VOL. 10, NO. 21, NOVEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

DOUBLE INTEGRATOR MODEL AND INVARIANT MANIFOLD THEORY ALGORITHM FOR AN X4-AUV

Z. M. Zain[1], M. R. Arshad[2] and K. A. A. Rahim[3]
[1]Robotics and Unmanned Research Group, Control Engineering, Cluster, Faculty of Electrical and Electronics Engineering, Universiti Malaysia Pahang, Pekan, Pahang, Malaysia
[2,3] Underwater, Control and Robotics Research Group, School of Electrical and Electronic Engineering, Universiti Sains Malaysia, Nibong Tebal, Pulau Pinang, Malaysia
E-Mail: zainah@ump.edu.my

ABSTRACT

Autonomous underwater vehicle (X4-AUV) with four inputs and 6 degrees of freedom (DOFs) is an underactuated system and has a nonholonomic features. There exist various studies on nonholonomic underatuated control so far, but most of them are confined into the case of systems with two inputs and therefore there are a few studies for the systems with three or more inputs. Control approaches for nonholonomic systems have utilized canonical forms. A nonholonomic double integrator model is the one of canonical forms for nonholonomic systems. In this paper an algorithm for an extended double integrator with four inputs is presented. Then a control law for an X4-AUV in extended double integrator model is derived using invariant manifold theory. It is expected that each state of the controlled object will be converge smoothly to the origin by using this type of control.

Keywords: invariant manifold, nonholonomic system, underactuated system, extended double integrator.

INTRODUCTION

Control of underactuated systems, i.e., with less control inputs than coordinates is emerging as an important topic in control theory and applications. The potentials of a complete understanding of this problem are enormous. For example, the possibility of building mechanism that can perform complex tasks using a small number of actuators will reduce cost, weight as well as occurrences of failures. One of the main obstacles in the development of a comprehensive methodology for controlling such systems is the fact that linear control techniques cannot be used for these problems. These are 'inherently nonlinear' systems which require new approached taking into consideration their nonlinear character. One challenging aspect of these systems is that they are controllable but not stabilizable by smooth static or dynamic state feedback control law [1].

Control of underactuated systems often fall under the area of control of nonholonomic systems. The nonholonomic system has some constraints of velocity or acceleration. Many of control methods of a nonholonomic system convert the controlled system into a canonical form first. A canonical form of a symmetry affine system which has a velocity constraint is a chained form, a power form, and a nonholonomic double integrator. A 4th-order symmetry affine system with 2 inputs can convert it into the chained form. There are discontinuous control [1] and switching control [2-4] as a control method based on a canonical form. A control method for the power form composed of two inputs and n states is a switching control and a quasi-continuous exponential stabilization utilized the invariant manifold [5]. A canonical form of the system that has the acceleration constraint or torque input is an extended nonholonomic double integrator [6][7], a chained form, and an extended power form. The advantage of a nonholonomic system has lightening, a small energy making, and the cost reduction.

Note however that among them major research is for controlled object with two-inputs [8] and therefore there is restricted research for controlled object with three or more inputs [9-11]. One of causes is that there is no definite method of transforming the original model into a canonical model to the case of the controlled system with three or more inputs. In this research, in order to expand the application of underactuated control to the controlled system of 3 or more inputs, it aims at establishing a control technique for an X4-AUV, which is an underactuated system with four inputs and six outputs.

In this paper, after transforming the controlled system with four inputs into the double integrator form model of chained form in accordance with the method of Watanabe *et al.* [12], a switching control technique is proposed to stabilize the origin by an invariant manifold. It is expected that the switching control based on the invariant manifold assures that all the states smoothly converge to the origin [13],[14].

COORDINATE SYSTEM

A special reference frame must establish in order to describe the motion of the underwater vehicle. There are two coordinate systems: i.e., an inertial coordinate system (or fixed coordinate system) and motions coordinate system (or body-fixed coordinate system). The coordinate frame $\{E\}$ is composed of the orthogonal axes $\{E_x\ E_y\ E_z\}$ and is called as an inertial frame. This frame is commonly placed at a fixed place on Earth. The axes E_x and E_y form a horizontal plane, and E_z is the direction of the field of gravity. The body-fixed frame $\{B\}$ is composed of the orthogonal axes $\{X, Y, Z\}$ and is attached to the vehicle. The body axes, two of which coincide with the principle axes of inertia of the vehicles are defined by

VOL. 10, NO. 21, NOVEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

Fossen (Fossen and Sagatun, 1991) as follows: *X* is the longitudinal axis (directed from aft to fore); *Y* is the transverse axis (directed to starboard); *Z* is the normal axis (directed from top to bottom). Figure-1 shows the coordinate systems of an AUV, which consist of a right-hand inertial frame $\{E\}$ in which the downward vertical direction is to be positive, and a right-hand body frame $\{B\}$.

Letting $\boldsymbol{\xi} = [x \quad y \quad z]^T$ denote the centre of mass of the body in the inertial frame, and defining the rotational angles of the *X*, *Y*, and *Z* axes as $\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T$, the rotational matrix *R* from the body frame $\{B\}$ to the inertial frame $\{E\}$ is reduced as:

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

where *c*α denotes cos α and *s*α is sin α.

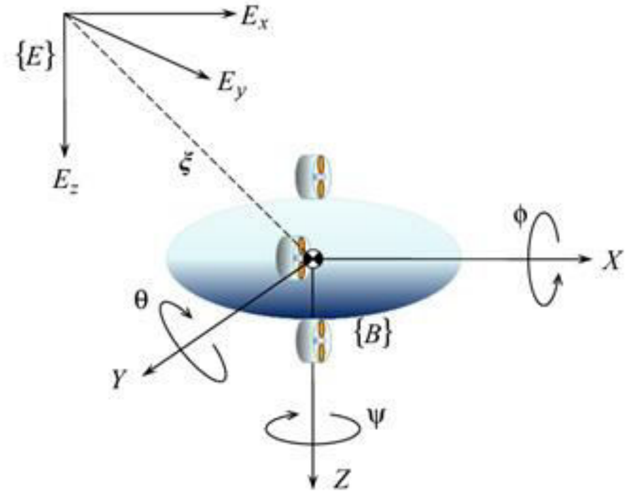

Figure-1. Coordinate systems of AUV.

X4-AUV DYNAMIC MODEL

Following a Lagrangian method, this section describes the dynamic model of the X4-AUV with the assumption of balance between buoyancy and gravity. The kinetic energy formula is:

$$T = T_{trans} + T_{rot} \quad (2)$$

here T_{trans} and T_{rot} are the translational kinetic energy and the rotational kinetic energy is defined by:

$$T_{trans} = \frac{1}{2}\dot{\xi}^T M \dot{\xi} \quad (3)$$

$$T_{rot} = \frac{1}{2}\dot{\eta}^T J \dot{\eta} \quad (4)$$

in which *M* is the total mass matrix of the body, and *J* is the total inertia matrix of the body. From the characteristics of added mass, it can be written as:

$$M = \text{diag}\,(m_1, m_2, m_3) = m_b I + M_f \quad (5)$$

$$J = \text{diag}\,(I_x, I_y, I_z) = J_b + J_f \quad (6)$$

Here, m_b is a mass of the vehicle, J_b is an inertia matrix of the vehicle and I is a 3×3 identity matrix.

Letting ρ denote a density of the fluid and using the formulation of the added mass and inertia under the assumption of $r_1 = 5r_2$ and $r_2 = r_3 = r$, where r_1, r_2 and r_3 the added mass matrix M_f and the added inertia matrix J_f are:

$$M_f = \text{diag}\,(0.394\rho\pi r^3, 5.96\rho\pi r^3, 5.96\rho\pi r^3) \quad (7)$$

$$J_f = \text{diag}\,(0, 24.2648\rho\pi r^5, 24.2648\rho\pi r^5) \quad (8)$$

From the assumption of the balance between the buoyancy and the gravity, i.e., the potential energy $U = 0$, the Lagrangian can be written as:

$$\begin{aligned} L &= T - U \\ &= T_{trans} + T_{rot} \end{aligned} \quad (9)$$

The dynamic model of X4-AUV summarized as:

$$\begin{aligned} m_1\ddot{x} &= \cos\theta \, \cos\psi \, u_1 \\ m_2\ddot{y} &= \cos\theta \, \sin\psi \, u_1 \\ m_3\ddot{z} &= -\sin\theta \, u_1 \\ I_x\ddot{\phi} &= \dot{\theta}\dot{\psi}(I_y - I_z) + u_2 \\ I_y\ddot{\theta} &= \dot{\phi}\dot{\psi}(I_z - I_x) - J_t\dot{\psi}\Omega + lu_3 \\ I_z\ddot{\psi} &= \dot{\phi}\dot{\theta}(I_x - I_y) + J_t\dot{\theta}\Omega + lu_4 \end{aligned} \quad (10)$$

where $u_{1,} u_2, u_3, and\ u_4$ are the control inputs for the translational (x, y, and z-axis) motion, the roll (ϕ-axis) motion, the pitch (θ-axis) motion, and yaw (ψ-axis) motion, respectively. A detailed derivation for dynamics model (10) given in [15].

Defining that b is a thrust factor, d is a drag factor, taken from $\tau_{Mi} = d\omega_i^2$ then $\Omega, u_{1,} u_2, u_3, and\ u_4$ are given by:

VOL. 10, NO. 21, NOVEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$$\begin{aligned}\Omega &= (\omega_2+\omega_4-\omega_1-\omega_3)\\ u_1 &= f_1+f_2+f_3+f_4\\ &= b(\omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2)\\ u_2 &= d(-\omega_2^2-\omega_4^2+\omega_1^2+\omega_3^3)\\ u_3 &= f_1-f_3 = b(\omega_1^2-\omega_3^2)\\ u_4 &= f_2-f_4 = b(\omega_2^2-\omega_4^2)\end{aligned} \tag{11}$$

The dynamic model (10) can be rewritten in a state-space form $\dot{X}=f(X,U)$ by introducing $X=(x_1\cdots x_{12})^T\epsilon\,\Re^{12}$ as state vector of the system as follows:

$$f(X,U)=\begin{pmatrix} x_2\\ \cos\theta\cos\psi\dfrac{1}{m_1}u_1\\ x_4\\ u_y\dfrac{1}{m_2}u_1\\ x_6\\ u_z\dfrac{1}{m_3}u_1\\ x_8\\ x_{10}x_{12}\,a_1+b_1u_2\\ x_{10}\\ x_8x_{12}\,a_2-\ a_3x_{12}\Omega+b_2u_3\\ x_{12}\\ x_8x_{10}\,a_5+\ a_4x_{10}\Omega+b_3u_4\end{pmatrix} \tag{12}$$

Where

$a_1=\frac{(I_y-I_z)}{I_x}$, $a_2=\frac{(I_z-I_x)}{I_y}$, $a_3=\frac{J_t}{I_y}$, $a_4=\frac{J_t}{I_z}$, $a_5=\frac{(I_x-I_y)}{I_z}$, $b_1=\frac{1}{I_x}$, $b_2=\frac{l}{I_y}$, $b_3=\frac{l}{I_z}$, $u_y=\cos\theta\sin\psi$, $u_z=-\sin\theta$.

INVARIANT MANIFOLD FOR EXTENDED NONHOLONOMIC DOUBLE INTEGRATOR WITH 4-INPUTS SYSTEM

Let the controlled object be represented by the following extended nonholonomic double integrator system:

$$\begin{aligned}\dot{x}_1 &= y_1\\ \dot{x}_2 &= y_2\\ \dot{x}_3 &= y_3\\ \dot{x}_4 &= y_4\\ \dot{x}_5 &= x_1y_2-x_2y_1\\ \dot{x}_6 &= x_1y_3-x_3y_1\\ \dot{x}_7 &= x_1y_4-x_4y_1\\ \dot{x}_8 &= x_2y_3-x_3y_2\\ \dot{x}_9 &= x_2y_4-x_4y_2\\ \dot{x}_{10} &= x_3y_4-x_4y_3\\ \dot{y}_1 &= u_1\\ \dot{y}_2 &= u_2\\ \dot{y}_3 &= u_3\\ \dot{y}_4 &= u_4\end{aligned} \tag{13}$$

and consider a stabilizing problem such that $\boldsymbol{x}(t)=[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ y_1\ y_2\ y_3\ y_4]^T$ is settled to zero as $t\to\infty$.

To derive an invariant manifold for this systems, assume that the following state feedback law is applied to Equation (13).

$$\begin{aligned}u_1(t)&=-2ky_1(t)-k^2x_1(t)\\ u_2(t)&=-2ky_2(t)-k^2x_2(t)\\ u_3(t)&=-2ky_3(t)-k^2x_3(t)\\ u_4(t)&=-2ky_4(t)-k^2x_4(t)\end{aligned} \tag{14}$$

Now, defining the state vector of the linear partial system in (1) as $\boldsymbol{x}_s(t)\underline{\underline{\Delta}}\,[x_1\ x_2\ x_3\ x_4\ y_1\ y_2\ y_3\ y_4]^T$ its closed loop linear partial becomes

$$\dot{\boldsymbol{x}}_s(t)=A\boldsymbol{x}_s(t) \tag{15}$$

Where

$$A=\begin{bmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ -k^2&0&0&0&-2k&0&0&0\\ 0&-k^2&0&0&0&-2k&0&0\\ 0&0&-k^2&0&0&0&-2k&0\\ 0&0&0&-k^2&0&0&0&-2k\end{bmatrix}$$

Then, the time response of Equation (3) is described by

$$\boldsymbol{x}_s(t)=e^{At}\boldsymbol{x}_s(t) \tag{16}$$

where e^{At} is

$$e^{-kt}\begin{bmatrix} 1+kt&0&0&0&t&0&0&0\\ 0&1+kt&0&0&0&t&0&0\\ 0&0&1+kt&0&0&0&t&0\\ 0&0&0&1+kt&0&0&0&t\\ -k^2t&0&0&0&1-kt&0&0&0\\ 0&-k^2t&0&0&0&1-kt&0&0\\ 0&0&-k^2t&0&0&0&1-kt&0\\ 0&0&0&-k^2t&0&0&0&1-kt\end{bmatrix}$$

Therefore, the closed-loop linear partial system is reduced to

$$\begin{aligned}x_1(t)&=x_1(0)\left[e^{-kt}+kte^{-kt}\right]+y_1(0)te^{-kt}\\ x_1(t)&=x_2(0)\left[e^{-kt}+kte^{-kt}\right]+y_2(0)te^{-kt}\\ x_1(t)&=x_3(0)\left[e^{-kt}+kte^{-kt}\right]+y_3(0)te^{-kt}\\ x_1(t)&=x_4(0)\left[e^{-kt}+kte^{-kt}\right]+y_4(0)te^{-kt}\\ y_1(t)&=x_1(0)\left[-k^2te^{-kt}\right]+y_1(0)\left[e^{-kt}-kte^{-kt}\right]\\ y_2(t)&=x_2(0)\left[-k^2te^{-kt}\right]+y_1(0)\left[e^{-kt}-kte^{-kt}\right]\\ y_3(t)&=x_3(0)\left[-k^2te^{-kt}\right]+y_1(0)\left[e^{-kt}-kte^{-kt}\right]\\ y_4(t)&=x_4(0)\left[-k^2te^{-kt}\right]+y_1(0)\left[e^{-kt}-kte^{-kt}\right]\end{aligned} \tag{17}$$

The time response of nonlinear term becomes

$$x_5(t) = x_5(0) + \left(x_1(0)y_2(0) - y_1(0)x_2(0)\int_0^t e^{-2k\tau} d\tau \right) = x_5(0) - \left(\frac{x_1(0)y_2(0)}{2k} - \frac{y_1(0)x_2(0)}{2k} \right)\left(e^{-2kt} - 1\right) \tag{18}$$

Utilizing constant term of Equation (18),

$$S_1(t) = x_5(t) + \frac{1}{2k}x_1(t)y_2(t) - \frac{1}{2k}x_2(t)y_1(t) \tag{19}$$

are selected to the candidate of invariant manifold. The differentiation of Equation (19) is

$$\dot{S}_1(t) = \dot{x}_5(t) + \frac{1}{2k}\left[\dot{x}_1(t)y_2(t) + x_2(t)\dot{y}_2(t)\right] - \frac{1}{2k}\left[\dot{x}_2(t)y_1(t) + x_2(t)\dot{y}_1(t)\right] \equiv 0 \tag{20}$$

Then it is found that $S_1(t)$ is an invariant manifold. From other nonlinear terms, invariant manifolds can be selected as

$$\begin{aligned} S_2(t) &= x_6(t) + \frac{1}{2k}x_1(t)y_3(t) - \frac{1}{2k}x_3(t)y_1(t) \\ S_3(t) &= x_7(t) + \frac{1}{2k}x_1(t)y_4(t) - \frac{1}{2k}x_4(t)y_1(t) \\ S_4(t) &= x_8(t) + \frac{1}{2k}x_2(t)y_3(t) - \frac{1}{2k}x_3(t)y_2(t) \\ S_5(t) &= x_9(t) + \frac{1}{2k}x_2(t)y_4(t) - \frac{1}{2k}x_4(t)y_2(t) \\ S_6(t) &= x_{10}(t) + \frac{1}{2k}x_3(t)y_4(t) - \frac{1}{2k}x_4(t)y_2(t) \end{aligned} \tag{21}$$

The differentiation of Equation (21) is

$$\begin{aligned} \dot{S}_2(t) &= \dot{x}_6(t) + \frac{1}{2k}\left[\dot{x}_1(t)y_3(t) + x_1(t)\dot{y}_3(t)\right] - \frac{1}{2k}\left[\dot{x}_3(t)y_1(t) + x_3(t)\dot{y}_1(t)\right] \equiv 0 \\ \dot{S}_3(t) &= \dot{x}_7(t) + \frac{1}{2k}\left[\dot{x}_1(t)y_4(t) + x_1(t)\dot{y}_4(t)\right] - \frac{1}{2k}\left[\dot{x}_4(t)y_1(t) + x_4(t)\dot{y}_1(t)\right] \equiv 0 \\ \dot{S}_4(t) &= \dot{x}_8(t) + \frac{1}{2k}\left[\dot{x}_2(t)y_3(t) + x_2(t)\dot{y}_3(t)\right] - \frac{1}{2k}\left[\dot{x}_3(t)y_2(t) + x_3(t)\dot{y}_2(t)\right] \equiv 0 \\ \dot{S}_5(t) &= \dot{x}_9(t) + \frac{1}{2k}\left[\dot{x}_2(t)y_4(t) + x_2(t)\dot{y}_4(t)\right] - \frac{1}{2k}\left[\dot{x}_4(t)y_2(t) + x_4(t)\dot{y}_2(t)\right] \equiv 0 \\ \dot{S}_6(t) &= \dot{x}_{10}(t) + \frac{1}{2k}\left[\dot{x}_3(t)y_4(t) + x_3(t)\dot{y}_4(t)\right] - \frac{1}{2k}\left[\dot{x}_4(t)y_3(t) + x_4(t)\dot{y}_3(t)\right] \equiv 0 \end{aligned} \tag{22}$$

Then, $S_i(t)$ is invariant manifold because $S_i(t)$ converges to zero.

CONCLUSIONS

In this paper, a new underactuated control method has been proposed for nonholonomic underactuated X4-AUV by applying a double integrator model and invariant manifold theory. At present, only the stabilization control problem was considered for an X4-AUV dynamic model. It is expected that each state of the controlled object will be converge mootly to the origin by using this type of control.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge all the people who contributed directly or indirectly and this work is supported in part by the Research Acculturation Collaborative Effort (RACE) Grant Scheme under grant no. RDU141301.

REFERENCES

[1] Astolfi A. 1996. Discontinuous control of nonholonomic systems. Systems and Control Letters, Vol. 27-1. pp. 37-45.

[2] Hespanha J. P. and Morse A. S. 1999. Stabilization of nonholonomic integrators via logic-based switching. Automatica Vol. 35-3. pp. 385-393.

[3] Bloch A. and Drakunov S. 1996. Stabilization and tracking in the nonholonomic integrator via sliding mode. Systems and Control Letters, vol. 29-2. pp. 91-99.

[4] Xu W. L. and Huo W. 2000. Variable structure exponential stabilization of chained systems based on the extended nonholonomic integrator. Systems and Control Letters, Vol. 41-4. pp. 225-235.

[5] Tsiotras P., Corless M. and Longuski J. M. 1993. Invatiant manifold techniques for attitude control of symmetric spacecraft. Proc of 32nd Conference on Decision and Control. pp. 1470-1475.

[6] Rehman F. 2005. Steering control of nonholonomic systems with drift: the extended nonholonomic double integrator example. Nonlinear Analysis, Vol. 62. pp. 1498-1515.

[7] Escobar G., Ortega R and Reyhanoglu M. 1998. Regulation and tracking of the nonholonomic double integrator: a field-oriented control approach. Automatica, Vol. 34-3. pp. 125-131.

[8] Beale M. H., Hagan M. T. and Dewuth H. B. 2006. Fundamentals of machine and machine tools. 2nd Ed. Marcel Dekker Inc. New York, USA. pp. 160-168.

[9] Mita T. 2000. Introduction to nonlinear control. Shokodo, Tokyo.

[10] Watanabe K., Tanaka K., Izumi K., Okamura K. and Syam R. 2007. Discontinuous control and backstepping method for the underactuated control of VTOL aerial robots with four rotors. Proc. of International Conference on Intelligent Unmanned Systems (ICIUS2007). pp. 224–231.

[11] Watanabe K., Tanaka K., Izumi K. and Okamura K. 2008. An underactuated control for VTOL aerial robots with four rotors via a chained form transformation. Proc. of the 13th Int. Symposium on Artificial Life and Robotics (AROB 13th '08). pp. 775–778.

[12] Watanabe K., Izumi K. and Okamura K. 2009. Kinematics-based control of underactuated vehicles with four-inputs and six states by applying invariant manifolds. IEEE Int. Conf. on Networking, Sensing and Control. pp. 846–848.

[13] Watanabe K., Izumi K., Okamura K. and Syam R. 2008. Discontinuous underactuated control for lateral X4 autonomous underwater vehicles. Proc. of the 2nd International Conference on Underwater System Technology: Theory and Applications 2008 (USYS '08). Paper ID 14

[14] Watanabe K., Yamamoto T., Izumi K. and Maeyama S. 2010. Underactuated control for nonholonomic mobile robots by using double integrator model and invariant manifold theory. Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS2010). pp. 2862–2867.

[15] Zain Z. M., Watanabe K., Danjo T. and Nagai I. 2010. Modeling an autonomous underwater vehicle with four-thrusters. Proc. Of The Fifteenth International Symposium on Artificial Life and Robotics (AROB 15th'10). pp. 934–937.