0.0201 \end{bmatrix}, \quad L_r = \begin{bmatrix} 0.0025 \\ 0 \\ 0 \\ -0.0005 \\ 0.0001 \end{bmatrix}, \quad L_f = \begin{bmatrix} 0 \\ 0 \\ 0.0001 \\ 0.0046 \\ 0.0201 \end{bmatrix},$$

The MROF has been simulated to demonstrate the capability to track the position of cart system and maintain the inverted pendulum in upright position. In the simulation, after 1 second the system is force to move 0.02m from its initial condition by following the reference input which is the unit step input. The switching surface is designed as $G = [-15.3 \quad -86 \quad 125 \quad -51.4564 \quad 20]$. Then the performance of the DSMC is compared with the discrete linear quadratic regulator (DLQR).

Thus, the designs gained for the DLQR controller are $K = [-42.1696 \quad 125.9181 \quad -40.3888 \quad 13.5309]$. Figure-2 shows the simulation result for the cart position with MROF and DLQR. Although all of the controllers are capable to control the cart position, but the output of each controller gave the different characteristic. The output of these three controllers can be referred as in Table-1.

Among all the data the MROF gives the faster settling time and lower overshoot as compared to DLQR controller.

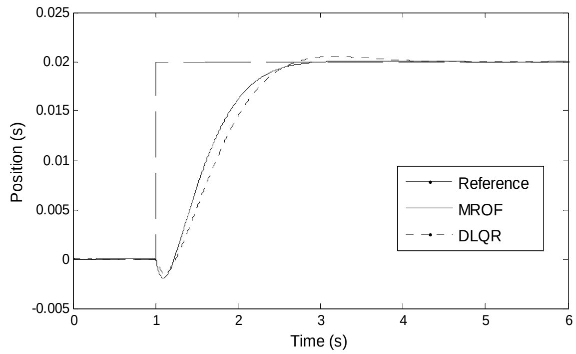

Figure-2. Tracking performance of cart position.

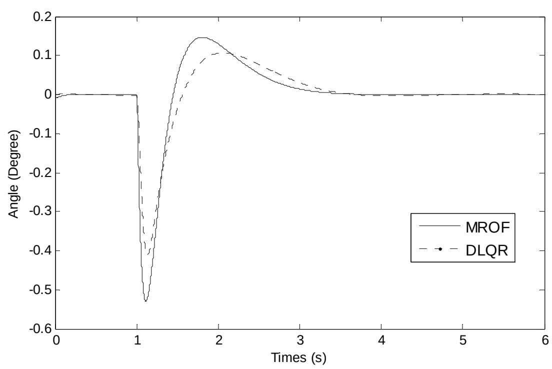

Figure-3. Angle of inverted pendulum system.

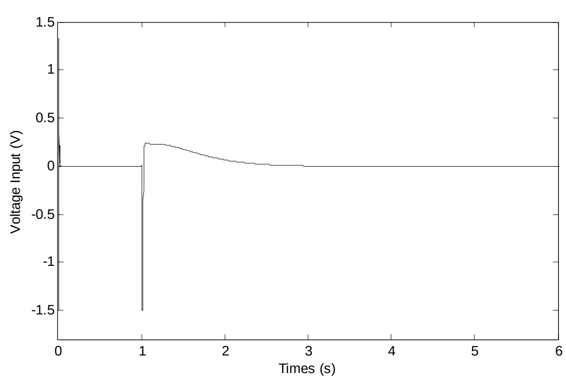

Figure-4. Input voltage.

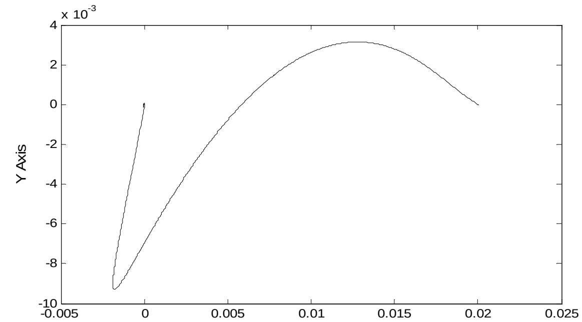

Figure-5. Phase plot.

The angle of inverted pendulum is shown in Figure-3 where the MROF give the faster response compared to the PID controller. Meanwhile Figure-4 shows the control input which is the input voltage of the system. Figure-5 shows the XY plot of the system.

Table-1. Performance characteristics of cart position with different types of controller.

Characteristic	MROF	PID
Rise time (s)	0.96	0.97
Settling Time (s)	1.95	2.59
Percentage of overshoot (%)	0	2.5

CONCLUSIONS

In this paper, MROF based DSMC is considered to control inverted system. The MROF is designed based on the existence of external disturbance in the sytem and the stability condition during sliding. Only by the usage of output state, the position of cart motor can be tracked and not depend on the state feedback. Based on the comparison of performance characteristic, the MROF provided better result compared to DLQR controller.

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