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# THE EFFECT OF PIEZOELECTRIC ACTUATION ON STRESS DISTRIBUTION IN ALUMINUM PLATE WITH CIRCULAR HOLE

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**ABSTRACT**

Structures with holes have been widely used in engineering designs. However, these structures have the tendency to develop critical crack growth because of stress concentration. This paper investigates the stress concentration factor (SCF) of an aluminum plate with circular hole and adhesively bonded piezoelectric actuator above the hole. The plate was subjected to uniaxial remote tensile stress while the actuator was excited with different voltages. The effect of the piezoelectric excitation on the stress in the host plate was analyzed using the finite element method (FEM). The stress distribution was evaluated by the stress concentration factor as an effectiveness criterion. The results indicated that the SCF is strongly affected by the applied voltage. Negative voltage tends to increase the SCF while the positive voltage tends to reduce the SCF. However, these effects are also dependent on the hole diameter and the size of the actuator.

**Keywords:** stress analysis, SCF, FEM, piezoelectric actuator.

## INTRODUCTION

Research activities in the area of smart structures incorporating active devices such as piezoelectric transducers and actuators are getting attention in the present time. These smart structures are widely used in various industrial and commercial fields which include aircrafts, spaceships and satellites [1,2] Piezoelectric ceramics have been widely used as sensors and actuators for their intrinsic multi-physics fields coupling behavior. This coupling trait is due to the effect of interaction between electrical and mechanical properties of the given material.

Some of the main reasons to use piezoelectric materials are their fast electro-mechanical response, relatively low power requirements and high generative forces. When an electric field is applied to a piezoelectric material, it deforms mechanically that is mostly seen as a change in dimension of the material. The direction of the dimensional change can be inverted by reversing the applied electric field.

Generally, it is known that the integrated piezoelectric actuator will generate compressive or tensile stresses in the host structure when excited. For structures with holes, the applied electric field to the actuator will either increase or decrease the SCF depending on the mode of actuation. Many of the previous studies investigated the relationship between the piezoelectric material and actuation parameters and the stress on the host plate [1,2]. However, these studies do not include the effects of holes or any pre-existing intermittence in the host structure and consequently do not have the capacity to foresee the singular stresses in these regions.

Stress concentration factor is one of the important design parameter for structural engineers, especially when dealing with members that exhibit geometrical change and subjected to external loads. The stress concentration factor is defined as the ratio between the peak stress at the root of the notch and the nominal stress which would be present if stress concentration did not occur [3]. When high SCF is left unattended at any point within the structure, cracks will initiate and propagate, ultimately resulting in catastrophic failure. Stress concentration in the vicinity of geometrical discontinuities can be reduced by smoothening the stress flow lines around these positions.

In recent years, the problem of piezoelectric plate containing defects has drawn intensive attentions [4-13]. Nevertheless, many of the previous studies are limited to investigating defects in the piezoelectric material itself and only few research works investigated the integrated structure with discontinuity. Recently, the use of piezoelectric material as an integral part in structural components has been found to affect the properties of host structure. Shah et al [14] investigated the effect of piezoelectric patches around a hole in an isotropic plate. Sekine [15], studied the repair of cracks in aircraft panels through piezoelectric patches. Liu [16, 17] presented work on the repairing process on a cracked beam using piezoelectric actuator based on linear elastic fracture mechanics. However, to the best knowledge of the authors there is no numerical investigation on the stress-concentration factor in isotropic plate with circular hole and adhesively bonded piezoelectric actuator above the hole.

In this paper the effects of the piezoelectric actuators on the stress concentration factor of a rectangular plate containing circular holes were studied. The piezoelectric actuator was placed above the circular hole and adhesively bonded with the help of epoxy. The key idea is to investigate the effects of the compression and tension stress generated by the piezoelectric actuators in the vicinity of the hole. Finite element analysis (FEA) was used to obtain the SCF under static uniaxial tension.

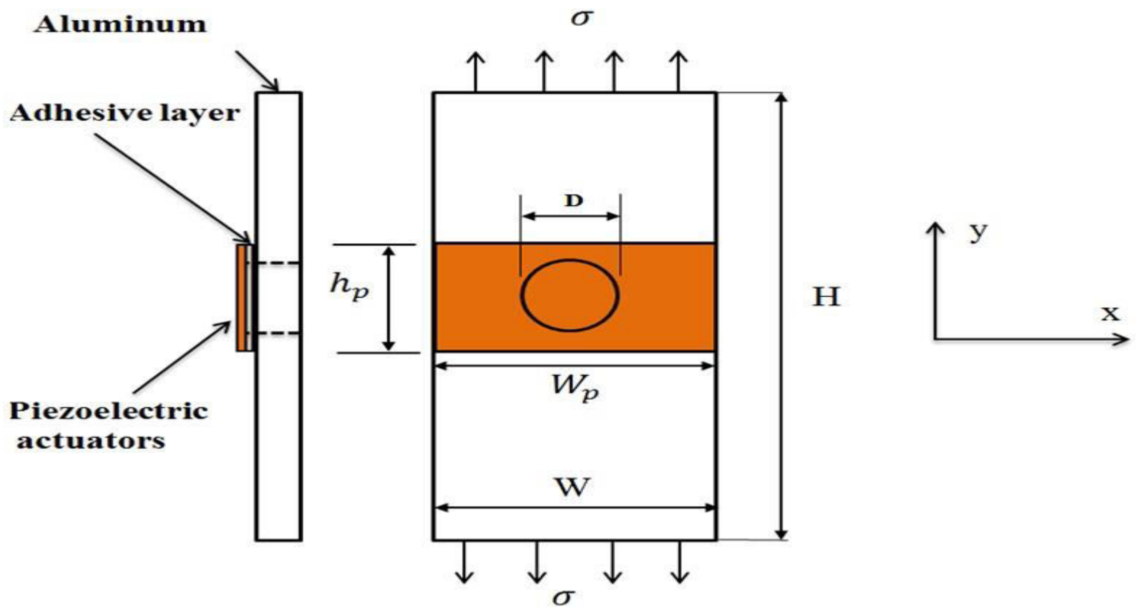


**Figure-1.** Rectangular plate with circular holes and adhesively bonded piezoelectric actuator.

The analysis was carried out using ANSYS to determine the plate's stresses that was later used to calculate the SCF.

## FORMULATION OF THE PROBLEM

The rectangular plate containing central circular hole bonded with piezoelectric patch loaded in a uniaxial remote tension and applied electric field is shown in Figure-1. The piezoelectric actuator was placed above the hole of the host plate. In this work, the stress concentration factor was calculated using the nominal stress at the minimum section, when the uniform tension is applied to the plate. The theoretical stress concentration factor $K_t$ without the piezoelectric patch is [3, 18]**:**

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \tag{1}$$

where $\sigma_{max}$ and $\sigma_{nom}$ are maximum and nominal stress respectively. The idea is to investigate the singularity of the stress in the vicinity of the hole. For this aim the SCF will be determined with positive and negative actuation voltage.

**Table-1.** Materials properties of aluminium, PIC151 and adhesive.

| Properties Parameter | symbols | Host structure | PI 151 patch | Adhesive |
|---|---|---|---|---|
| Density | ρ | 2715 $kg/m^3$ | 7800 $kg/m^3$ | 1000 $kg/m^3$ |
| Poisson ratio | $v$ | 0.33 | 0.34 | 0.3 |
| Young's modulus | E | 68.0 e9 $N/m^2$ | | 5.09 e9 $N/m^2$ |
| Compliance matrix | $S_t$ | | $S_{11}$ 19.0 $\times 10^{-12}\, m^2/N$ | |
| | | | $S_{33}$ 15.0 $\times 10^{-12}\, m^2/N$ | |
| Electric permittivity | $\varepsilon^T$ | | $\varepsilon_{11}^T$ 1977 | |
| coefficient | | | $\varepsilon_{33}^T$ 2395 | |
| Piezoelectric strain | $d$ | | $d_{31}$ − 2.10 $\times 10^{-10}\, m/V$ | |
| coefficient | | | $d_{32}$ 2.10 $\times 10^{-10}\, m/V$ | |

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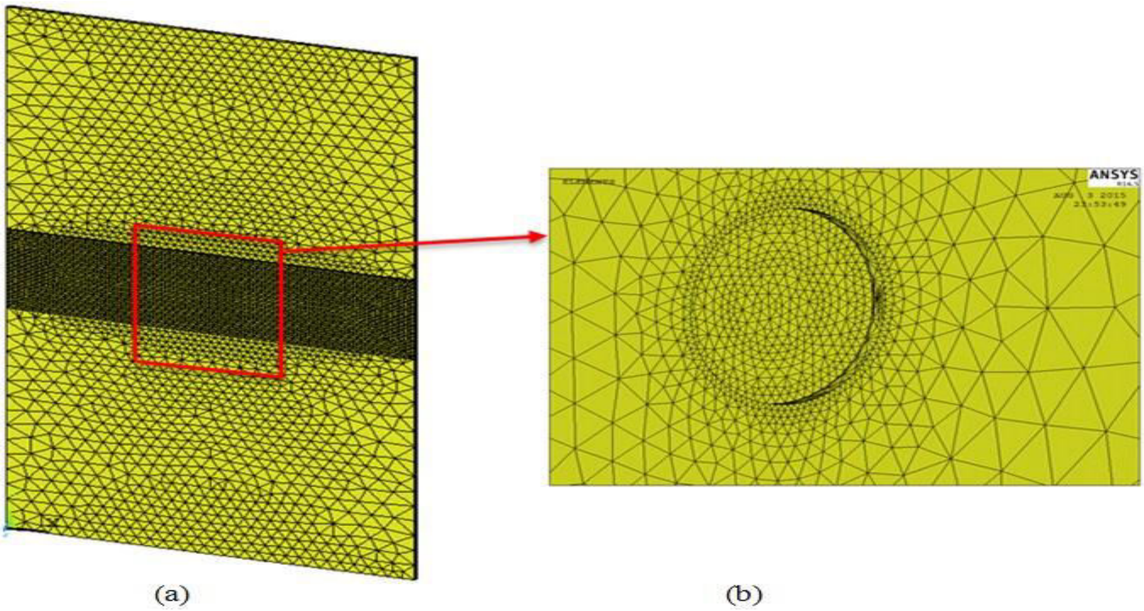



**Figure-2.** (a) Finite element model of the host structure and piezoelectric actuator (b) Rear view of finite element model.

## PIEZOELECTRIC CONSTITUTIVE EQUATIONS

A piezoelectric material is applicable both as a sensor and actuator. The constitutive equations governing the electromechanical response are given below [19].

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{32}\\ \sigma_{31}\\ \sigma_{12}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{22}&C_{23}&0&0&0\\ C_{13}&C_{23}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{32}\\ 2\varepsilon_{31}\\ 2\varepsilon_{12}\end{Bmatrix}-\begin{bmatrix}e_{11}&0&0\\ e_{12}&0&0\\ e_{13}&0&0\\ 0&0&0\\ 0&0&e_{35}\\ 0&e_{26}&0\end{bmatrix}\begin{Bmatrix}E_1\\ E_2\\ E_3\end{Bmatrix}$$

$$\begin{Bmatrix}D_1\\ D_2\\ D_3\end{Bmatrix}=\begin{bmatrix}e_{11}&e_{12}&e_{13}&0&0&0\\ 0&0&0&0&0&e_{26}\\ 0&0&0&0&e_{35}&0\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\\ 2\varepsilon_{12}\end{Bmatrix}+\begin{bmatrix}\in_{11}&0&0\\ 0&\in_{22}&0\\ 0&0&\in_{33}\end{bmatrix}\begin{Bmatrix}E_1\\ E_2\\ E_3\end{Bmatrix}$$

where $C$ the elasticity constant, $D$ is the electric displacement, $E$ is the electric field vector, $e$ is the piezoelectric coefficient matrix and $\in$ is the dielectric permittivity matrix.

## FINITE ELEMENT ANALYSIS

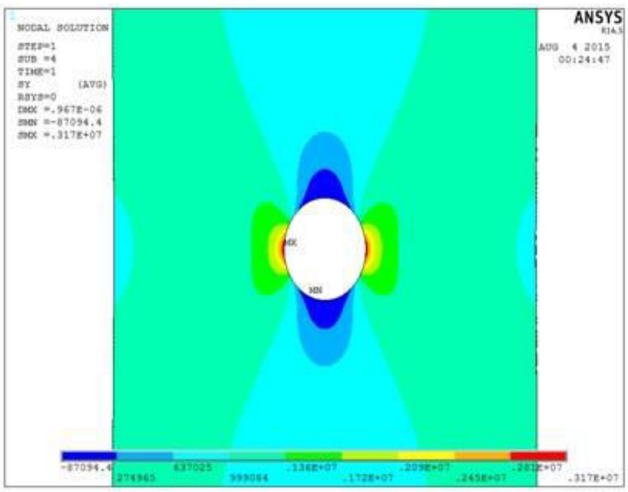
### Geometrical model

The dimensions of the aluminum plate considered in this work are H=200mm, w=100mm, and made up a central circular hole of radius of r=10mm and thickness t=1mm. The PIC151 [20] piezoelectric patch with adhesive has dimensions of $h_P=h_{ad}$=30mm, $w_P= w_{ad}$=w, and thickness of $t_p$=0.5t , and $t_{ad}$=0.025mm. Table 1 lists the material properties for the aluminum plate, adhesive, and piezoelectric actuator.

It is important to keep in mind that good adhesive response can only be observed for thickness ranging from 0.12mm to 0.25 mm [21].

### Finite element model and analysis

The SCF of a specimen can be determined from stress or the displacement near the discontinuous area using experimental or numerical methods such as finite element. Generally, the stresses and the deformation fields have high gradients around the discontinuous area. To generate such stress/strain gradients the mesh must have a specific characteristic such that the element near discontinuous area must be very fine.




(a)

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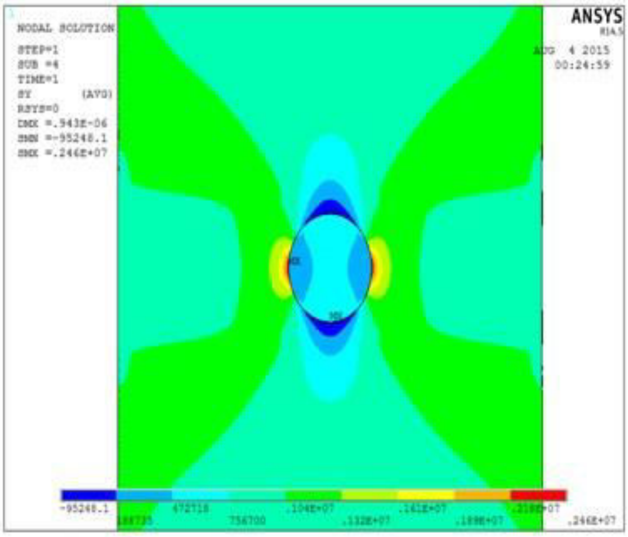


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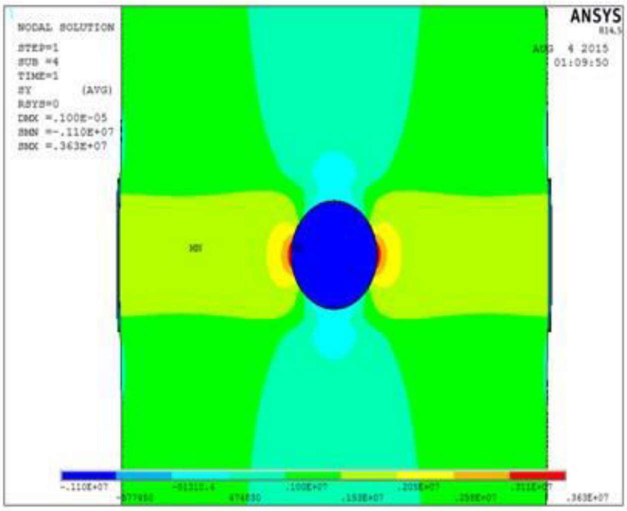


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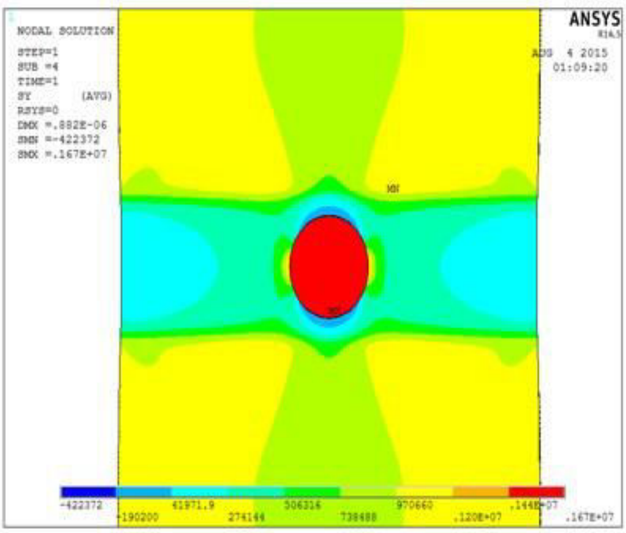


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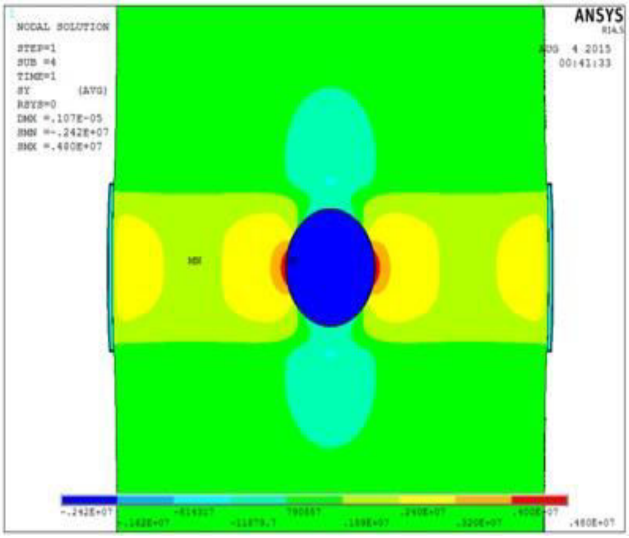


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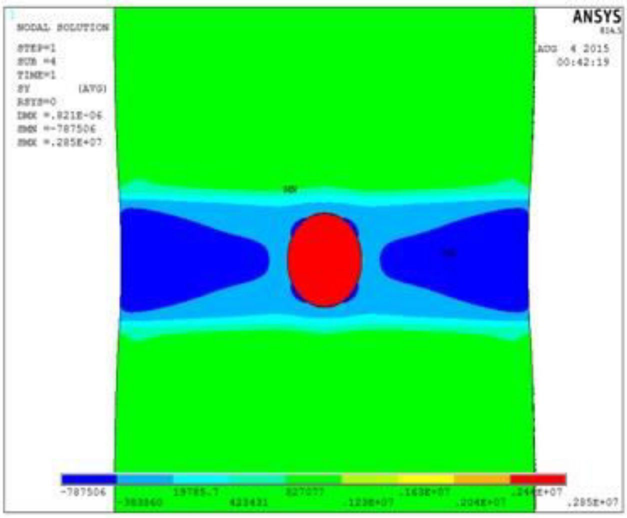


(f)

**Figure-3.** Nodal stress distributions (a) without piezoelectric actuator (b) passive piezoelectric actuator, (c) active with -50 V, (d) active with +50 V, (e) active with -100 V, (d) active with -100V.

To verify the finite element model, an aluminum plate without the piezoelectric actuator under static load was initially considered. A fine resolution 3D mesh was constructed near the hole. The numerical stress was used to calculate the SCF. The stress concentration factor for 1MPa load was found to be $K_t = 2.512$ and this was compared with the theoretical values of $K_t$ extracted from the works of Pilkey [3] and Tada [18]. The result shows good agreement with 1.5% discrepancy in regards to the analytical solution.

For the integrated structure, 3D element SOLID186 was used to model the host plate and SOLID226 for the piezoelectric, while the adhesive was modelled using SOLID186. Appropriate boundary conditions were applied to the host plate. The finite-element model of the plate with the integrated piezoelectric actuator is shown in the Figure-2.

## RESULT AND DISCUSSION

The distributions of the nodal stress in the vicinity of the plate's hole without and with piezoelectric actuator with selected voltages (0V, 50V,-50V,100V,-100V) are shown in Figure-3 in order to illustrate the effects of different actuation modes. The figure shows that the nodal stresses for the plate increase with the application of negative voltage and decrease with positive voltage. It can be observed that induced compression/tension stress by the active piezoelectric actuator redistributes the high-stress gradient around the hole and this results in reduction/increase of the SCF.

From Figure-3, it is also observed that the reduction of the stress concentration at the vicinity of the hole is accompanied by changes in the locations of high and minimum stresses in the plate as well as increase in the adhesive stress. This is due to the stress redistribution caused by the compression stress from the piezoelectric actuator.

Alternatively, the tensile stress produced by the piezoelectric actuator has small effects on the redistribution of the stresses in vicinity of the hole's minimum stresses.

Figure-3(b) shows that there is a reduction in the plate's stress distribution when there is no voltage applied to the actuator. This is due to the reinforcement of the hole resulting from the increased stiffness contributed by the actuator. However, in the case of the passive piezoelectric actuator the locations of the maximum and minimum stresses were not affected. Also, the plate's stress distribution only showed minor change.

The stress concentration factor at the plate's hole for various positive and negative voltages are illustrated in Figures-4 and 5 respectively. The results in Figure 4 show that stress concentration factor decreases with the increase in the positive voltage. This is attributed to the increase in the opposite stresses produced by the piezoelectric actuator in the surrounding compression and tensile region. It is observed that the maximum stress with actuator at 0V is less compared to the stress level without actuator. This is attributed to the passive increases of the integrated structure's stiffness due to adhesion of the piezoelectric actuator. Further increase in the voltage resulted in a linear decrease in the stress concentration factor. Also, it is important to mention that the interface stress between the piezoelectric actuator and the host plate increases linearly with applied voltage.

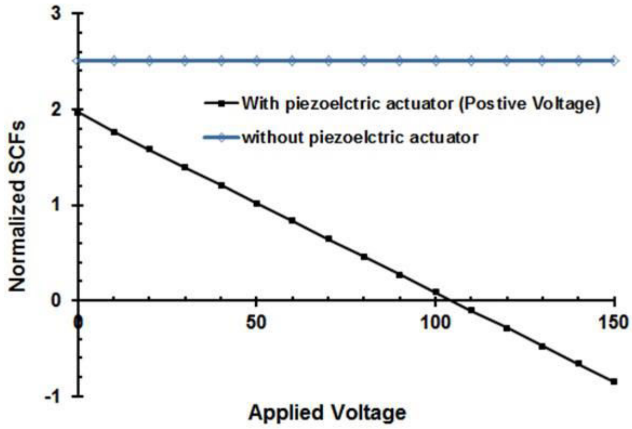


**Figure-4.** The variation of SCF with positive voltage.

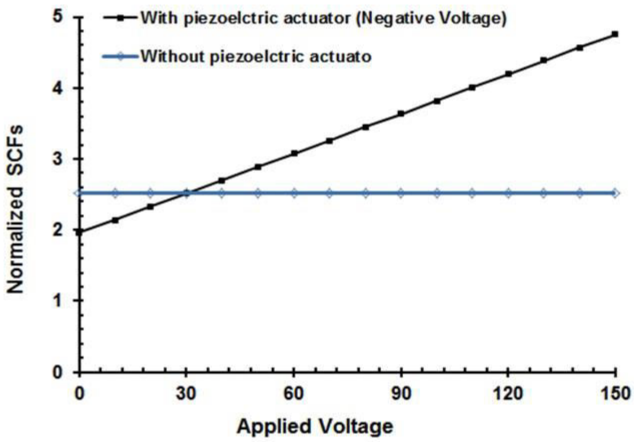


**Figure-5.** The variation of SCF with negative voltage.

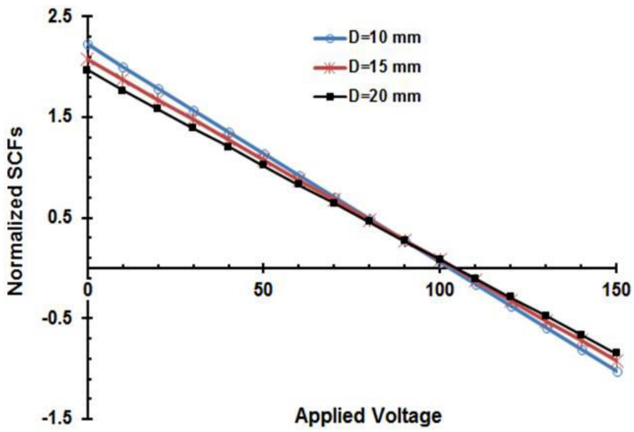


**Figure-6.** The variation of SCF with positive voltage and hole diameter, D.

On the contrary, Figure-5 shows that the stress concentration factor increases linearly with negative electric field. This is due to superposition of the tensile stresses produce by the piezoelectric actuators and the far field tension stresses.

Figures-6 and 7 show the variation of stress concentration factor for various hole diameter D under negative and positive voltages respectively. The results show that the reduction or increase of the stress concentration factor with the application of voltages in the actual term is not absolute, and it is effected by the hole diameter, the applied voltage and the distance between the piezoelectric actuator's edge and the hole. This is attributed to the actuator's effective distance, as it is indicating that the stress induced varies inversely with distance from the piezoelectric actuator edges. However, in the case of the high positive voltage the variation of the stress concentration factor is more obvious compare to high negative voltage.

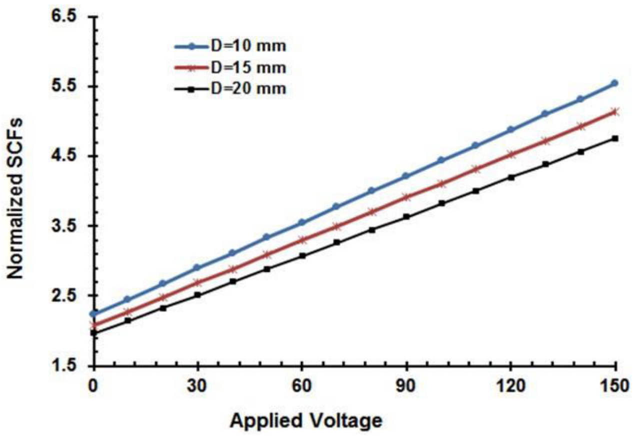


**Figure-7.** The variation of SCF with negative voltage and hole diameter, D.

High positive voltage raises the interface stress between the host structure and the piezoelectric actuators beyond the initial stress concentration without piezoelectric actuator.

In fact, from this study, it is found that it is better to avoid the extensional operation mode of the piezoelectric actuator as it can lead to an increase in the stress concentration factor.

## CONCLUSIONS

The effects of different piezoelectric actuation mode on the SCF have been studied for a rectangular plate containing circular hole under uniaxial tension. The piezoelectric actuator was placed above the circular hole and adhesively bonded with the help of epoxy. The results demonstrated that the piezoelectric actuator affects the SCF of the plate and this depends mainly on the piezoelectric actuation voltage. Negative voltage tends to increase the SCF while the positive voltage tends to reduce the SCF. However, this effect is also dependent of the plate hole diameter and the distance between the piezoelectric actuator's edge and the hole. The reduction or increase in stress concentration factor varied linearly with the applied voltages. However, very high positive voltage will not benefit the host structure as the interface stress will rise beyond the initial stress concentration.

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## REFERENCE

[1] E. F. Crawley and J. De Luis.1987. Use of piezoelectric actuators as elements of intelligent structures, AIAA Journal, vol. 25, pp. 1373-1385, 1987/10/01.

[2] B. M. Badr and W. G. Ali. 2011. Applications of Piezoelectric Materials, Advanced Materials Research, vol. 189-193, pp. 3612-3620.

[3] W. D. Pilkey. 2008. Peterson's Stress Concentration Factors, Second Edition: Wiley Online Library.

[4] M. Zhao, Q. Zhang, E. Pan, and C. Fan. 2014. Fundamental solutions and numerical modeling of an elliptical crack with polarization saturation in a transversely isotropic piezoelectric medium, Engineering Fracture Mechanics, vol. 131, pp. 627-642, 11.

[5] Y.-J. Wang, C.-F. Gao, and H. Song. 2015. The anti-plane solution for the edge cracks originating from an arbitrary hole in a piezoelectric material, Mechanics Research Communications, vol. 65, pp. 17-23, 4.

[6] Y.-J. Wang and C.-F. Gao. 2012. Thermoelectroelastic Solution for Edge Cracks Originating from an Elliptical Hole in a Piezoelectric Solid," Journal of Thermal Stresses, vol. 35, pp. 138-156, 2012/01/01.

[7] L. Jinxi, L. Xianglin, and Z. Yongbin. 2001. Green's functions for anisotropic magnetoelectroelastic solids with an elliptical cavity or a crack, International Journal of Engineering Science, vol. 39, pp. 1405-1418.

[8] C.-F. Gao, Y.-T. Zhao, and M.-Z. Wang. 2002. An exact and explicit treatment of an elliptic hole problem in thermopiezoelectric media, International journal of solids and structures, vol. 39, pp. 2665-2685.

[9] C.-F. Gao and W.-X. Fan. 1999. Exact solutions for the plane problem in piezoelectric materials with an elliptic or a crack, International Journal of Solids and Structures, vol. 36, pp. 2527-2540.

[10] L. Dai, W. Guo, and X. Wang.2006. Stress concentration at an elliptic hole in transversely isotropic piezoelectric solids," International journal of solids and structures, vol. 43, pp. 1818-1831.

[11] M. Chung and T. Ting. 1996. Piezoelectric solid with an elliptic inclusion or hole, International Journal of Solids and Structures, vol. 33, pp. 3343-3361.

[12] C.-R. Chiang. 2014. Subsurface crack problems in a cubic piezoelectric material, Engineering Fracture Mechanics, vol. 131, pp. 656-668, 11// 2014.

[13] H.-s. Chen, W.-y. Wei, J.-x. Liu, and D.-n. Fang. 2014. Propagation of a semi-infinite conducting crack in piezoelectric materials: Mode-I problem, Journal of the Mechanics and Physics of Solids, vol. 68, pp. 77-92, 8.

[14] S. D, C. W, and J. S. 1993. FINITE ELEMENT ANALYSIS OF PLATES WITH PIEZOELECTRIC LAYERS, in 34th Structures, Structural Dynamics and Materials Conference, ed: American Institute of Aeronautics and Astronautics.

[15] S. Hideki. 2006. Advances in smart-patch repair of aircraft panels, in 12th Unites States/Japan Conf. on Composite Materials (Univ. Michigan Dearborn, Dearborn, MI).

[16] T. J.-C. Liu. 2008. Crack repair performance of piezoelectric actuator estimated by slope continuity and fracture mechanics, Engineering Fracture Mechanics, vol. 75, pp. 2566-2574.

[17] T. J. C. Liu. 2007. Fracture mechanics and crack contact analyses of the active repair of multi-layered piezoelectric patches bonded on cracked structures, Theoretical and Applied Fracture Mechanics, vol. 47, pp. 120-132, .

[18] H. Tada, P. Paris, and G. Irwin. 2000. The analysis of cracks handbook: New York: ASME Press.

[19] "IEEE Standard on Piezoelectricity," ANSI/IEEE Std 176-1987, p. 0_1, 1988.

[20] "Designing with Piezoelectric Transducers.2005.Nanopositioning Fundamentals (Karlsruhe: Tutorial)." Physik Instrumente (PICeramic) GmbH & Co. KG.

[21] K. Madani, S. Touzain, X. Feaugas, M. Benguediab, and M. Ratwani.2009."Stress distribution in a 2024-T3 aluminum plate with a circular notch, repaired by a graphite/epoxy composite patch," International Journal of Adhesion and Adhesives, vol. 29, pp. 225-233, 4.