VOL. 10, NO 22, DECEMBER, 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

ON NEW SPATIAL FILTERS USING ABSTRACT CELLULAR COMPLEX

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ABSTRACT

In this paper, new-type Mean and Median spatial filters, to reduce various noises on images based on the notions of Abstract Cellular Complex (ACC) are proposed. The proposed Mean and Median filters using ACC are implemented through MATLAB. The experimental results showed that the performance of proposed filters are better than the standard Mean and Median filters in terms of the quantitative measures.

Keywords: spatial filter, mean filter, median filter, RMSE, PSNR, Kovalevsky's 2D cellular complex, smallest open neighborhood (SON), closure.

1. INTRODUCTION

Digital topology is the study of topological properties on digital images and it plays a very important role in digital image processing. Digital images are generally corrupted by several noise during image acquisition process. It causes degradation of image spatial resolution, loss of image details and distortion of important image features. Therefore it is essential to correct corrupted images that is, image denoising before using them in any application such as recognition, compression, tracing, etc. Image denoising filters are classified into linear and non-linear filters. There were many filters proposed for reducing noise but all of them are based on 4- or 8- neighborhood shape of pixels.

In digital image processing, the choice of good representation of an image is essential. Several approaches have been proposed to define a topologically consistent representation. Rosenfeld [20] introduced the adjacency relation among pixels to investigate the topological notions on digital image. Thus, considering the digital plane as a graph whose vertices are the pixels and an edge of the graph corresponds to each pair of adjacent pixels. Such a graph is called neighborhood graph. The most commonly used graphs are the 4- and 8- neighborhood graphs. But, basically the 4- and 8- connected neighborhoods have connectivity paradox and boundary contradictions which are discussed in [7]. Substantial efforts to overcome these difficulties have been made by [2, 5, 17, 23]. But, all these attempts provided unsatisfactory solutions.

Kovalevsky [8] introduced the concept of Abstract Cellular Complex (ACC) to study the topological notions of the digital images. The notion of ACC has no contradictions and paradox and it is very easy to develop a simple and comprehensible algorithms in ACC. This motivates us to propose some basic filters using ACC.

In this paper, new Mean filter and Median filter for noise reduction on various images using ACC are proposed. In chapter 2, basic notions on ACC which are useful for our study are recalled. In chapter 3, the process of transforming the digitalized image into an ACC are explained clearly. Further, a 2×2 kernel (window of size 2×2) is designed using the notion of smallest open neighborhood (SON) in ACC and a 3×3 kernel (window of size 3×3) is developed using the notions closure of a 2-cell and SON of the 0-cell in ACC. Using the designed windows new spatial filters namely Mean and Median filter with the window of size 2×2 and window of size 3×3 are proposed.

In Chapter 4, the proposed filters are applied to the different images with Gaussian noise and Salt and Pepper noise and their performances are analyzed and compared with existing standard Mean and Median filters through Root Mean Square Error (RMSE) value and Peak to Signal Noise Ratio (PSNR) values which are given by

$$RMSE=\sqrt{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(X(i,j)-P(i,j)\right)^2} \quad PSNR=20\log_{10}\left(\frac{255}{RMSE}\right)$$

Where M and N are dimensions of the image, X- original image and P- filtered image

The experimental results showed that the Mean and Median filters developed through window of size 2×2 are efficient in removing low density noises in the images and the performance of Mean and Median filters with window size 3×3 are better than the existing filters.

2. BASIC NOTIONS OF ACC

In this section, we recall some details about Abstract Cellular Complex.

Definition 2.1 [8]: An abstract cellular complex(ACC) $C = (E,B, dim)$ is a set E of abstract elements provided with an anti-symmetric, irreflexive, and transitive binary relation $B \subset E \times E$ called the bounding relation, and with a dimension function $dim : E \rightarrow I$ from

E into the set *I* of non-negative integers such that $\dim(e') < \dim(e'')$ for all pairs $(e', e'') \in B$.

Definition 2.2[8]: A subcomplex *S* of *C* is called open in *C* if for every element e' of *S* all elements of *C* which are bounded by e' are also in *S*.

Definition 2.3[8]: The smallest subset of a set *S* which contains a given cell $c \in S$ and is open in *S* is called smallest neighborhood of *c* relative to *S* and is denoted by *SON(c,S)*.

Definition 2.4 [8]: The smallest subset of a set *S* which contains a given cell $c \in S$ and is closed in *S* is called the closure of *c* relative to *S* and denoted by *Cl(c, S)*.

Definition 2.5 [8]: Coordinate assignment rule: Each pixel F of a 2D image receives one 0-cell assigned to its "own cell". This is the 0-cell P lying in the corner of F which is the nearest to the origin of the coordinates. Two 1-cells incident to F and P are also declared to be "own cell" of F. Thus each pixel gets three own cells of lower dimensions. All own cells of F are assigned the same standard coordinates as F. They can be distinguished by their type.

3. DESIGN AND IMPLEMENTATION OF FILTERS IN ACC

In this chapter, the process of converting the digital image to ACC is explained and the new spatial Mean filter and Median filter to reduce various noise on the images with various window size are introduced on the images on Kovalevsky's 2D cellular complex.

3.1 Digitalized image into ACC

The digitalized image can be transformed to ACC by the following process.

Step 1: Defining the lower dimensional cells namely 0-cells (points), 1-cells (cracks) and 2-cells (pixels) in it with the bounding relation

Step 2: Identify the own cells for each pixel using the coordinate assigning rule given in definition2.5, such that each pixel has a point lying in the corner of the pixel which is the nearest to the origin in the coordinate system and two cracks incident to the point and pixel as its own cell. Thus, each pixel gets three own cells of lower dimensions.

Step 3: Padding the pixel values in the boundary in order to process the cells in the boundary of an image.

3.2 Designing the window of size 2×2 using ACC

After converting the digitalized image into ACC, the kernel (window) with various sizes are designed using different notions of ACC.

3.2.1 Window of size 2×2

The window of size 2×2 is designed based on the concept SON of a 0-cell in an ACC. The SON of a 0-cell (point) in an ACC is a subcomplex (named it as window of size 2×2) which contains four 2-cells, four 1-cells and one 0-cell as illustrated in Figure-1. Since the intensity values are assigned to the 2-cell (pixels) of an image, hence only four 2-cells (pixels) which belong to SON of a 0-cell are considered for the construction of a window of size 2×2

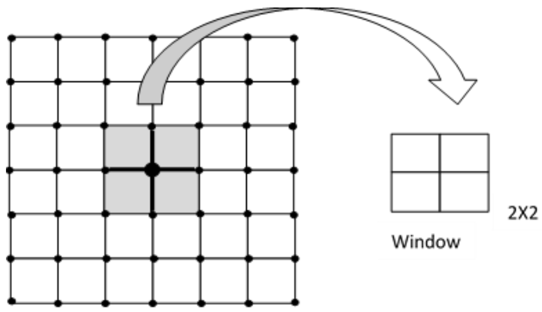

Figure-1. smallest open neighborhood of the 0-cell(shaded portion) ; window of size 2×2 .

Mean and Median filters using the window of Size 2×2 are constructed and formal descriptions of the algorithms are given.

3.2.2 Mean filter

The proposed Mean filter algorithm using ACC processes each and every pixel through points in an image. Hence, scan the image row by row through points and apply a window of size 2x2 on each processing point. Then, calculate the average of the four pixels values which are in the window and assign it to the processing point. This process will continues, until all the points in an image are processed. This entire process is illustrated below in Figure-2(a) and (b).

VOL. 10, NO 22, DECEMBER, 2015 ISSN 1819-6608
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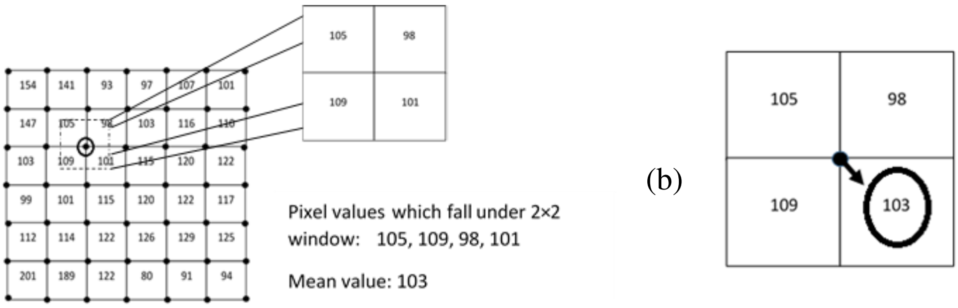

Figure-2. (a) window of size 2x2 on processing point and pixels values which fall under the window; (b)'own' pixel of the processing point is replaced by the mean value.

The formal description of the algorithm is given below:

Step 1: Scan the image row by row through points and apply 2×2 window on each point in an image.

Step 2: Compute the average of pixel values which are in the window.

Step 3: Assign value computed in step 2 to the processing point

Step 4: Repeat the above steps until all the points in the image are processed.

3.2.3 Median filter

The proposed Median filter algorithm using ACC processes each and every pixel through points in an image. Hence, scan the image row by row through points and apply a window of size 2x2 on each processing point. Then, calculate the median of the four pixels values which fall under the window and assign it to the processing point. This process continues until all the points in an image are processed. This entire process is illustrated below in Figure-3(a) and (b).

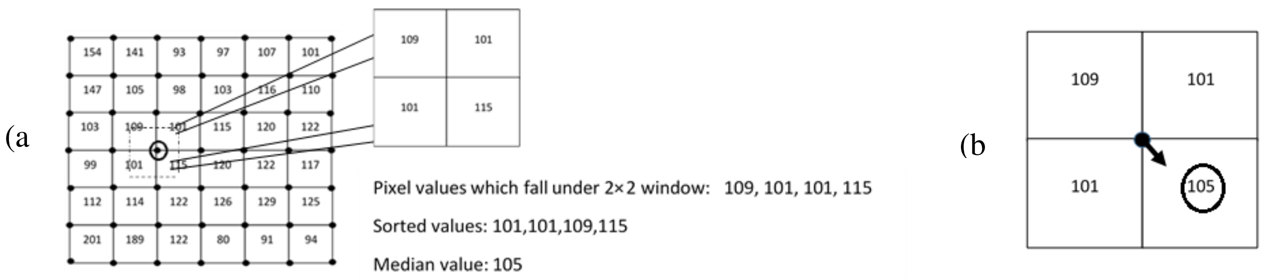

Figure-3. (a) window of size 2x2 on processing point and pixels values which fall under the window; (b) 'own' pixel of the processing point is replaced by the median value.

The formal description of the algorithm is given below:

Step 1: Scan the image row by row through points and apply 2×2 window on each point in an image.

Step 2: Sort the pixel values which fall under the window in ascending order and Compute the median value.

Step 3: Assign the median value to the processing point

Step 4: Repeat the above steps until all the points in the image are processed.

3.3 Designing of spatial filters using the window of size 3×3:

The standard Mean and Median filters are based on the window of size 3×3. Hence, here we extend the window size by considering the different notions of abstract cellular complex.

3.3.1 Window of size 3×3

The window of size 3×3 is designed by considering the notion of smallest open neighborhood of the 0-cell in the closure of a pixel. The SON of all points of the closure of a pixel in an ACC (window of size 3x3) contains nine pixels, twelve cracks (1-cells) and four points which are illustrated in Figure-4. Since the intensity value is assigned only to the pixels in an image, hence only the nine pixels which belong to the SON of a point are considered.

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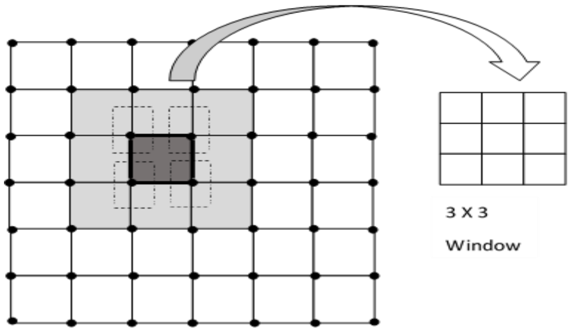

Figure-4. smallest open neighborhood of all 0-cells (light shade and dark shade portions covered by dotted lines) of the closure of a 2-cell (dark shaded portion) and four 2×2 sub windows (dotted line squares); window of size 3×3.

3.3.2 Mean filter

This proposed Mean filter algorithm using abstract cellular complex processes each and every pixel directly in an image. The closure of the pixel contains four corner points namely c_1,c_2, c_3 andc_4 at each c_i(i=1,2,3,4) consider the 2x2 window as in section 3.2.1 and assign the mean value m_i to each c_i (i=1,2,3,4) as in section 3.2.2. Now compute the mean value of the calculated mi's namely m. That is m = mean {m_1, m_2, m_3, m_4} and assign it to the corresponding pixel. This process continues until all the pixels in an image are processed. The entire process is illustrated below in Figure-5(a) and (b).

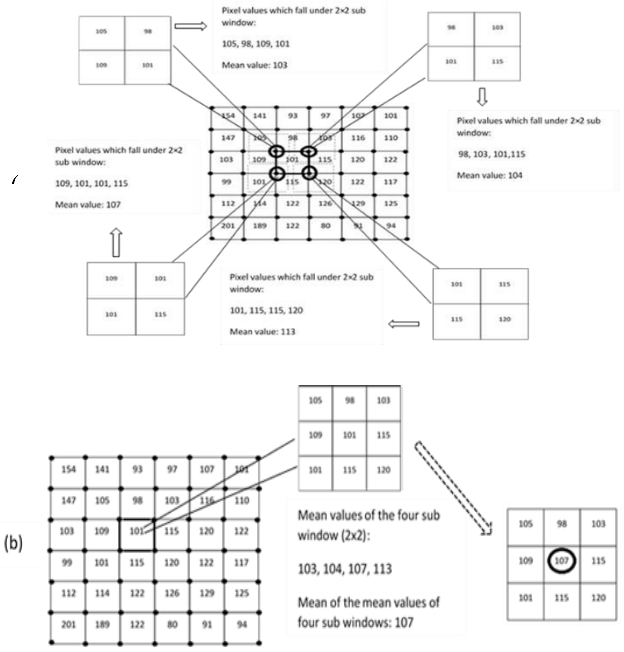

Figure-5. (a) The four sub windows of size 2×2 in a large window (3×3) and its mean values; (b) processing pixel value is replaced by its resultant mean value in a large window.

The formal description of the algorithm is given by:

Step 1: Scan the image row by row through pixels and construct 3×3 window on each pixel in an image

Step 2: Construct closure of each pixel and consider the four corner points of the closure.

Step 3: At each corner points construct 2×2 window as in section 3.2.1 and assign the mean value to corner points as in as in section 3.3.1.

Step 4: Compute mean of the assigned value at each corner points and assign it to the pixel.

Step 5: Repeat Step1-Step4 until all the pixels of the image are processed.

3.3.3 Median filter

The proposed Median filter algorithm using abstract cellular complex processes each and every pixel directly in an image. The closure of the pixel contains four corner points namely c_1,c_2, c_3 andc_4 at each c_i(i=1,2,3,4) consider the 2x2 window size as in section 3.2.1 and assign the median value m_i to each c_i (i=1, 2, 3, 4) as in section 3.2.3. Now compute the median value of the calculated m_i's namely m. That is m = median {m_1, m_2, m_3, m_4} and assign it to the corresponding pixel. This process continues until all the pixels in an image are processed. This entire process is illustrated below in Figure-6(a) and (b).

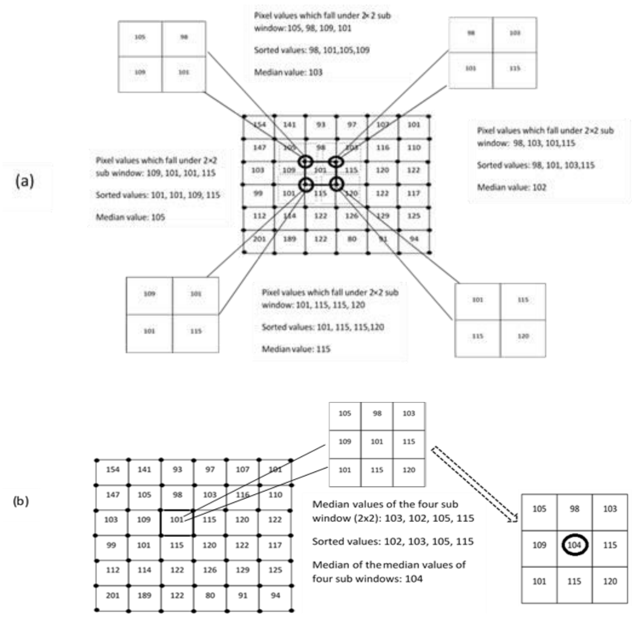

Figure-6. (a) The four sub windows of size 2×2 in a large window (3×3) and its median values; (b) processing pixel value is replaced by its resultant median value in a large window.

The formal description of the algorithm is given by:

Step 1: Scan the image row by row through pixels and construct 3×3 window on each pixel in an image.

Step 2: Construct closure of each pixel and consider the four corner points of the closure.

Step 3: At each corner points construct 2×2 window as in section 3.2.1 and assign the median value to corner points as in section 3.3.1.

Step 4: Compute median value of the assigned value at each corner points and assign it to the pixel.

Step 5: Repeat Step1 -Step4 until all the pixels of the image are processed.

4. EXPERIMENTAL RESULT AND DISCUSSION

The proposed algorithms are implemented through MATLAB. The standard images like headct, Lena, eight and cameraman shown in Figure-7 with various noise range are used for performance evaluation.

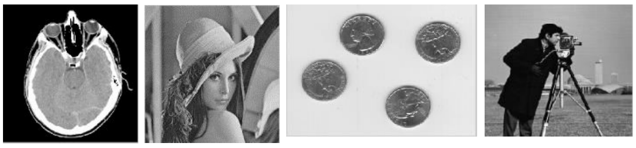

Figure-7. Original test images.

The test images are corrupted by Gaussian noise and salt and pepper noise with various noise densities. The Mean and median filters with window size 2×2 are proposed. This mean filter gives high PSNR value (70.919987) for the image corrupted by Gaussian noise and the Median filter gives high PSNR value (74.9450) for the image corrupted by Salt and pepper noise. The resultant images of these filters are illustrated below in Figures 8 and 9.

Figure-8. From left to right: Noisy image with μ=0 and σ^2=0.001Gaussian noise level; filtered image of Mean filter with 2×2 window.

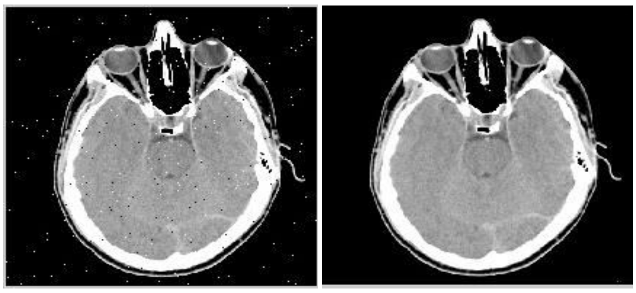

Figure-9. From left to right: Salt and pepper noise image noise level and the filtered image of Median filter with 2×2 window.

4.1 Comparison

The proposed Mean and Median filters with window size 3×3 are tested and compared with standard Mean and Median filters. The performance of the filtering technique is analyzed using the quality metrics RMSE and PSNR as well as in terms of visual quality of the images. The results of this comparison on various noise image are shown in Figures 10, 11, 13 and 14.

Figure-10. From left to right: Noisy image with μ=0 and σ^2=0.009 noise level; filtered image of Mean filter with 3×3 window and filtered image of standard Mean filter.

Figure-11. From left to right: Noisy image with μ=0 and σ^2=0.03 noise level; filtered image of Mean filter with 3×3 window and filtered image of standard Mean filter.

The experimental results of the comparison between the Mean and Standard Mean filters are illustrated in Table-1 and Figure-12.

Table-1. PSNR values of Mean filter and standard mean filter for the original test image corrupted by Gaussian noise on different variances.

Filter	Gaussian noise with mean(0) and variance (σ^2)		
	σ^2=0.009	σ^2=0.01	σ^2=0.03
	PSNR	PSNR	PSNR
Mean	73.1887	73.1710	72.1026
Standard mean	71.8178	71.7976	71.0840

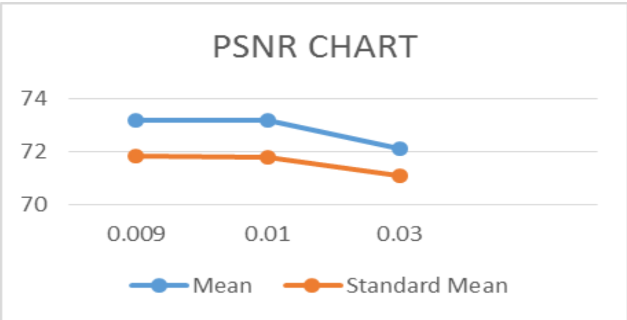

Figure-12. Comparative analysis of mean and standard Mean filters for the original test image corrected by Gaussian Noise on different variances using PSNR values.

Figure-13. From left to right: Salt and pepper noisy image with noise level (0.01); filtered image of Median filter with 3×3 window and filtered image of standard median filter.

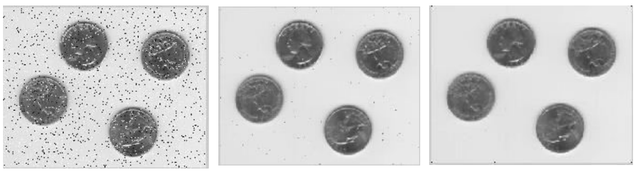

Figure-14. From left to right: Salt and pepper noisy image with noise level (0.05); filtered image of median filter with 3×3 window and filtered image of standard median filter.

The experimental results of the comparison between the Median and standard Median filters are illustrated in Table-2 and Figure-15.

Table-2. PSNR values of Median filter and standard median filter for the original test image corrupted by salt and pepper noise on different variances.

Filter	Salt and pepper noise		
	0.01	0.03	0.05
	PSNR	PSNR	PSNR
Median	81.0188	80.3504	78.3255
Standard median	78.4769	78.3190	77.8267

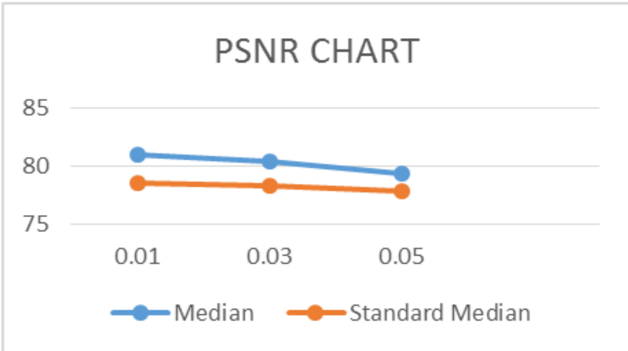

Figure-15. Comparative analysis of median and standard Median filters for the original test image corrupted by Salt and Pepper noise on different variances using PSNR values.

5. CONCLUSION AND FUTURE WORK

In this paper, Mean and Median filters with the different window sizes (2×2 and 3×3) based on the concept of ACC are proposed. The proposed filter are applied on the images with Gaussian noise and salt and pepper noise. The developed filters are compared with the existing standard Mean and Median filters. The experimental results showed that Mean, Median filters with window size 2x2 efficiently handles low density noises and Mean and Median filters with window size 3x3 performs better than the standard Mean and Standard Median filters by giving higher PSNR value. The merits of the proposed filters are the existing filters are developed only using the window of size (2n+1)× (2n+1) but the proposed filters are developed using both window of size (2n)× (2n) and (2n+1)× (2n+1) and the proposed filters are free from contradictions and boundary paradox.

In future, using the different notions of ACC, the dimension of window size can be increased and we can develop the filters using higher dimensions. The proposed work can be extended to 3D-images also.

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