VOL. 10, NO. 21, NOVEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

DISCOVERING RAZI ACCELERATION IN MULTIPLE ROTATING COORDINATE FRAMES SYSTEM

A. S. Mohd Harithuddin[1] and P. M. Trivailo[2]
[1]Department of Aerospace Engineering, Universiti Putra Malaysia, Serdang, Malaysia
[2]Schools of Aerospace, Mechanical and Manufacturing Engineering, RMIT University, Bundoora, Australia
E-Mail: a_salahuddin@upm.edu.my

ABSTRACT

This paper outlines the procedure of calculating inertial acceleration in a kinematic system involving more than two relatively rotating coordinate frames. The equation of relative motion between two frames – one inertial and one rotating – exhibits inertial acceleration terms like centripetal, Coriolis and tangential accelerations. It is shown that by introducing a third relatively-rotating frame, the Razi acceleration term appears in the equation, complementing the three aforementioned inertial acceleration terms. This is demonstrated by extending the application of Euler derivative transformation formula to include multiple coordinate frames. To show the appearance of the Razi acceleration, an experimental investigation is conducted using a multiple-axis robotic arm to simulate an "enclosed rotation" motion. A sensor is attached to the end-effector of the robotic arm to measure its acceleration. A comparison of the result with the equation produced by the derivative transformation formula is presented to show the effect of the Razi acceleration on a rigid body undergoing rotation about multiple axes.

Keywords: razi acceleration, rigid body kinematics, coordinate transformation, multiple coordinate system kinematics.

INTRODUCTION

The problem of gyroscopic motion, or enclosed rotation, and its mechanical effects has been long investigated [1-5]. Hirschberg and Mendelson [1] asserts that when there rotation axis is itself rotated, a structure under such motion experiences force acting parallel to the rotation axis. We describe the term "enclosed rotation" as "rotation about two or more axes simultaneously" and the acceleration induced by such motion as "inertial acceleration".

Comparing to the application-based papers on this topic, the work on the actual kinematics of such motion – analysing and calculating velocity and acceleration terms – is sparse. Kane [4] and Sherby and Chmielewski [5] are some of the early researchers who focused on this problem from vector mathematics point of view. Modern works on vector derivatives of multiple reference frames can be found in Jazar [6], in which the mathematical expression for the Razi acceleration is shown.

This paper derives the expression for the second-order derivative of a position and angular velocity vector measured in a system of n relatively rotating coordinate frames. The appearance of the Razi acceleration term in the expression is demonstrated by an experimental investigation using a multiple-axis motion simulator.

EULER DERIVATIVE TRANSFORMATION FORMULA

Let $A(OXYZ)$ and $B(Oxyz)$ be two arbitrary, relatively rotating coordinate frames sharing a common origin O. Let ${}^{B}\mathbf{r}$ be a generic vector expressed in the frame B and ${}_{A}\boldsymbol{\omega}_{B}$ be the angular velocity of the frame B with respect to frame A. The time derivative of the ${}^{B}\mathbf{r}$ vector as seen from frame A is expressed as:

$$ {}^{B}_{A}\dot{\mathbf{r}} = {}^{B}_{B}\dot{\mathbf{r}} + {}^{B}_{A}\boldsymbol{\omega}_{B} \times {}^{B}\mathbf{r} \tag{1} $$

where all vectors are expressed in the coordinate of frame B (left superscript). Equation (1) is known as the Euler derivative transformation formula [6]. Differentiating the ${}^{B}\mathbf{r}$ vector twice from frame A yields:

$$ {}^{B}_{AA}\ddot{\mathbf{r}} = {}^{B}_{BB}\ddot{\mathbf{r}} + {}^{B}_{AA}\boldsymbol{\alpha}_{B} \times {}^{B}\mathbf{r} + 2\,{}^{B}_{A}\boldsymbol{\omega}_{B} \times {}^{B}_{B}\dot{\mathbf{r}} \tag{2} $$

$$ + {}^{B}_{A}\boldsymbol{\omega}_{B} \times \left({}^{B}_{A}\boldsymbol{\omega}_{B} \times {}^{B}\mathbf{r} \right) $$

The first term on the right hand side of Equation (2) is the absolute acceleration of the ${}^{B}\mathbf{r}$ vector in its B-frame, and the second, third and fourth terms are the inertial accelerations; tangential acceleration, Coriolis acceleration, and centripetal acceleration, respectively.

APPLICATION IN MULTIPLE COORDINATE FRAMES SYSTEM

To extend the Euler derivative transformation to account for arbitrary number of moving coordinate frames, we derive the general formula for the composition of relative angular velocity vectors and angular acceleration vectors. This is useful for a system with multiple rotating coordinate frames as in the case of enclosed rotation.

Consider n angular velocities representing a set of n non-inertial, rotating coordinate frames. The composition of the relative angular velocities satisfies the relation.

$$ {}^{0}_{0}\boldsymbol{\omega}_{n} = \sum_{i=1}^{n} {}^{0}_{i-1}\boldsymbol{\omega}_{i} \tag{3} $$

For the purpose of this paper, we can assume that frame 0 is the inertial-fixed frame. Using the Euler

 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

derivative transformation formula, it can be shown that the first time-derivative of Equation (3) (angular accelerations), if it is differentiated from an arbitrary frame *f* in the set, satisfies the relation:

$$ {}_{f,0}^{\;\;0}\boldsymbol{\alpha}_n = \sum_{i=1}^{n} {}_{i-1}^{\;\;0}\boldsymbol{\alpha}_i + {}_{f}^{0}\boldsymbol{\omega}_i \times {}_{i-1}^{\;\;0}\boldsymbol{\omega}_i \quad (4) $$

Derivations of Equation (3) and Equation (4) can be found in [7].

Therefore, given a set of *n* non-inertial rotating coordinate frames in enclosed rotations, with ${}_{i-1}\boldsymbol{\omega}_i$ representing the angular velocity of ***i***-th coordinate frame with respect to the preceding coordinate frame ***i***-1, we present the general expression for the acceleration (the double derivative of a vector) of the *n*-th coordinate frame as seen from the inertial coordinate frame 0.

$$ {}_{00}^{\;n}\ddot{\mathbf{r}} = {}_{nn}^{\;n}\ddot{\mathbf{r}} + \sum_{i=1}^{n} {}_{i-1}^{\;\;n}\boldsymbol{\alpha}_i \times {}^{n}\mathbf{r} + \sum_{i=1}^{n} \left({}_{0}^{n}\boldsymbol{\omega}_i \times {}_{i-1}^{\;\;n}\boldsymbol{\omega}_i \right) \times {}^{n}\mathbf{r} $$
$$ + 2\sum_{i=1}^{n} {}_{i-1}^{\;\;n}\boldsymbol{\omega}_i \times {}_{n}^{n}\dot{\mathbf{r}} + \sum_{i=1}^{n} {}_{i-1}^{\;\;n}\boldsymbol{\omega}_i \times \left(\sum_{i=1}^{n} {}_{i-1}^{\;\;n}\boldsymbol{\omega}_i \times {}^{n}\mathbf{r} \right) \quad (5) $$

The third acceleration term on the right hand side, called the Razi acceleration, appears when the non-inertial rotating frame used in the kinematic system is more than one.

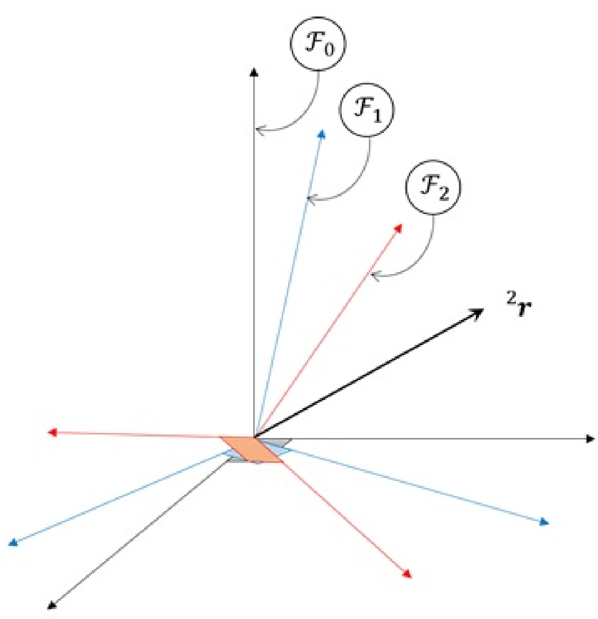


Figure-1. Three relatively rotating frames. The F_2-frame is rotating in the F_1-frame, and, in turn, the F_2-frame is rotating in the inertial F_0-frame. The ${}^{2}\mathbf{r}$ vector is in the F_2-frame.

To demonstrate the derivation of the Razi acceleration, we provide an example of an enclosed rotation motion with two non-inertial rotating frames as shown in Figure-1. The general form of the second derivative of the vector ${}^{C}\mathbf{r}$ as differentiated from the inertial frame *A*, following the convention of Equation (5), is:

$$ {}_{00}^{\;2}\ddot{\mathbf{r}} = {}_{22}^{\;2}\ddot{\mathbf{r}} + \sum_{i=1}^{2} {}_{i-1}^{\;\;2}\boldsymbol{\alpha}_2 \times {}^{2}\mathbf{r} + \sum_{i=1}^{2} \left({}_{0}^{2}\boldsymbol{\omega}_i \times {}_{i-1}^{\;\;2}\boldsymbol{\omega}_i \right) \times {}^{2}\mathbf{r} $$
$$ + 2\sum_{i=1}^{2} {}_{i-1}^{\;\;2}\boldsymbol{\omega}_i \times {}_{2}^{2}\dot{\mathbf{r}} + \sum_{i=1}^{2} {}_{i-1}^{\;\;2}\boldsymbol{\omega}_i \times \left(\sum_{i=1}^{2} {}_{i-1}^{\;\;2}\boldsymbol{\omega}_i \times {}^{2}\mathbf{r} \right) \quad (6) $$

For brevity, it can be also expressed as:

$$ {}_{00}^{\;2}\ddot{\mathbf{r}} = {}_{22}^{\;2}\ddot{\mathbf{r}} + {}_{0}^{2}\boldsymbol{\alpha}_2 \times {}^{2}\mathbf{r} + \left({}_{0}^{2}\boldsymbol{\omega}_1 \times {}_{1}^{2}\boldsymbol{\omega}_2 \right) \times {}^{2}\mathbf{r} $$
$$ + 2\,{}_{0}^{2}\boldsymbol{\omega}_2 \times {}_{2}^{2}\dot{\mathbf{r}} + {}_{0}^{2}\boldsymbol{\omega}_2 \times \left({}_{0}^{2}\boldsymbol{\omega}_2 \times {}^{2}\mathbf{r} \right) \quad (7) $$

The Razi acceleration term, in this case, is represented by the term $({}^{2}{}_{0}\boldsymbol{\omega}_1 \times {}^{2}{}_{1}\boldsymbol{\omega}_2) \times {}^{2}\mathbf{r}$.

EXPERIMENTAL INVESTIGATION

To test the theory of derivative kinematics and the appearance of the Razi acceleration, we utilize a multi-joint robot arm to simulate the enclosed rotation motion. A sensor is attached at a point on the manipulator to record its acceleration and the angular rates. To validate the derivative transformation formula method and to show the appearance of the Razi acceleration, we compare the acceleration signals with the formulaic calculation (using the angular rates of the motion) to show that the Razi term describes the acceleration in the out-of-plane direction.

An industrial-grade six-axis robotic arm, the ABB IRB 1400, is used as the motion simulator to replicate the simultaneous rotations about multiple axes. The IRB-1400 robotic manipulator consists of a floor-mounted base and six arms, connected by active revolute joints (shown in Figure-2). Each axis provides one rotary degree-of-freedom. The type of motion and range of motion for each axis is shown in Table-1. The sixth arm is the end-effector to which a motion sensor is attached.

Figure-2. The ABB IRB 1400 robotic manipulator.

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Table-1. ABB IRB 1400 range of motion.

Robot axis	Motion classification	Range of rotation	Maximum speed
1	Rotation	-170° to 170°	120 °/s
2	Arm motion	-100° to 20°	120 °/s
3	Arm motion	-65° to 70°	120 °/s
4	Wrist motion	-150° to 150°	280 °/s
5	Bend motion	-115° to 115°	280 °/s
6	Turn motion	-300° to 300°	280 °/s

An inertial measurement unit (IMU) called x-IMU from x-io Technologies Limited (http://www.x-io.co.uk/) is used to measure the motion of the end-effector. The sensor consists of a tri-axis accelerometer, a tri-axis gyroscope, and a tri-axis magnetometer. It has the capabilities to track translational and rotational motions, and orientation with respect to the direction of gravity. The magnetometer feature is particularly useful for this experiment as it provides the Earth-based reference frame which is used as the inertial coordinate frame in this experiment. The accelerometer has a selectable measurement range of up to +/- 8 g. The gyroscope is capable of detecting angular speed of up to +/- 2000 °/s. Sensor data and instantaneous orientation values from the sensor are transmitted wirelessly via Class 1 Bluetooth with an effective transmitting and receiving range of 100 m. The sampling rate is set at 256 Hz for all experiments.

The IMU is made secure at the end-effector with a small piece of foam to absorb the small vibration due to the robotic arm motion. Preliminary tests showed that the foam did not add any significant reduction in vibration. The IMU's coordinate frame is aligned parallel to the F_2-frame, which in turn is rotating about the x_1-axis of the F_1-frame. The coordinate frames setup is depicted in Figure-3. The z_1-axis and the x_2-axis are rotated simultaneously by Axis 1 and Axis 4 respectively to provide two rotating frames. The angular velocities and the directions of rotation of the two frames are given Table-2. The acceleration of the end effector is expressed by Eqn. (7). The acceleration signal from the IMU is then compared to the plotted Eqn. (7), which is calculated from the angular velocities of both $F_1$and F_2 coordinate frames.

Table-2. Measured angular velocity of each axis of rotation.

Robot axis	Coordinate frame axis	Angular velocity	Range of rotation (deg)
1	z_1-axis (F_1)	128 °/s	0° to 164.5°
4	x_2-axis (F_2)	210 °/s	-145° to 145°

The motion is repeated several times to get more readings. Figure-4 shows the acceleration reading from the IMU for five repeated enclosed rotations. The signals are separated into three orthogonal axes with the static acceleration (gravity) filtered out. Only the dynamic acceleration, i.e. acceleration caused by motion, is recorded. Some minor vibrations, which is apparent in the acceleration reading on the *y*.direction and the *z*-direction, are disregarded. The spikes at the start and the end of each cycle are due to the jerking motion of the robotic arm.

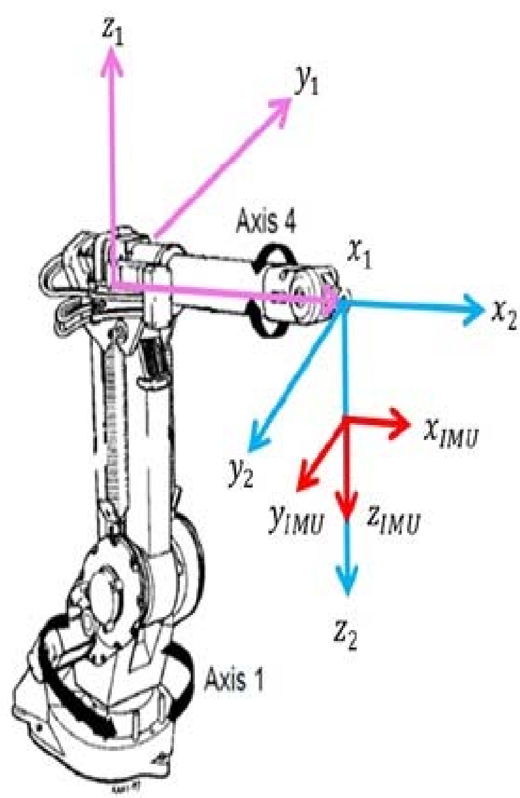


Figure-3. The F_1-frame (purple) is attached at the base of the first rotating arm with the F_2-frame (blue) is attached at the end at a distance of 870 mm along the x_1-axis.The IMU-frame (red) is attached at the end of the second rotating arm with a distance of 266 mm along the z_2-axis. The inertial frame is not shown.

VOL. 10, NO. 21, NOVEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

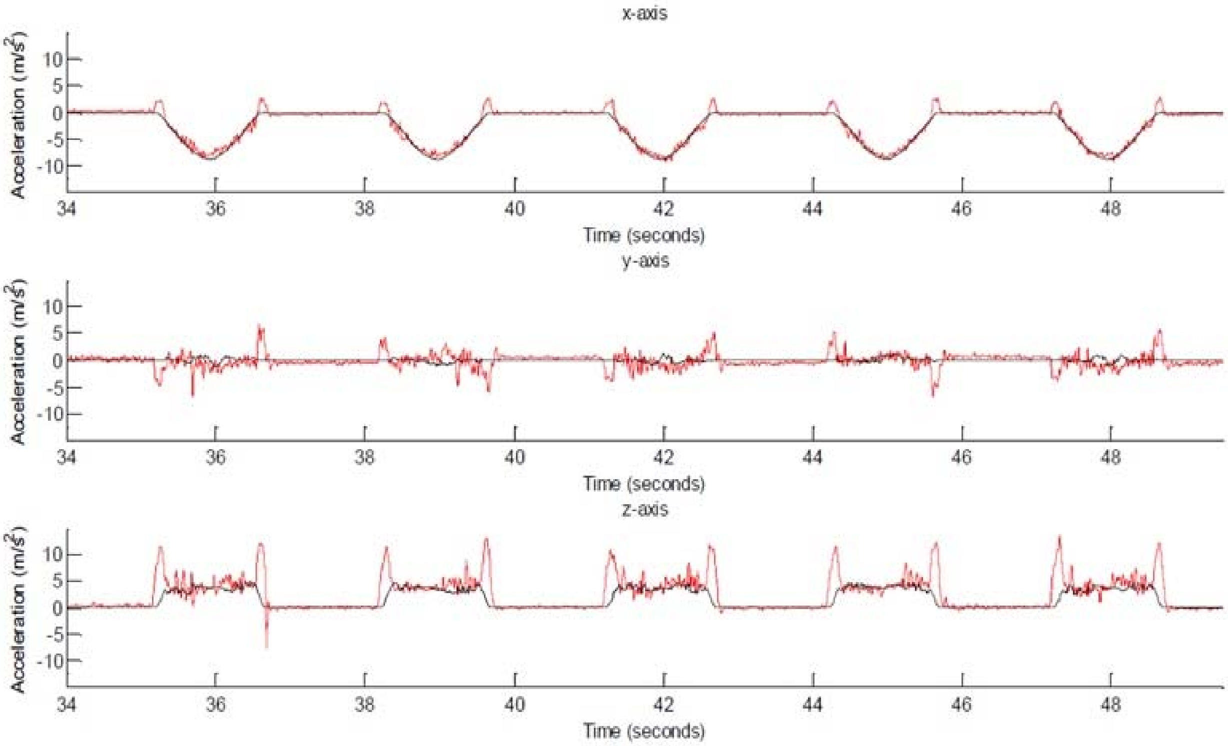
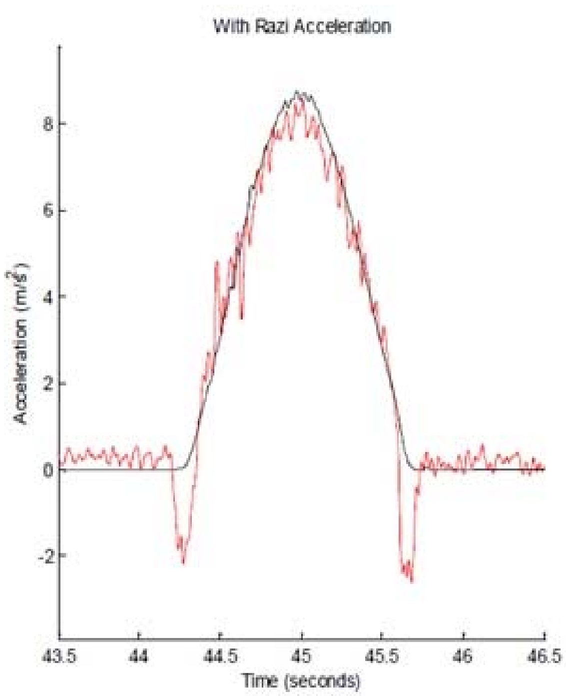


Figure-4. Comparison of the measured acceleration signal (red) and the acceleration calculated by Equation (7) (black).



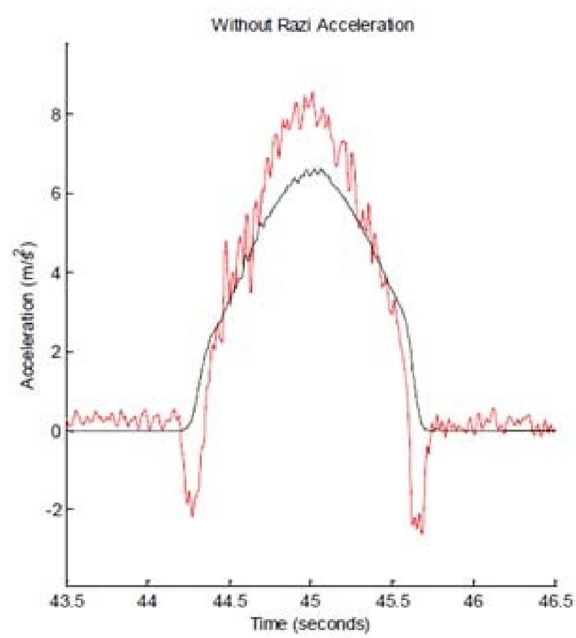


Figure-5. Theory of derivative kinematics (black) compared against the measured acceleration of the rigid body in motion (red). The above figure shows the acceleration calculated with the inclusion of the Razi acceleration term and the bottom figure shows the effect when it is ignored.

The jerk consideration is not included in our acceleration model in Equation (5). The plots show the agreement of the plotted acceleration model with the IMU-recorded acceleration signal. This shows that the general expression for acceleration for multiple rotating coordinate frames in the previous section can predict the acceleration acting on a rigid body in compound/enclosed rotation motion.

The Razi acceleration is expected to act parallel to the axis of rotation of the F_2-frame (out-of-plane direction) according to the cross product sequence in the Razi term $({}^2_0\boldsymbol{\omega}_1 \times {}^2_1\boldsymbol{\omega}_2) \times {}^2\mathbf{r}$**.** The acceleration in this direction is also contributed by the centripetal acceleration caused by the rotation of the F_1-frame. To show, more explicitly, the influence of the Razi acceleration in the simulated motion, we omit the Razi term in our calculation and show in Figure-5 the difference in acceleration magnitude. It is apparent that the omission of the Razi term here has shown a significant error in predicting the acceleration caused by multiple simultaneous rotations. In this test, there is about 2 m/s^2 difference in predicting the maximum magnitude. This error can be considered huge since the maximum acceleration in the *x*-direction is around 8 m/s^2. It is stressed here that the numbers andpercentages of error given are not typical for all cases; however the subtle inertial acceleration caused by multiple relative motion, such as the Razi acceleration, must be carefully analysed before any simplification is made in order to avoid seriousmodelling and calculation errors.

CONCLUSIONS

The general expression for the second-order derivative of a position and angular velocity vector measured in a system of n relatively rotating coordinate frames is presented. It can be used to develop a procedure to generate equations of motion of a system that involves rotation about multiple axes. The expression presents explicitly the inertial acceleration in compound rotation motion without neglecting the small acceleration due to the relative rotation. The focus of this study is on the importance and effectiveness of a particular acceleration term called the Razi acceleration. The magnitude and sensitivity of the Razi acceleration, in a system with two simultaneous rotations, depends on the two angular velocities, the angle between the angular velocity vectors, and the radii of rotations. An experiment using a multi-axis robotic arm is set up to show the appearance of the Razi acceleration. The magnitude and direction of the Razi acceleration is demonstrated by comparing the plotted acceleration equation and the acceleration signals, which shows a good agreement between the theoretical and experimental data. It is also demonstrated in this example that the excluding the Razi acceleration term may lead to a serious error in predicting the acceleration acting on a complex rotating system.

REFERENCES

[1] Hirschberg M. H. and A. Mendelson. 1958. Analysis of stresses and deflections in a disk subjected to gyroscopic forces. National Advisory Committee for Aeronautics. Lewis Flight Propulsion Laboratory, Cleveland, Ohio. TN-4218.

[2] Barlow J. 1976. Inertia loading in finite element analysis of structures subject to compound motion. International Journal of Numerical Methods in Engineering, Vol. 10, No. 1, pp. 197-209.

[3] Sakata M., Kimura K., Park S.K. and Ohnabe H. 1989. Vibration of bladed flexible rotor due to gyroscopic moment. Journal of Sound and Vibration, Vol. 131, No. 3, pp. 417-430.

[4] Kane T. R., Likins P. W. and Levinson D. A. 1983. Spacecraft dynamics. New York, USA. McGraw-Hill.

[5] Sherby T. A. and Chmielewski J. F. 1968. Generalized vector derivatives for systems with multiple relative motion. Journal of Applied Mechanics, Vol. 35, No. 1, pp. 20-24.

[6] Jazar R. N. 2012. Derivative and coordinate frames. Nonlinear Engineering, Vol. 1, No. (1-2), pp. 25-34.

[7] Beatty M.F. 1986. Principles of engineering mechanics, i. kinematics – geometry of motion. Plenum Press, New York, USA.