



CORRECTION BIAS ERROR TOOTH FLANK OF A HELICAL GEAR GENERATED BY CNC GEAR SHAVING MACHINE

Van-The Tran

Department of Mechanical Engineering, Hung Yen University of Technology and Education, Hung Yen City, Vietnam

E-Mail: vanc4.hut@gmail.com

ABSTRACT

On CNC shaving machine, the tooth flank of work gear can be crowned longitudinally by varying the plunge motion of shaving cutter. However, which will induce a bias error on tooth flank of the shaved work gear. In this paper, therefore, we propose a new CNC shaving method by modifying work gear rotation angle and without varying the plunge motion of shaving cutter during the gear shaving process that can be reduced the bias error of the tooth flank on shaved work gear. A numeral example is presented to illustrate and verified the merits of the proposed CNC gear shaving method in longitudinal crowning to obtain the tooth flank of work gear surface without bias error.

Keywords: CNC gear shaving, longitudinal crowning, bias error tooth flank.

INTRODUCTION

The finish process for generation of involute helical gear is usually performed by gear shaving machine. Wherein, two helical gears are meshed with together in crossed axes. One gear of the gear pair is shaving cutter with gashed teeth to obtain cutting edges. These cutting edges act as blades and cut very thin pieces material of teeth of work gear. In this process, the longitudinal crowning of work gear surface can be achieved by plunging motion. However, which will induce twisted tooth flanks on the shaved work gear. Therefore, we propose a methodology for longitudinal crowning by modification of work gear rotation angle and without variation of the plunge during the gear shaving process.

Basic mesh conditions for the involute helical gear set have been presented in some textbooks [1, 2]. In 1990, Endoy [3] and Dugas [4] have shown some important discussions of the shaving process, such as the approximate life cycle time and the even contact. A stochastic model to predict the effect of shaving cutter performance on the finished tooth form was proposed by Moriwaki [5] and Moriwaki and Fujita [6]. In 1996, a MS-Windows application program has been developed to design the specification of the shaving cutter from the given shaved gear by Kim and Kim [7]. Tooth profile errors of the plunge shaving cutter due to errors from the setup of grinding machines were explored by Koga *et al.* [8]. And Miao and Koga [9] have developed a mathematical model of a theoretical tooth profile for the plunge shaving cutter. Subsequently, Seol and Litvin [10] proposed a modification of geometry of involute spur and helical gears with parallel and crossed axes on gear shaving for localization and stabilization of bearing contact and reduction of noise and vibration. More recently, a numerical method to simulate the auxiliary crowning in the parallel gear shaving process has developed by Hsu [11]. Chang *et al.* [12] and Hung *et al.*

[13] developed a mathematical model of the shaved gear with auxiliary crowning taking into account the setting parameters of the gear shaving machine and cutter assembly errors. This model is considered at standard pitch cylinder. However, the meshing point between work gear and shaving cutter should be considered at operating pitch cylinder. Therefore, a mathematical model considering at operating pitch cylinder for the tooth profile of work gear finished by the parallel shaving process with the auxiliary crowning mechanism is proposed by Hsu and Fong [14]. Litvin *et al.* [15, 16] proposed and developed a method for a new topology of modified helical gear tooth surfaces that localization of the bearing contact under misalignment conditions based on meshing of a double-crowned gear with a standard helical involute gear. Finally, Hsu and Su [17] investigated the topologies, contact ellipses and transmission errors of the double-crowned work gear pairs generation by modified hob in gear-hobbing process. Recently, Tran *et al.* [18] proposed a novel hobbing method to generate anti-twist tooth flanks of the involute helical gear in longitudinal tooth crowning by supplementing an additional rotation angle of work gear during its hobbing process.

In this paper, to reduce the bias error of tooth flank on the helical gear surface, a new method for longitudinal crowning by modification of work gear rotation angle and without variation of the plunge motion during the gear shaving process is proposed. A numeral example is presented to illustrate and verify the merits of the proposed gear shaving method in longitudinal tooth crowning of helical gears.

MATHEMATICAL MODEL OF THE STANDARD SHAVING CUTTERS

Basically, the theoretical tooth profile of a standard shaving cutter is an involute helical gear, as shown in Figure-1(a). Accordingly, the position vector and



unit normal vector of the standard shaving cutter's right-hand side profile can be expressed in coordinate system $S_i(x_i, y_i)$ as follows:

$$\mathbf{r}_i(u) = [x_i(u), y_i(u)]^T, \quad (1)$$

$$\text{and } \mathbf{n}_i(u) = [n_{xi}(u), n_{yi}(u)]^T, \quad (2)$$

where

$$x_i = r_{bs} \cos(u + \mu_s) + u \sin(u + \mu_s), \quad (3)$$

$$y_i = r_{bs} \sin(u + \mu_s) - u \cos(u + \mu_s), \quad (4)$$

$$n_{xi} = \sin(u + \mu_s), \quad (5)$$

$$n_{yi} = \cos(u + \mu_s), \quad (6)$$

$$\text{and } \mu_s = \left(\frac{2\pi}{N_s} - \frac{s_{pts}}{2r_{ps}} \right) - \text{inv}\alpha_{pts}, \quad (7)$$

where u is the involute profile parameter, r_{bs} is the radius of the base cylinder of shaving cutter, s_{pts} is the cross

section space width on the pitch cylinder of shaving cutter, N_s is the number of teeth of the shaving cutter, r_{ps} is the shaving cutter pitch radius, and α_{pts} is the profile angle in transverse section, as shown in Figure-1(a).

By transforming coordinate system of the helical motion from S_i to S_s , as shown in Figure-1(b), the position vector and unit normal vector of the standard shaving cutter's surface can be expressed in coordinate system $S_s(x_s, y_s, z_s)$ as follows:

$$\mathbf{r}_s(u, \theta) = \begin{bmatrix} x_s(u, \theta) \\ y_s(u, \theta) \\ z_s(u, \theta) \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & t_{ps}\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i(u) \\ y_i(u) \\ 0 \\ 1 \end{bmatrix}, \quad (8)$$

$$\mathbf{n}_s(u, \theta) = \begin{bmatrix} n_{xs}(u, \theta) \\ n_{ys}(u, \theta) \\ n_{zs}(u, \theta) \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_{xi}(u) \\ n_{yi}(u) \\ 1 \end{bmatrix}, \quad (9)$$

where θ is the longitudinal parameter, $t_{ps} = r_{ps} \cos \beta_{ps}$ is the helix parameter, β_{ps} is the pitch helix angle of the shaving cutter surface.

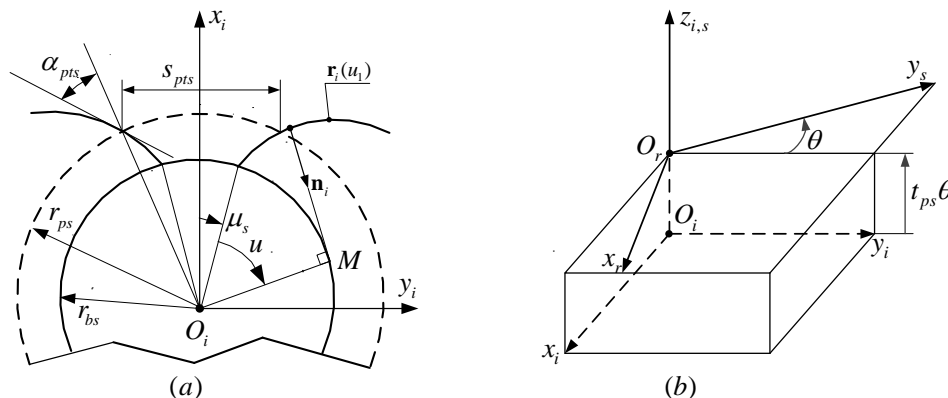


Figure-1. Generation process of the standard shaving cutter surface.

MATHEMATICAL MODEL OF THE CROWNED WORK GEAR

The coordinate systems for the shaving process of helical gears with longitudinal crowning teeth are shown in Figure-2, where coordinate systems $S_s(x_s, y_s, z_s)$ and $S_l(x_l, y_l, z_l)$ are rigidly connected to the shaving cutter and work gear, respectively, while the coordinate system $S_b(x_b, y_b, z_b)$ is rigidly connected to the frame of shaving

machine, and $S_c(x_c, y_c, z_c)$, $S_d(x_d, y_d, z_d)$, $S_e(x_e, y_e, z_e)$, $S_f(x_f, y_f, z_f)$ and $S_g(x_g, y_g, z_g)$ are auxiliary coordinate systems for simplification of coordinate transformation. On a modern gear shaving machine, there are two shaving cutter's movements: traverse movement along the axis of the work gear l and radial feed-in $E_{os}^{(l)}$ along the center distance between the



shaving cutter and work gear. The crossed angle γ_s of the shaving cutter and work gear axes is usually a machine-tool setting. For conventional longitudinal tooth crowing of a work gear, it is accomplished by modifying the radial feed-in $E_{os}^{(l)}$ to a second order polynomial, such as $E_{os}^{(l)} = E_{os}^{(0)} - al^2$, as shown in Fig. 2. However, this modification causes a bias error of the tooth flank on the helical gear. Therefore, this study proposes a novel shaving method to reduce bias error of tooth flanks on the crowned helical gear. The rotating angle of work gear, $\phi_1(\phi_s, l)$, is set as a function of shaving cutter's rotation

angle, ϕ_s , and the amount of shaving cutter's traverse feed, l .

The position vector $\mathbf{r}_s(u, \theta)$ (see Eq. (8)) and unit normal $\mathbf{n}_s(u, \theta)$ (see Eq. (9)) present locus of shaving cutter surface, respectively. By applying the homogeneous coordinate transformation matrix equation from S_s to S_1 , the locus of shaving cutter can be represented in coordinate system S_1 as follows:

$$\mathbf{r}_1(u, \theta, \phi_s, l) = \mathbf{M}_{1s}(\phi_s, l) \cdot \mathbf{r}_s(u, \theta), \quad (10)$$

where

$$\mathbf{M}_{1s}(\phi_s, l) = \begin{bmatrix} \cos \gamma_s \sin \phi_s \sin \phi_1 - \cos \phi_s \cos \phi_1 & \cos \gamma_s \cos \phi_s \sin \phi_1 + \sin \phi_s \cos \phi_1 & -\sin \gamma_s \sin \phi_1 & E_{os}^{(l)} \cos \phi_1 \\ -\cos \gamma_s \sin \phi_s \cos \phi_1 - \cos \phi_s \sin \phi_1 & \sin \phi_s \sin \phi_1 - \cos \gamma_s \cos \phi_s \cos \phi_1 & \sin \gamma_s \cos \phi_1 & E_{os}^{(l)} \sin \phi_1 \\ \sin \gamma_s \sin \phi_s & \sin \gamma_s \cos \phi_s & \cos \gamma_s & -l \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

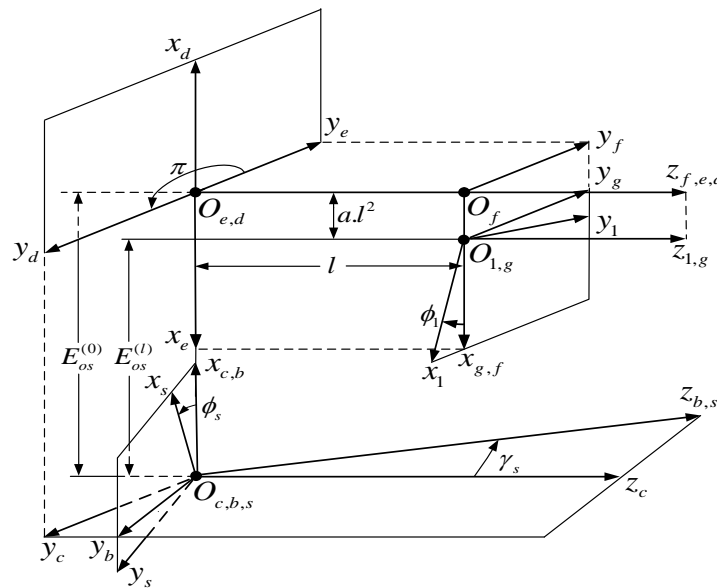


Figure-2. Coordinate systems for the CNC shaving machine.

After some mathematical operations, the locus of shaving cutter surface can be simplified as follows:

$$\mathbf{r}_1(u, \theta, \phi_s, l) = [x_1(u, \theta, \phi_s, l), y_1(u, \theta, \phi_s, l), z_1(u, \theta, \phi_s, l), 1]^T, \quad (12)$$

where

$$x_1 = \cos \phi_1 (E_{os}^{(l)} - x_s \cos \phi_s + y_s \sin \phi_s) + \sin \phi_1 [-z_s \sin \gamma_s + \cos \gamma_s (y_s \cos \phi_s + x_s \sin \phi_s)], \quad (13)$$

$$y_1 = \cos \phi_1 [z_s \sin \gamma_s - \cos \gamma_s (y_s \cos \phi_s + x_s \sin \phi_s)] + \sin \phi_1 (E_{os}^{(l)} - x_s \cos \phi_s + y_s \sin \phi_s), \quad (14)$$



$$\text{and } z_1 = -l + z_s \cos \gamma_s + (y_s \cos \phi_s + x_s \sin \phi_s) \sin \gamma_s. \quad (15)$$

There are two independent kinematic parameters l and ϕ_s in the CNC shaving process. Therefore, there are two equations of meshing between the work gear and the shaving cutter as follows:

$$f_1(u, \theta_1, \phi_s, l) = \mathbf{n}_1 \cdot \frac{\partial [x_1(u, \theta, \phi_s, l), y_2(u, \theta, \phi_s, l), z_2(u, \theta, \phi_s, l)]}{\partial \phi_s} = 0, \quad (16)$$

and

$$f_2(u, \theta, \phi_s, l) = \mathbf{n}_1 \cdot \frac{\partial [x_1(u, \theta, \phi_s, l), y_1(u, \theta, \phi_s, l), z_1(u, \theta, \phi_s, l)]}{\partial l} = 0, \quad (17)$$

where

$$\mathbf{n}_1(u, \theta, \phi_s, l) = \mathbf{L}_{1s}(\phi_s, l) \cdot [n_{x_s}(u, \theta), n_{y_s}(u, \theta), n_{z_s}(u, \theta)]^T, \quad (18)$$

The transformation matrix $\mathbf{L}_{1s}(\phi_s, l)$ is the sub-matrix of \mathbf{M}_{1s} by deleting the last column and row.

The rotational relationship between the work gear and shaving cutter is defined as

$$F_2(\phi_s, l) = \frac{1}{2!} \cdot [(\phi_s - \phi_{s0})^2 \cdot (\phi_1)''_{\phi_s}(\phi_{s0}, l_0) + 2 \cdot (\phi_s - \phi_{s0}) \cdot (l - l_0) \cdot ((\phi_1)'_{\phi_s})'_l(\phi_{s0}, l_0) + (l - l_0)^2 \cdot (\phi_1)''_{l_l}(\phi_{s0}, l_0)], \quad (22)$$

$$F_n(\phi_s, l) = \frac{1}{n!} \cdot [(\phi_s - \phi_{s0})^n \cdot (\phi_1)^{(n)}_{\phi_s}(\phi_{s0}, l_0) + n \cdot (\phi_s - \phi_{s0})^{n-1} (l - l_0) \cdot ((\phi_1)^{(n-1)}_{\phi_s})'_l(\phi_{s0}, l_0) + \dots + n \cdot (\phi_s - \phi_{s0}) (l - l_0)^{n-1} \cdot ((\phi_1)'_{\phi_s})^{(n-1)}_l(\phi_{s0}, l_0) + (l - l_0)^n \cdot (\phi_1)^{(n)}_{l_l}(\phi_{s0}, l_0)], \quad (23)$$

and $R^n(\phi_s, l)$ is the remainder of the Taylor series, which can be omitted for simplicity. By substituting Eqs. (21)–(23) into Eq. (20) and performing certain mathematical operations, we can then represent Eq. (20) in second-order Taylor polynomials as follows:

$$\phi_1(\phi_s, l) = \frac{N_s}{N_1} \phi_s - \frac{\tan \beta_{o1}}{r_{o1}} l + a_0 + a_1 \phi_s + a_2 l + a_3 \phi_s^2 + a_4 \phi_s l + a_5 l^2, \quad (24)$$

where a_0, \dots, a_5 are coefficients of the proposed modified rotation angle for the helical work gear. These coefficients can be obtained directly from Eq. (20) by solving their respective derivatives. The generated tooth surface depends on the motion functions of the rotating angle ϕ_s and the traverse movement l . The coefficients of Eq. (24)

$$\phi_1 = \frac{N_s}{N_1} \phi_s - \frac{\tan \beta_{o1}}{r_{o1}} l \quad (19)$$

where symbols N_s and N_1 indicate the number of teeth of the shaving cutter and gear, respectively. Because the work gear rotation angle $\phi_1(\phi_s, l)$ is a linear function of the shaving cutter's rotation angle ϕ_s and the traverse movement of work gear l along the axis of shaving cutter, it can be approximated in terms of the rotating angle ϕ_s and traverse movement l by n th-order Taylor polynomials, expanded at an arbitrary point $(\phi_s, l) = (\phi_{s0}, l_0)$, as a function dependent on two variables. The rotational relation among the work gear, shaving cutter, and the Taylor series can then be expressed by

$$\phi_1(\phi_s, l) = F_1(\phi_s, l) + F_2(\phi_s, l) + \dots + F_n(\phi_s, l) + R^n(\phi_s, l), \quad (20)$$

where

$$F_1(\phi_s, l) = \phi_1(\phi_{s0}, l_0) + \phi_s \cdot (\phi_1)'_{\phi_s}(\phi_{s0}, l_0) + l \cdot (\phi_1)'_{l_l}(\phi_{s0}, l_0) \approx \frac{N_s}{N_1} \phi_s - \frac{\tan \beta_{o1}}{r_{o1}} l + \phi_1(\phi_{s0}, l_0), \quad (21)$$

can then be used in the gear shaving process to generate a crowned helical gear with bias error of the tooth flanks. The tooth surface of shaved work gear can be defined by solving Equations (10), (16) and (17), simultaneously.

RESULT COMPARISONS AND DISCUSSIONS

To reduce the bias error of tooth flank on crowned helical gear surfaces, an index of evenness of lengthwise crowning along the longitude of work gear is proposed. This index is called the crowning evenness ratio, R_{ce} , and it is defined by the smallest crowning amount divided by the largest crowning amount at different position on each longitude of work gear surfaces. Therefore, it is preferable if the crowning evenness ratio approaches one. The purpose of this example is to validate the proposed longitudinal crowning method. The basic data for work gear, shaving cutter and radial feed



modification coefficient are given in Table-1. The shaving meshing conditions as illustrated in Ref. [1, 2]. machine setup data are calculated according to the basic

Table-1. Basic data of the gear and shaving cutter.

Work gear data	
Number of teeth (N_1)	36
Normal module (m_{pn})	2.65
Normal circular-tooth thickness (s_{pn1})	5.163 mm
Normal pressure angle (α_{pn})	20°
Helix angle (β_{p1})	10°R.H.
Face width (F_{w1})	20 mm
Shaving cutter data	
Number of teeth (N_s)	73
Helix lead angle (λ_{ps})	82° R.H.
Normal circular-tooth thickness (s_{pns})	2.502 mm
Shaving process data	
Center distance variation coefficient a	$1.22 \times 10^{-4} \text{ mm}^{-1}$

According to the algorithm presented in Ref. [18], the modifying work gear rotation angle, expressed in

Eq. (24), for the helical gear with longitudinal tooth crowning is determined as follows:

$$\phi_1(\phi_s, l) = \frac{73}{36} \phi_s - 364.042 \times 10^{-5} l + 5.349 \times 10^{-5} \phi_s - 126.1 \times 10^{-5} \phi_s^2 + 2.066 \times 10^{-5} \phi_s l - 6.917 \times 10^{-7} l^2, \quad (25)$$

The normal deviations and simulated tooth surface topographies of the crowned helical gear, with and without modifying work gear rotation angle for the shaved work gear, are shown in Tables 2-3 and Figures 3-4, respectively. The maximum bias error of tooth flank is about $3.39 \mu\text{m}$ for the generated work gear tooth flank by applying conventional shaving method. And after modifying work gear rotation angle in gear shaving process, the maximum bias error of tooth flank is reduced from $3.39 \mu\text{m}$ to a negligible amount of $0.11 \mu\text{m}$. It is

verified that the bias error of the tooth flank is reduced significantly by modifying work gear rotation angle proposed in this study. The longitudinal crowning evenness ratio for the tooth flank surface, as shown in Table 2 and Fig. 3 ($R_{ce} = 0.59$), is much smaller than that of the tooth flank surface, generated by modifying work gear rotation angle, as shown in Table-3 and Figure-4 ($R_{ce} = 0.98$).

Table-2. Normal deviations of crowned work gear surfaces generated with original formula.

	Top Land (Unit: μm)								
E	-5.0	-2.6	-1.0	-0.1	-0.1	-0.9	-2.5	-5.0	-8.4
D	-5.5	-3.0	-1.2	-0.2	0.0	-0.6	-2.1	-4.5	-7.7
C	-6.2	-3.5	-1.5	-0.4	0.0	-0.4	-1.7	-3.9	-6.9
B	-6.9	-4.1	-1.9	-0.6	0.0	-0.3	-1.3	-3.3	-6.1
A	-8.1	-5.0	-2.6	-1.0	-0.2	-0.1	-0.9	-2.6	-5.1
	1	2	3	4	5	6	7	8	9
Gear Root									



maximum tooth flank twist = $3.39\mu m$ and longitudinal crowning evenness ratio $R_{ce} = 0.59$

Table-3. Normal deviations of crowned work gear surfaces generated with modifying work gear rotation angle.

	Top Land									(Unit: μm)
E	-7.0	-4.1	-2.1	-0.9	-0.4	-0.9	-2.1	-4.1	-7.0	
D	-6.8	-3.9	-1.8	-0.6	-0.2	-0.6	-1.8	-3.9	-6.7	
C	-6.6	-3.7	-1.7	-0.4	0.0	-0.4	-1.7	-3.7	-6.6	
B	-6.6	-3.7	-1.6	-0.4	0.0	-0.4	-1.6	-3.7	-6.5	
A	-6.9	-4.0	-1.9	-0.7	-0.2	-0.7	-1.9	-3.9	-6.8	
	1	2	3	4	5	6	7	8	9	
Gear Root										
maximum tooth flank twist = $0.11\mu m$ and longitudinal crowning evenness ratio $R_{ce} = 0.98$										

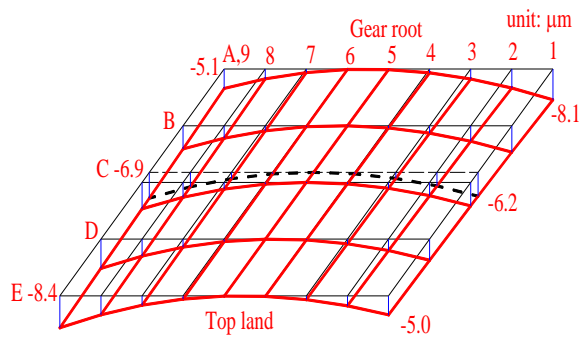


Figure-3. Simulated topography of crowned work gear surfaces generated with original formula.

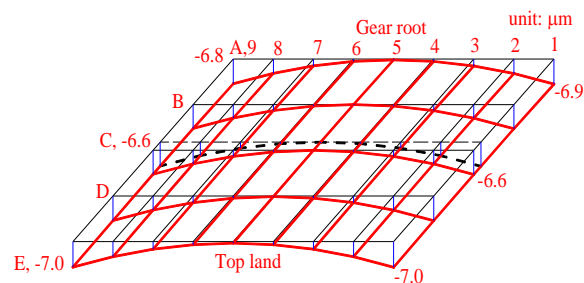


Figure-4. Simulated topography of crowned work gear surfaces generated with modifying work gear rotation angle.

CONCLUSIONS

This paper proposes a new CNC shaving method for longitudinal crowning tooth flank by modifying the work gear rotation angle and without varying the center distance during the gear shaving process. The tooth surface topographies of the conventional and proposed shaved helical gear are simulated and compared. It reveals

that the bias error of tooth flank on helical gear is really significantly reduced by applying our proposed shaving method.

Nomenclature

s_{jki}	circular tooth thickness, $j = b, o, p$; $k = t, n$; $i = 1, 2$
β_{jki}	helix angle, $j = b, o, p$; $k = t, n$; $i = 1, 2$
γ	crossed angle
λ_{jki}	lead angle, $j = b, o, p$; $k = t, n$; $i = 1, 2$
m_{jki}	module, $j = b, o, p$; $k = t, n$; $i = 1, 2$
p_{on}	normal circular pitch measured at the operating pitch plane
N_i	number of teeth, $i = 1, 2$
E_{os}	operating distance
α_{jki}	pressure angle, $j = b, o, p$; $k = t, n$; $i = 1, 2$
r_{jki}	radius of the pitch circle, $j = b, o, p$; $k = t, n$; $i = 1, 2$
a	center distance variation coefficient

Subscripts

b	base circle
n	measured in normal section
t	measured in transverse section
o	operating pitch circle
p	pitch circle
s	shaving
l	work gear

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