REACTION WHEEL CONFIGURATIONS FOR HIGH AND MIDDLE INCLINATION ORBITS

Zuliana Ismail and Renuganth Varatharajoo
Department of Aerospace Engineering, Universiti Putra Malaysia, Malaysia
E-Mail: zuliana.ismail@gmail.com

ABSTRACT

The purpose of this paper is to identify the low-power Reaction Wheel (RW) configuration for a 3-axis satellite attitude control at high and middle inclination orbits. All of the proposed RW configurations are evaluated through the numerical simulations with respect to an identical reference mission. The simulations are tested for two different orbit positions; first, at a high inclination (e.g., 83°), second, at a middle inclination (e.g., 53°). All configurations are analysed in terms of their total torques and attitude performances. The stable attitude accuracies (≈0.001°) are achieved in all the configurations either at 83° or 53° inclinations. Results also revealed that the change of orbit inclination slightly influences the determination of the low-power RW configurations. This research provides a quick summary on a possible low-power arrangement of reaction wheels onboard a small satellite.

Keywords: reaction wheel, Satellite attitude control, control torque.

INTRODUCTION

Most of the sophisticated satellite missions rely on the use of Reaction Wheels (RWs) for precision satellite attitude controls [1]. RWs act as a source of action-reaction energy to generate the control torques. When a satellite rotates one way due to the disturbance torques (i.e., solar pressure, aerodynamic drag, etc.), the RWs will be counter-rotated to produce the same magnitude reaction torque in order to correct the attitude. Practically, a set of two, three or four RWs configuration with the suitable attitude controllers are employed for a full 3-axis satellite attitude control as discussed by Kim et al. [2]. Therefore, the 3-axis satellite attitude control using the RWs is indeed an important subject of research [3-4].
For a small satellite, it is rather challenging to adopt multiple RWs due to the power limitations problem. There are a number of researches which investigate the issue of minimizing the power consumed by RWs onboard small satellite such as RWs miniaturization [5-6] and controller optimization [7]. A torque efficient attitude control system is indeed desirable in many recent innovative space systems [8-12]. Basically, the total torque as well as the power consumed by the RWs can be lowered by particularly arranging the RWs' orientation on-board the satellite[13]. However, the available literature on wheel configuration issues proves that the results are difficult to compare and adopt as they were all tested with different parameters and conditions [14-15]. Moreover, the earlier study was only focused on a single configuration optimization without the inclination variations [13]or was limited to three RWs' configuration [16].

In contrast, in this work, all the possible RW configurations for a 3-axis satellite attitude control are introduced and tested under an identical reference mission with different inclinations, making them unique in comparison to all the existing works. This study is done for two configurations, the first for three RWs and the second for four RWs. Firstly, the standard mathematical models of the satellite attitude control system with RWs are described, whereby the standard PD-type (proportional-derivative) controller is adopted.

The suitable RW orientation that produces a minimum total control torque can be identified by estimating the total torques required to maintain the 3-axis satellite attitude control. The simulations are performed for two different inclinations which are the high orbit inclination (e.g., 83°) and the middle orbit inclination (e.g., 53°). Note that these inclinations are proposed as examples to facilitate the analysis herein.

METHODOLOGY

Attitude dynamics and kinematics

Normally, the satellite's equations of motion are linearized when the Euler's angles are assumed to be small. According to this work, the satellite's equations of motion are not linearized in order to ensure the system is applicable even for the large Euler's angles. The non-linear satellite's dynamic equation with RWs can be written as [2]:

$$\dot{\omega}_x = \frac{T_x - \left(I_y - I_z\right)\omega_z\omega_y + h_{wz}\omega_y - h_{wy}\omega_z}{I_x}$$
$$\dot{\omega}_y = \frac{T_y - \left(I_z - I_x\right)\omega_x\omega_z + h_{wx}\omega_z - h_{wz}\omega_x}{I_y}$$
$$\dot{\omega}_z = \frac{T_z - \left(I_x - I_y\right)\omega_y\omega_x + h_{wy}\omega_x - h_{wx}\omega_y}{I_z} \quad (1)$$

Assuming that the external torques consist of the aerodynamic torques and solar torques, thus the total disturbance torques may be written as:

$$\mathbf{T}_{\mathrm{d}} = \mathbf{T}_{\mathrm{aero}} + \mathbf{T}_{\mathrm{solar}} \quad (2)$$

where each of them are modelled as the sum of constant and harmonic quantities as follows:

$$\mathbf{T}_\mathrm{d} = \mathbf{T}_\mathrm{constant} + \mathbf{T}_\mathrm{harmonic} \cdot sin(\omega_o t) \tag{3}$$

For attitude kinematics, quaternion method is adopted because of its numerical advantages and avoidance of singularities. Thus, the derivatives of the Euler parameters can be updated using the kinematics equation as follows [17].

$$\dot{\mathbf{q}} = \frac{1}{2}\Omega(\boldsymbol{\omega}_{LVLH/B})\mathbf{q} \tag{4}$$

where $\mathbf{q}$ is an attitude quaternion that represents the attitude of the satellite relative to the local-vertical-local-horizontal (LVLH) frame and Ω is the skew symmetric matrix.

Reaction wheel control strategy

The block diagram of the RW control strategy with a PD controller is presented in Figure-1.

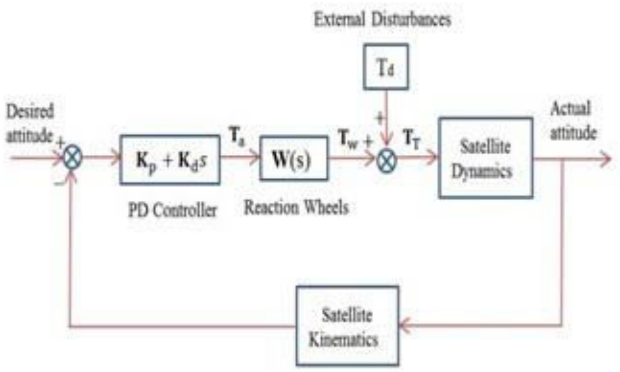


Figure-1. Block diagram for satellite attitude control using reaction wheels.

As the command control torques depends on the quaternion and angular rate errors, the control law can be represented as [2]:

$$\mathbf{T}_\mathrm{a} = 2\mathbf{K}_\mathrm{p}\boldsymbol{q}_e q_{e4} + \mathbf{K}_\mathrm{d}\boldsymbol{\omega}_e \tag{5}$$

where the error quaternion $\boldsymbol{q}_e q_{e4}$ is the quaternion difference between the reference quaternion $\boldsymbol{q}_r$ and the current quaternion $\boldsymbol{q}_c$. Whereas, $\boldsymbol{\omega}_e$ is the angular rate error.

From Equation (5), it is found that the system is based on the second order dynamic system, thus the equations to find the proportional and derivative gains are $\mathbf{K}_\mathrm{p} = \omega_n^2\mathbf{I}$ and $\mathbf{K}_\mathrm{d} = 2\xi\omega_n\mathbf{I}$, respectively. These control gains are the functions of dynamic characteristics, i.e., the natural frequency ω_n and the damping ratio ξ.

In addition, the derivation of Euler angles error $[\phi, \ \theta, \ \psi]^T$ from the attitude quaternion error is as follows [18].

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arctan\left(\dfrac{2(q_1q_4 + q_2q_3)}{1-2(q_1^2 + q_2^2)}\right) \\ \arcsin\left(2(q_4q_2 - q_3q_1)\right) \\ \arctan\left(\dfrac{2(q_4q_3 + q_1q_2)}{1-2(q_2^2 + q_3^2)}\right) \end{bmatrix} \tag{6}$$

From Figure-1, considering that a maximum of four RWs are installed onboard the satellite, the applied 3-axis control torque $\mathbf{T}_\mathrm{W}$ from the RWs can be calculated as:

$$\mathbf{T}_\mathrm{w} = [\mathbf{A}_\mathrm{w}] \cdot \mathbf{T}_\mathrm{c} \tag{7}$$

where $\mathbf{A}_\mathrm{W}$ is the RW configuration matrix and $\mathbf{T}_\mathrm{c}$ is the wheel control torque.

For example, if there are three RWs aligned along the primary axis of the satellite and a redundant wheel tilted at equal distances from the others, $\mathbf{A}_\mathrm{W}$ can be defined as

$$\mathbf{A}_\mathrm{w} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{8}$$

In order to determine the magnitudes of wheel control torque $\mathbf{T}_\mathrm{c}$, the pseudo-inverse of the RW configuration matrix $[\mathbf{A}_\mathrm{W}]^{-1}$ can be multiplied with the commanded control torques $\mathbf{T}_\mathrm{a}$ obtained in Eq. (5).

$$\mathbf{T}_\mathrm{c} = [\mathbf{A}_\mathrm{w}]^{-1} \cdot \mathbf{T}_\mathrm{a} \tag{9}$$

The configuration matrix in Eq. (8) is representing just one of the possible RW orientations that can be implemented on-board a satellite. Actually, there are more possible orientations available without degrading the 3-axis attitude control. The different configurations of the RW that are proposed in this study both for three RWs set and four RWs are summarized in the Figure-2 (a) – (j) and Figure-3, respectively. Among those configurations, the suitable RW orientation that consumes a minimum current can be identified by calculating their total minimum torques required to maintain the 3-axis satellite attitude control.

Numerical simulations

In order to simulate the satellite attitude control performance, a reference mission is proposed as in Tables 1 and 2. All the RWs are assumed to be identical and their configurations will be simulated using the reference mission. Thus, the governing equations and the reference missions' parameters are implemented in the Matlab Simulink^{TM} codes. Then, numerical treatments are performed, which allow an assessment of the each configuration's merit.

Table-1. Orbit parameters.

Parameters	Values	
Inclination, i	Position 1: 83°	Position 2: 53°
Right Ascension, Ω	15.7°	
Altitude, **h**	470 km	
Simulation time, **t**	5640 s (1 orbit)	

Table-2. Satellite specifications.

Parameter	Value	Unit
Moments of inertia matrix	$I = diag[4.2 \quad 4.4 \quad 4.2]$	kg. m^2
Proportional gain	$Kp = [0.672 \quad 0.704 \quad 0.672]$	Nm/rad
Derivative gain	$Kd = [2.352 \quad 2.464 \quad 2.352]$	Nms/rad
Maximum External Disturbance Torque	$Td_x = 8\times10^{-5}(\sin\omega_o t)$ $Td_y = 8\times10^{-6} + 8\times10^{-5}(\sin\omega_o t) + 5\times10^{-5}(\cos\omega_o t)$ $Td_z = 8\times10^{-6} + 5\times10^{-5}(\cos\omega_o t)$	Nm
Initial Attitude Errors	$[\phi, \quad \theta, \quad \psi] = \pm5^o$	deg

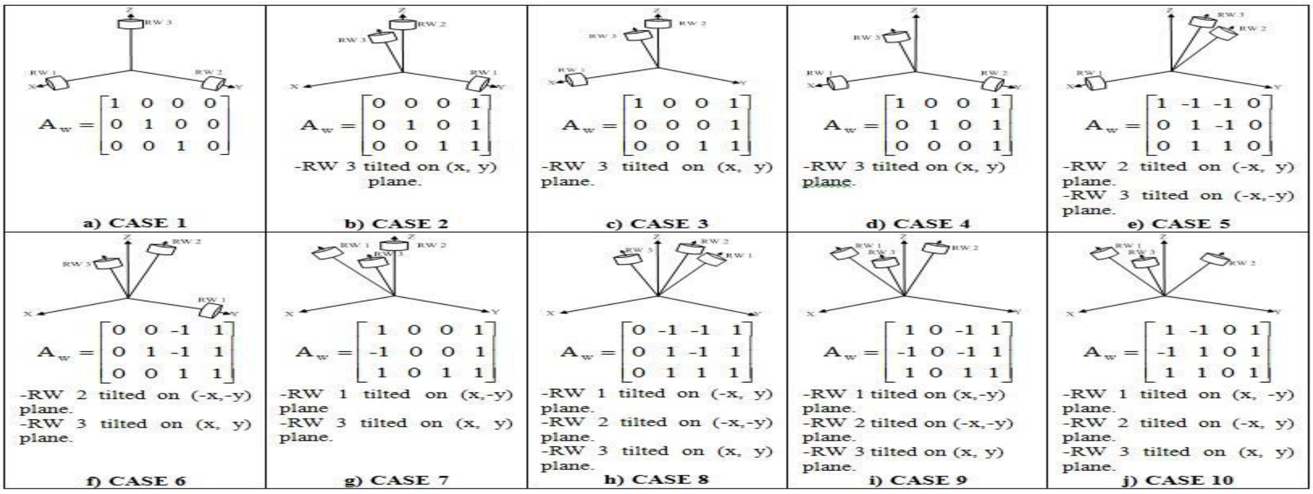


Figure-2. (a) – (j). Configuration matrix of 3 reaction wheels.

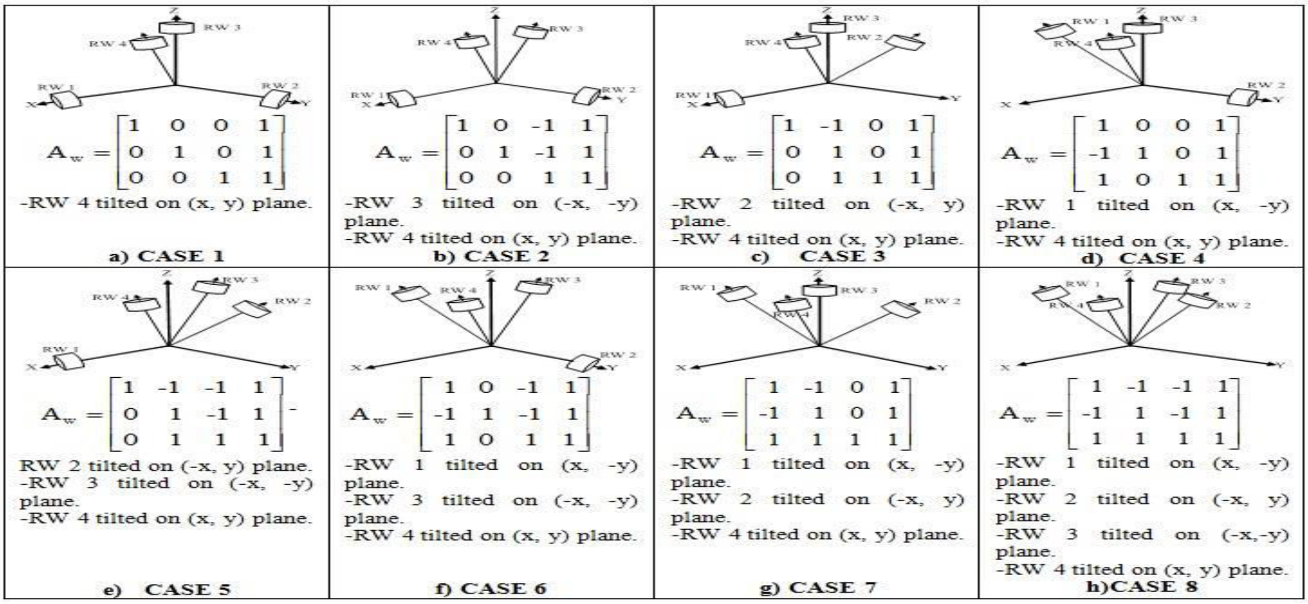


Figure-3. (a) – (h). Configuration matrix of 4 reaction wheels.

RESULTS

Performance analysis

The simulation result pertaining to each configuration is carefully evaluated in terms of their torques and attitude performances. The RW configuration that has a minimum total control torque level is identified in order to determine a minimum power intake configuration. The performance analysis for all the test cases corresponding to three and four RWs for the simulation at inclinations of 83° and 53° are summarized in Tables 3 and 4, respectively.

For the three RWs at 83° of inclination, the total minimum torque is in Case 5, $T_{w_total} = 1.25 \times 10^{-5}$Nm, where one wheel is aligned along x axis and the other two wheels are tilted; see Figure 2 (e). While, for three RWs at 53° of inclination, the total minimum torque is in Case 7$T_{w_total} = 1.08 \times 10^{-5}$Nm, where one wheel is aligned along z axis and the others are tilted; see Figure-2(g).

For the four RWs both at 83° and 53° of inclination, the total minimum torque is in Case 3, where two wheels are aligned along x and z axes and the other two wheels are tilted, see Figure-3 (c). The total minimum torque for inclination of 83° and 53° are $T_{w_total} = 8.57 \times 10^{-6}$Nmand $T_{w_total} = 9.20 \times 10^{-6}$Nm, respectively. These minimum torque configurations correspond to the minimum power intake configurations as well.

By comparing the results at these two different inclinations (i.e.; 83° and 53°), the total minimum torque for four RWs configuration is retained in the same case as in Case 3. However, the total minimum torque for three RWs configuration resulting in the different cases, whereby the best cases are in Case 5 and Case 7 for 83° and 53° of inclinations, respectively. These results convey that the change of orbit inclinations, actually, influences the satellite's attitude control performances, i.e., the generation of attitude control torques and pointing accuracies. Looking at the pointing performances, all the configurations have a similar total attitude pointing accuracy of about 0.001°. Nevertheless, the best attitude pointing (<0.001°) is achieved in Case 1 of three RWs at 83° of inclination. However, this configuration fundamentally inherits a catastrophic failure in the case of one wheel failure compared to all the other configurations at 83°; and therefore, it is not a suitable configuration for satellite missions [17].

Attitude control performances

Indeed, the full 3-axis satellite attitude controls are achieved in all the test cases; thus, only each case of the three and four RWs configuration attitude performance plot is shown for each of 83° and 53° of inclinations. Figures-4 (a)-(b) show the satellite attitude performances at 83° of inclination for Case 5 of the three RWs configuration and Case 3 of the four RWs configuration. Whereas Figures-4 (c)-(d) represent the satellite attitude performances at 53° of inclination for Case 7 of the three RWs configuration and Case 3 of the four RWs configuration.

Table-3. Performance analysis for 83° of inclination.

3 Reaction Wheels						
Case	Reaction Wheel Torques [Nm]	Attitude Errors [deg]		Case	Reaction Wheel Torques [Nm]	Attitude Errors [deg]
Case 1	$\lvert T_{wx}\rvert_{max} = 3.3\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 1.3\times10^{-5}$ $\lvert T_{wz}\rvert_{max} = 9.3\times10^{-6}$ $T_{w\_total} = 2.56\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0002$ $\lvert\theta\rvert_{max} = 0.0001$ $\lvert\psi\rvert_{max} = 0.0001$ Total < 0.001		Case 6	$\lvert T_{wx}\rvert_{max} = 2.6\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 1.1\times10^{-5}$ $\lvert T_{wz}\rvert_{max} = 7.2\times10^{-6}$ $T_{w\_total} = 2.10\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0002$ $\lvert\theta\rvert_{max} = 0.0002$ $\lvert\psi\rvert_{max} = 0.0003$ Total ≈ 0.001
Case 2	$\lvert T_{wx}\rvert_{max} = 5.6\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 5.4\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 7.0\times10^{-6}$ $T_{w\_total} = 1.80\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0003$ $\lvert\theta\rvert_{max} = 0.0004$ $\lvert\psi\rvert_{max} = 0.0004$ Total ≈ 0.001		Case 7	$\lvert T_{wx}\rvert_{max} = 1.8\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 5.1\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.2\times10^{-5}$ $T_{w\_total} = 1.85\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0002$ $\lvert\theta\rvert_{max} = 0.0005$ $\lvert\psi\rvert_{max} = 0.0001$ Total ≈ 0.001
Case 3	$\lvert T_{wx}\rvert_{max} = 2.5\times10^{-8}$ $\lvert T_{wy}\rvert_{max} = 6.5\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 8.4\times10^{-6}$ $T_{w\_total} = 1.49\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0004$ $\lvert\psi\rvert_{max} = 0.0002$ Total ≈ 0.001		Case 8	$\lvert T_{wx}\rvert_{max} = 3.4\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 6.4\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 9.1\times10^{-6}$ $T_{w\_total} = 1.89\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0003$ $\lvert\psi\rvert_{max} = 0.0010$ Total ≈ 0.001

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Case 4	$\left\|T_{wx}\right\|_{max} = 3.5\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 5.0\times10^{-6}$ $\left\|T_{wz}\right\|_{max} = 9.5\times10^{-6}$ $T_{w\_total} = 1.80\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0002$ $\left\|\theta\right\|_{max} = 0.0005$ $\left\|\psi\right\|_{max} = 0.0002$ $Total \approx 0.001$	Case 9	$\left\|T_{wx}\right\|_{max} = 5.20\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 3.10\times10^{-6}$ $\left\|T_{wz}\right\|_{max} = 1.96\times10^{-5}$ $T_{w\_total} = 2.79\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0004$ $\left\|\theta\right\|_{max} = 0.0006$ $\left\|\psi\right\|_{max} = 0.0002$ $Total \approx 0.001$
Case 5	$\left\|T_{wx}\right\|_{max} = \mathbf{6.8\times10^{-6}}$ $\left\|T_{wy}\right\|_{max} = \mathbf{2.6\times10^{-6}}$ $\left\|T_{wz}\right\|_{max} = \mathbf{3.1\times10^{-6}}$ $T_{w\_total} = \mathbf{1.25\times10^{-5}}$	$\left\|\phi\right\|_{max} = 0.0001$ $\left\|\theta\right\|_{max} = 0.0004$ $\left\|\psi\right\|_{max} = 0.0009$ $Total \approx 0.001$	Case 10	$\left\|T_{wx}\right\|_{max} = 6.2\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 9.8\times10^{-6}$ $\left\|T_{wz}\right\|_{max} = 2.2\times10^{-6}$ $T_{w\_total} = 1.82\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0001$ $\left\|\theta\right\|_{max} = 0.0001$ $\left\|\psi\right\|_{max} = 0.0008$ $Total \approx 0.001$

Table-4. Performance analysis for 83° of inclination.

4 Reaction Wheels					
Case	Reaction Wheel Torques [Nm]	Attitude Errors [deg]	Case	Reaction Wheel Torques [Nm]	Attitude Errors [deg]
Case 1	$\left\|T_{wx}\right\|_{max} = 9.1\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 6.9\times10^{-7}$ $\left\|T_{wz}\right\|_{max} = 3.7\times10^{-6}$ $T_{w\_total} = 1.35\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0004$ $\left\|\theta\right\|_{max} = 0.0007$ $\left\|\psi\right\|_{max} = 0.0001$ $Total \approx 0.001$	Case 5	$\left\|T_{wx}\right\|_{max} = 9.6\times10^{-7}$ $\left\|T_{wy}\right\|_{max} = 9.2\times10^{-6}$ $\left\|T_{wz}\right\|_{max} = 9.2\times10^{-6}$ $T_{w\_total} = 1.94\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0002$ $\left\|\theta\right\|_{max} = 0.0003$ $\left\|\psi\right\|_{max} = 0.0002$ $Total \approx 0.001$
Case 2	$\left\|T_{wx}\right\|_{max} = 3.8\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 6.0\times10^{-6}$ $\left\|T_{wz}\right\|_{max} = 2.9\times10^{-6}$ $T_{w\_total} = 1.27\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0001$ $\left\|\theta\right\|_{max} = 0.0004$ $\left\|\psi\right\|_{max} = 0.0009$ $Total \approx 0.001$	Case 6	$\left\|T_{wx}\right\|_{max} = 6.0\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 1.54\times10^{-5}$ $\left\|T_{wz}\right\|_{max} = 1.34\times10^{-5}$ $T_{w\_total} = 3.48\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0001$ $\left\|\theta\right\|_{max} = 0.0002$ $\left\|\psi\right\|_{max} = 0.0010$ $Total \approx 0.001$
Case 3	$\left\|T_{wx}\right\|_{max} = \mathbf{4.1\times10^{-6}}$ $\left\|T_{wy}\right\|_{max} = \mathbf{3.8\times10^{-6}}$ $\left\|T_{wz}\right\|_{max} = \mathbf{6.4\times10^{-7}}$ $T_{w\_total} = \mathbf{8.57\times10^{-6}}$	$\left\|\phi\right\|_{max} = 0.0001$ $\left\|\theta\right\|_{max} = 0.0004$ $\left\|\psi\right\|_{max} = 0.0008$ $Total \approx 0.001$	Case 7	$\left\|T_{wx}\right\|_{max} = 3.84\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 3.6\times10^{-6}$ $\left\|T_{wz}\right\|_{max} = 1.24\times10^{-5}$ $T_{w\_total} = 1.99\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0002$ $\left\|\theta\right\|_{max} = 0.0005$ $\left\|\psi\right\|_{max} = 0.0001$ $Total \approx 0.001$
Case 4	$\left\|T_{wx}\right\|_{max} = 3.8\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 4.9\times10^{-6}$ $\left\|T_{wz}\right\|_{max} = 5.1\times10^{-6}$ $T_{w\_total} = 1.38\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0001$ $\left\|\theta\right\|_{max} = 0.0004$ $\left\|\psi\right\|_{max} = 0.0005$ $Total \approx 0.001$	Case 8	$\left\|T_{wx}\right\|_{max} = 1.7\times10^{-6}$ $\left\|T_{wy}\right\|_{max} = 1.0\times10^{-5}$ $\left\|T_{wz}\right\|_{max} = 6.7\times10^{-6}$ $T_{w\_total} = 1.84\times10^{-5}$	$\left\|\phi\right\|_{max} = 0.0002$ $\left\|\theta\right\|_{max} = 0.0007$ $\left\|\psi\right\|_{max} = 0.0001$ $Total \approx 0.001$

 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Table-5. Performance analysis for 53° of inclination.

3 Reaction Wheels						
Case	**Reaction Wheel Torques [Nm]**	**Attitude Errors [deg]**		**Case**	**Reaction Wheel Torques [Nm]**	**Attitude Errors [deg]**
Case 1	$\lvert T_{wx}\rvert_{max} = 7.95\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 7.4\times10^{-7}$ $\lvert T_{wz}\rvert_{max} = 1.2\times10^{-5}$ $T_{w\_total} = 2.07\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0004$ $\lvert\theta\rvert_{max} = 0.0008$ $\lvert\psi\rvert_{max} = 0.0001$ $Total \approx 0.001$		**Case 6**	$\lvert T_{wx}\rvert_{max} = 4.25\times10^{-7}$ $\lvert T_{wy}\rvert_{max} = 5.95\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 5.1\times10^{-6}$ $T_{w\_total} = 1.15\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0002$ $\lvert\theta\rvert_{max} = 0.0005$ $\lvert\psi\rvert_{max} = 0.0004$ $Total \approx 0.001$
Case 2	$\lvert T_{wx}\rvert_{max} = 2.0\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 1.0\times10^{-5}$ $\lvert T_{wz}\rvert_{max} = 5.6\times10^{-6}$ $T_{w\_total} = 1.76\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0002$ $\lvert\theta\rvert_{max} = 0.0003$ $\lvert\psi\rvert_{max} = 0.0003$ $Total \approx 0.001$		**Case 7**	$\mathbf{\lvert T_{wx}\rvert_{max} = 3.3\times10^{-6}}$ $\mathbf{\lvert T_{wy}\rvert_{max} = 4.4\times10^{-6}}$ $\mathbf{\lvert T_{wz}\rvert_{max} = 3.1\times10^{-6}}$ $\mathbf{T_{w\_total} = 1.08\times10^{-5}}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0005$ $\lvert\psi\rvert_{max} = 0.0005$ $Total \approx 0.001$
Case 3	$\lvert T_{wx}\rvert_{max} = 2.3\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 4.2\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.5\times10^{-5}$ $T_{w\_total} = 2.15\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0007$ $\lvert\psi\rvert_{max} = 0.0001$ $Total \approx 0.001$		**Case 8**	$\lvert T_{wx}\rvert_{max} = 6.0\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 1.2\times10^{-5}$ $\lvert T_{wz}\rvert_{max} = 6.1\times10^{-6}$ $T_{w\_total} = 2.41\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0004$ $\lvert\theta\rvert_{max} = 0.0002$ $\lvert\psi\rvert_{max} = 0.0002$ $Total \approx 0.001$
Case 4	$\lvert T_{wx}\rvert_{max} = 1.7\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 4.3\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.4\times10^{-5}$ $T_{w\_total} = 2.0\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0007$ $\lvert\psi\rvert_{max} = 0.0001$ $Total \approx 0.001$		**Case 9**	$\lvert T_{wx}\rvert_{max} = 4.2\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 1.4\times10^{-5}$ $\lvert T_{wz}\rvert_{max} = 1.1\times10^{-6}$ $T_{w\_total} = 1.93\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0004$ $\lvert\theta\rvert_{max} = 0.0001$ $\lvert\psi\rvert_{max} = 0.0004$ $Total \approx 0.001$
Case 5	$\lvert T_{wx}\rvert_{max} = 6.0\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 3.3\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 3.0\times10^{-6}$ $T_{w\_total} = 1.23\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0004$ $\lvert\psi\rvert_{max} = 0.0009$ $Total \approx 0.001$		**Case 10**	$\lvert T_{wx}\rvert_{max} = 3.5\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 7.1\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.6\times10^{-5}$ $T_{w\_total} = 2.66\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0007$ $\lvert\psi\rvert_{max} = 0.0003$ $Total \approx 0.001$

Table-6. Performance analysis for 53° of inclination.

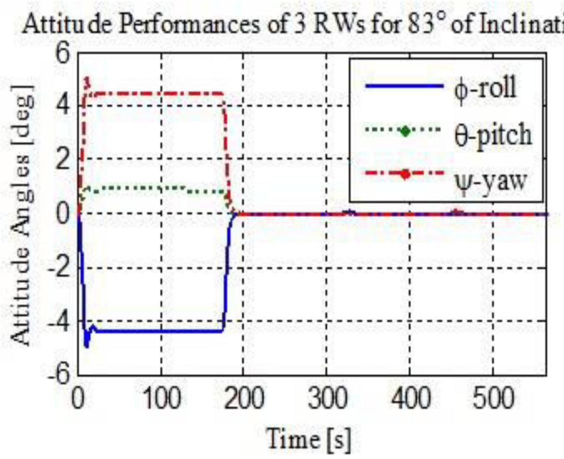
4 Reaction Wheels						
Case	**Reaction Wheel Torques [Nm]**	**Attitude Errors [deg]**		**Case**	**Reaction Wheel Torques [Nm]**	**Attitude Errors [deg]**
Case 1	$\lvert T_{wx}\rvert_{max} = 1.4\times10^{-5}$ $\lvert T_{wy}\rvert_{max} = 8.9\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 7.9\times10^{-6}$ $T_{w\_total} = 3.08\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0007$ $\lvert\theta\rvert_{max} = 0.0009$ $\lvert\psi\rvert_{max} = 0.0006$ $Total \approx 0.001$		**Case 5**	$\lvert T_{wx}\rvert_{max} = 5.76\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 3.87\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.42\times10^{-5}$ $T_{w\_total} = 2.38\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0003$ $\lvert\theta\rvert_{max} = 0.0006$ $\lvert\psi\rvert_{max} = 0.0001$ $Total \approx 0.001$
Case 2	$\lvert T_{wx}\rvert_{max} = 5.6\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 3.1\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.3\times10^{-6}$ $T_{w\_total} = 1.00\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0006$ $\lvert\psi\rvert_{max} = 0.0008$ $Total \approx 0.001$		**Case 6**	$\lvert T_{wx}\rvert_{max} = 1.67\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 1.08\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 2.77\times10^{-5}$ $T_{w\_total} = 3.05\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0003$ $\lvert\theta\rvert_{max} = 0.0009$ $\lvert\psi\rvert_{max} = 0.0006$ $Total \approx 0.001$
Case 3	$\mathbf{\lvert T_{wx}\rvert_{max} = 1.8\times10^{-6}}$ $\mathbf{\lvert T_{wy}\rvert_{max} = 4.5\times10^{-6}}$ $\mathbf{\lvert T_{wz}\rvert_{max} = 2.9\times10^{-6}}$ $\mathbf{T_{w\_total} = 9.20\times10^{-6}}$	$\lvert\phi\rvert_{max} = 0.0001$ $\lvert\theta\rvert_{max} = 0.0005$ $\lvert\psi\rvert_{max} = 0.0005$ $Total \approx 0.001$		**Case 7**	$\lvert T_{wx}\rvert_{max} = 1.44\times10^{-5}$ $\lvert T_{wy}\rvert_{max} = 1.03\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.39\times10^{-5}$ $T_{w\_total} = 2.93\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0007$ $\lvert\theta\rvert_{max} = 0.0009$ $\lvert\psi\rvert_{max} = 0.0003$ $Total \approx 0.001$
Case 4	$\lvert T_{wx}\rvert_{max} = 6.67\times10^{-6}$ $\lvert T_{wy}\rvert_{max} = 1.10\times10^{-5}$ $\lvert T_{wz}\rvert_{max} = 8.79\times10^{-6}$ $T_{w\_total} = 2.65\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0004$ $\lvert\theta\rvert_{max} = 0.0003$ $\lvert\psi\rvert_{max} = 0.0001$ $Total \approx 0.001$		**Case 8**	$\lvert T_{wx}\rvert_{max} = 1.71\times10^{-5}$ $\lvert T_{wy}\rvert_{max} = 1.83\times10^{-6}$ $\lvert T_{wz}\rvert_{max} = 1.94\times10^{-5}$ $T_{w\_total} = 3.83\times10^{-5}$	$\lvert\phi\rvert_{max} = 0.0009$ $\lvert\theta\rvert_{max} = 0.0008$ $\lvert\psi\rvert_{max} = 0.0001$ $Total \approx 0.001$

ARPN Journal of Engineering and Applied Sciences

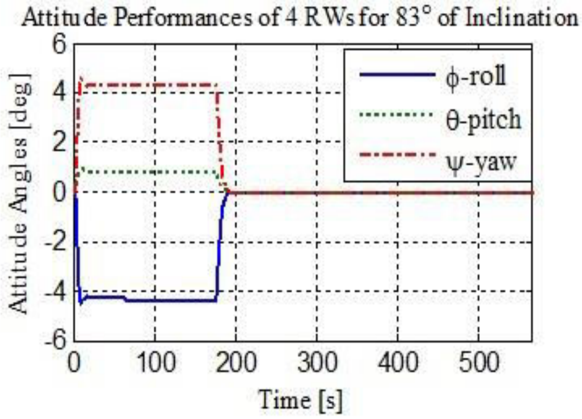
Figure 4 obviously shows that the roll-pitch-yaw attitudes converge to the stable pointing accuracies (≈0.001°) from the initial attitude errors (±5°) after about 0.035 orbits (197 s). In order to verify the effect of large initial attitude errors, three different initial errors (e.g., 30°, -45° and 60°) are introduced in a test case, i.e., Case 1 of the three RW configurations. The attitudes performances are shown in Figure-5 confirm that all the attitude errors converge to their steady state (≈0.001°) at about 197 s. Therefore, the attitude performances given in Figure 4 are valid even in the cases with large initial attitude errors.

CONCLUSIONS

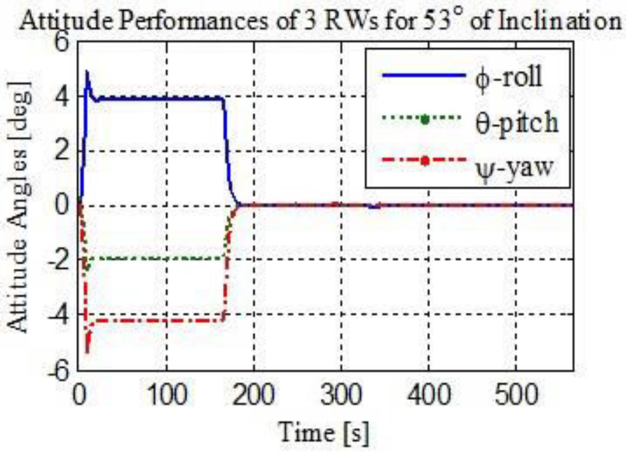
The satellite attitude control performances using three and four identical Reaction Wheels (RWs) for two different orbit positions have been successfully evaluated in this paper. The results show that a 3-axis satellite attitude control is achieved in all the proposed configurations even with large initial attitude errors both at 83° and 53° inclinations. Satellite attitude accuracies around 0.001° are achieved in all the configurations. For four RWs configuration, the configuration with a minimum total torque level resulted in the same case. However, for three RWs configuration, the configuration with a minimum torque level resulted in the different cases. These results showed that the satellite's attitude control performances are actually influenced by the satellite's orbit inclination and affected the determination of the low-power RWs configuration as well. In fact, numerical treatments in this work have revealed the RW configuration with a minimum total torque level based on the mission requirements. Therefore, the power intake for the attitude control system can be minimized by selecting the minimum torque configuration with respect to the defined reference mission as shown in this work.



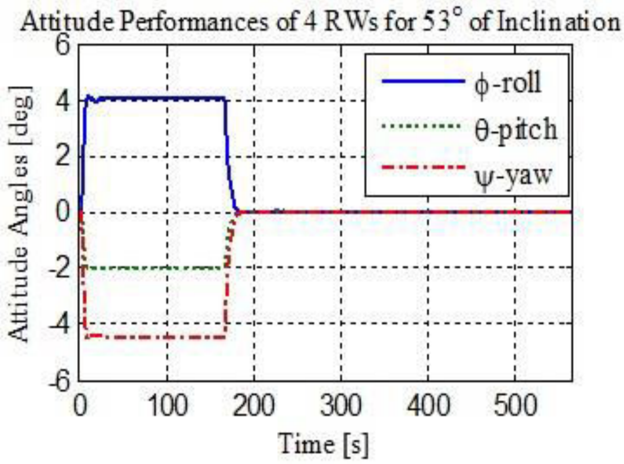
(a) Case 5 of 3 RWs configuration.



(b) Case 3 of 4 RWs configuration.



(c) Case 7 of 3 RWs configuration.



(d) Case 3 of 4 RWs configuration.

Figure-4. (a) – (d). Attitude performances.

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

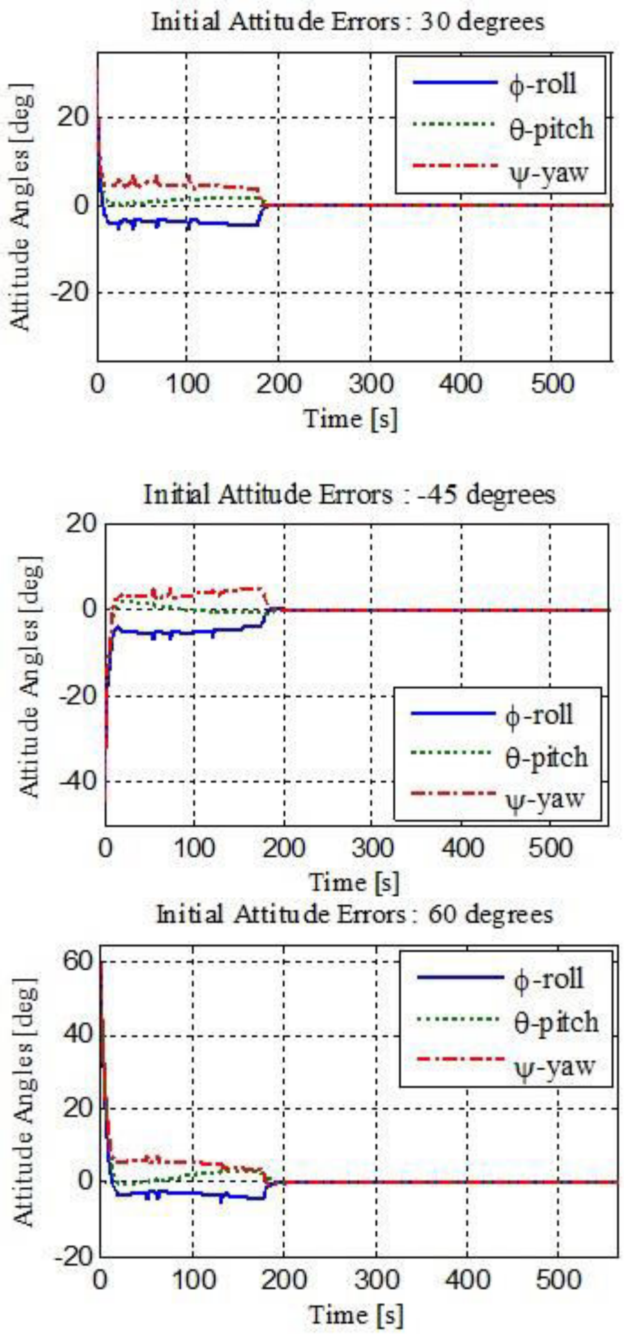


Figure-5. Attitude performances for different initial attitude errors.

REFERENCES

[1] F. F. Mobley, W. E. Radford and L. R. Kennedy. 1996. "MSX attitude determination and control hardware," Johns Hopkins APL Tech. Dig. (Applied Phys. Lab., Vol. 17, No. 2, pp. 153–159.

[2] B. Kim, H. Lee and S. Choi. 1996. "Three-Axis Reaction wheel Attitude Control System for Kitsat-3 Microsatellite," Sp. Technol. Res. Cent., Vol. 16, No. 5–6, pp. 291–296.

[3] S. Ge. 2006. "A Comparative Design of Satellite Attitude Control System with Reaction Wheel," First NASA/ESA Conf. Adapt. Hardw. Syst., pp. 359–364.

[4] S. S. Nudehi, U. Farooq, A. Alasty and J. Issa. 2008. "Satellite attitude control using three reaction wheels," 2008 Am. Control Conf., pp. 4850–4855, Jun.

[5] Y. Zhang, Y. Postrekhin, K. B. Ma and W.-K. Chu. 2002. "Reaction wheel with HTS bearings for mini-satellite attitude control," Superconductor Science and Technology, Vol. 15, No. 5. pp. 823–825.

[6] K. B. Ma, Y. Zhang, Y. Postrekhin and W. K. Chu. 2003. "HTS bearings for space applications: Reaction wheel with low power consumption for mini-satellites," in IEEE Transactions on Applied Superconductivity, Vol. 13, No. 2 II, pp. 2275–2278.

[7] S. Zhaowei, G. Yunhai, X. Guodong and H. Ping, "The combined control algorithm for large-angle maneuver of HITSAT-1 small satellite," Acta Astronaut., Vol. 54, No. 7, pp. 463–469.

[8] R. Varatharajoo. 2006. "Onboard errors of the combined energy and attitude control system," Acta Astronaut., Vol. 58, No. 11, pp. 561–563, Jun.

[9] R. Varatharajoo. 2006. "Operation for the combined energy and attitude control system," Aircr. Eng. Aerosp. Technol., Vol. 78, No. 6, pp. 495–501.

[10] R. Varatharajoo and F. Nizam. 2004. "Attitude Performance of the Spacecraft Combined Energy and Attitude Control System," J. Br. Interplanet. Soc., vol. 57, pp. 237–241.

[11] R. Varatharajoo and T. Ahmad. 2004. "Flywheel energy storage for spacecraft," Aircr. Eng. Aerosp. Technol., Vol. 76, pp. 384–390.

[12] E. Stoll, S. Jaekel, J. Katz, A. Saenz-Otero and R. Varatharajoo. 2012. "SPHERES interact-human-machine interaction aboard the International Space Station," J. F. Robot., Vol. 29, No. 4, pp. 554–575.

[13] Z. Ismail and R. Varatharajoo. 2010. "A study of reaction wheel configurations for a 3-axis satellite attitude control," Adv. Sp. Res., Vol. 45, No. 6, pp. 750–759, March.

[14] B. Lee, B. Lee, H. Oh, S. Lee and S. Rhee. 2005. "Time optimal attitude maneuver strategies for the agile spacecraft with reaction wheels and thrusters," J. Mech. Sci. Technol.

[15] J. Jin, S. Ko and C.-K. Ryoo. 2008. "Fault tolerant control for satellites with four reaction wheels,"

Control Eng. Pract., Vol. 16, No. 10, pp. 1250–1258, October.

[16] H. B. Hablani. 1994. “Sun-tracking commands and reaction wheel sizing with configuration optimization,” J. Guid. Control. Dyn., Vol. 17, No. 4, pp. 805–814, July.

[17] M. J. Sidi. 1997. Spacecraft dynamics and control: a practical engineering approach. Vol. 7. Cambridge university press.

[18] C. J. Van Beusekom, R. Lisowski, J. M. Fulton and C. Morand. 2003. “Three-axes attitude determination and control system design for low-cost micro-satellites,” in IEEE Aerospace Conference Proceedings, Vol. 6, pp. 2615–2628.