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INVARIANT MANIFOLD FOR EXTENDED NONHOLONOMIC DOUBLE INTEGRATOR SYSTEMS

Z. M. Zain¹, M. R. Arshad², M. M. Noh¹, D. Pebrianti¹, N. M. Zain³, and K. A. A. Rahim⁴

¹Robotics and Unmanned Research Group, Control Engineering Cluster, Faculty of Electrical and Electronics Engineering, Universiti Malaysia Pahang, Pekan, Pahang, Malaysia

² Underwater, Control and Robotics Research Group, School of Electrical and Electronic Engineering, Universiti Sains Malaysia, Nibong Tebal, Pulau Pinang, Malaysia

> ³Faculty of Agro-based Industry, University of Malaysia Kelantan, Locked, Jeli, Kelantan, Malaysia E-Mail: <u>zainah@ump.edu.my</u>

ABSTRACT

Nonholonomic underactuated systems are typically modelled as highly nonlinear ones, which becomes obvious as the dimension of the system increases. Since this system cannot be stabilized by a static continuous feedback with constant gains, there are several control methods by using a canonical form up to now such as a chained form, a power form, a goursat normal form and a double integrator model. In this paper, we consider extended nonholonomic double integrator systems obtained by extending the Brockett nonholonomic integrator in order to expand the application of underactuated control to control the system using invariant manifold approach.

Keywords: invariant manifold, nonholonomic double integrator, extended double integrator.

INTRODUCTION

Control of nonholonomic systems is theoretically challenging and practically interesting. Due to Brockett's Theorem [1], these systems cannot be stabilized to a point with pure smooth (or even continuous) state feedback control. As a consequence, the classical smooth theory and design mechanism of nonlinear control systems cannot be applied. This challenging problem becomes even more difficult when the system is underactuated, that is possesses fewer actuators than configuration degrees of freedom.

Various researches about the control technique of underactuated systems have been achieved up to now. Among them, it is very often to use canonical models such as a chained form, a power form, a double integrator model, etc. Astolfi [2] made a canonical model discontinuous, and then he proposed the technique of performing continuous feedback control. Khennouf et al. [3] carried out well use of the structure of a chained form, and proposed the switching control that performs two steps of control by an invariant manifold. By this technique, attractive control to an invariant manifold is performed first in the 1st step, and each state on the invariant manifold is stabilized in the 2nd step. Watanabe et al. [4][5][6][7][8][9][10] also proposed the invariant manifold techniques to control underactuated systems. Furthermore, Khennouf et al. [11] also proposed the technique called quasi-continuous exponential stabilization control. Such a technique is implemented after summarizing the control methods, which were divided into two steps as mentioned above, to one step. Note however that among them major research is for controlled object with two-inputs [12] and therefore there is restricted research for controlled object with three or more inputs [13][14][15]. One of causes is that there is no definite method of transforming the original model into a canonical model to the case of the controlled system with three or more inputs.

In this paper, in order to extend the application of underactuated control, we consider the transformation of extended nonholonomic double integrator model in case of two-inputs and four-inputs systems obtained by extending the Brockett nonholonomic integrator to control underactuated systems using invariant manifold approach.

RELATED WORK

In his pioneering paper, Brocket [1] introduced the system

$$\dot{x}_{1} = u_{1}$$

$$\dot{x}_2 = u_2 \tag{1}$$

$$\dot{x}_{3} = x_{1}u_{2} - x_{2}u_{1}$$

where $[x_1, x_2, x_3]^T \in \Re^3$ is the state vector and $[u_1, u_2]^T \in \Re^2$ is a two-dimensional input. This system, quoted in the literature as the nonholonomic integrator system or Heisenberg system, has been pointed out as a benchmark example of a first-order nonholonomic underactuated system. It mimics the kinematic model of wheeled mobile robot of the unicycle type and displays all the basic properties of first-order nonholonomic systems.

It is easy to see that, under transformation of coordinates, system (1) can be transformed into one of the following chained forms

$$\dot{x}_{1} = u_{1} \qquad \dot{x}_{1} = u_{1}
\dot{x}_{2} = u_{2} \qquad \text{or} \qquad \dot{x}_{2} = u_{2} \qquad (2)
\dot{x}_{3} = x_{2}u_{1}, \qquad \dot{x}_{3} = x_{1}u_{2}.$$

under the transformations of coordinates given, respectively by

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$$\widetilde{x}_3 = \frac{1}{2} (x_3 + x_1 x_2)$$
 and $\widetilde{x}_3 = -\frac{1}{2} (x_3 - x_1 x_2)$

In [2][3][4], conditions have been found in order to check if a given first-order nonholonomic system can be transformed, via feedback and coordinate transformations, into the form

$$\dot{x}_{1} = u_{1}
\dot{x}_{2} = u_{2}
\dot{x}_{3} = x_{2}u_{1}
\vdots$$
(3)

 $\dot{x}_n = x_{n-1}u_1$

where $[x_1, x_2, x_3, \cdots , x_n]^T \in \Re^n$ is the state of the

system and u_1 and u_2 are inputs. Such form is referred to in the literature as the first-order chained form or Goursat normal form.

In contrast to systems with first-order nonholonomic constraints, systems with second-order nonholonomic constraints include the drift-term (that is, a vector field describing the evolution not subject to controls) which complicates the analysis, but makes it more challenging. Many researchers have been working on the stabilization problem and on the tracking control problem for this class of systems (see for example, in [16][17][18]. A key procedure in some works is to convert the second order nonholonomic system under consideration, via feedback and coordinate transformations, into a special canonical system of the form

$$\ddot{x}_{1} = u_{1} \qquad \qquad \ddot{x}_{1} = u_{1} \ddot{x}_{2} = u_{2} \qquad \text{or} \qquad \qquad \ddot{x}_{2} = u_{2}$$

$$\ddot{x}_{3} = x_{2}u_{1}, \qquad \qquad \qquad \ddot{x}_{3} = x_{1}u_{2}.$$

$$(4)$$

where $[x_1, x_2, x_3]^r \in \mathbb{R}^3$ is the configuration of the system and u_1 , u_2 are the inputs. (see, for instance, [19][20][21]). Such canonical forms, known in the literature as the second-order chained forms, simplify considerably the dynamical equations of the system, so being much more suitable to deal with than the original dynamical equations.

We observe that the two forms in Equation. (4) can be seen as the analogues of the first-order chained forms in the 3-dimensional case (see Equation. (2)), reflecting similarities between the first-order nonholonomic constraints, respectively, $\dot{x}_3 = x_2 \dot{x}_1$ and $\dot{x}_3 = x_1 \dot{x}_2$, and the second-order nonholonomic constraint, respectively, $\ddot{x}_3 = x_2 \dot{x}_1$ and $\ddot{x}_3 = x_1 \dot{x}_2$. In this spirit, it would be natural to consider also the system

$$\ddot{x}_{1} = u_{1}$$

 $\ddot{x}_{2} = u_{2}$
 $\ddot{x}_{3} = x_{1}u_{2} - x_{2}u_{1}$
(5)

reflecting similarities between the first-order nonholonomic constraint $\dot{x}_3 = x_1 \dot{x}_2 - x_2 \dot{x}_1$ in system (1) and the second-order nonholonomic constraint $\ddot{x}_3 = x_1 \ddot{x}_2 - x_2 \ddot{x}_1$ in (5).

Extended nonholonomic double integrator (ENDI)

It is observed in [22] that the Brockett nonholonomic integrator (1) fails to capture the case where both the kinematics and dynamics of a wheeled robot must be taken into account. To tackle this realistic case, the authors propose to extend the nonholonomic integrator model. Actually, it is shown in that paper that the dynamic equations of motion of a mobile robot of the unicycle type can be transformed into the system

$$\ddot{x}_{1} = u_{1} \ddot{x}_{2} = u_{2} \ddot{x}_{3} = x_{1}\dot{x}_{2} - x_{2}\dot{x}_{1}$$
(6)

where $x \triangleq [x_1, x_2, x_3, \dot{x}_1, \dot{x}_2]^T \in \Re^s$ is the state vector and $u \triangleq [u_1, u_2]^T \in \Re^s$ is a two-dimensional control vector. This system, which can be viewed as an extension of the Brockett nonholonomic integrator (1), is quoted in the literature as the Extended Nonholonomic Double Integrator (ENDI). It is locally strongly accessible for any $x \in \Re^s$, controllable and small time local controllable (STLC) at any equilibrium $x_a \in \{\Re^s : \dot{x}_1 = \dot{x}_2 = 0\}$ [9].

Converting systems to the form (ENDI)

As already observed, it is shown in [23] that the dynamic equations of motion of a mobile robot of the unicycle type can be transformed into the system (ENDI) (see Equation. (6)). Rewriting that system as a first-order system in \Re^{s} we obtain the first-order system

$$\dot{x}_{1} = y_{1} \qquad \dot{y}_{1} = u_{1} \dot{x}_{2} = y_{2} \qquad \dot{y}_{2} = u_{2}$$

$$\dot{x}_{3} = x_{1}y_{2} - x_{2}y_{1}$$

$$(7)$$

with state $[x_1, x_2, x_3, y_1, y_2]^T \in \Re^5$.

INVARIANT MANIFOLD FOR NONHOLONOMIC DOUBLE INTEGRATOR SYSTEM

Let the controlled object be described by the following nonholonomic double integrator system:

$$\dot{x}_1 = u_1$$

 $\dot{x}_2 = u_2$ (8)
 $\dot{x}_3 = x_1 u_2 - x_2 u_1$

and consider a stabilizing control problem such that $\mathbf{x}(t) = [x_1 \ x_2 \ x_3]^T$ becomes zero as $t \to \infty$. Here, all the states are assumed to be measurable.

To derive an invariant manifold, the feedback law given by

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 $\dot{\mathbf{r}} - \mathbf{v}$

$$u_1 = -kx_1$$

 $u_2 = -kx_2, \qquad k > 0$
(9)

is assumed to be applied to Equation. (8).

Then, solving the time response of the closed-loop system gives

$$\begin{aligned} x_1(t) &= x_1(0)e^{-kt} \\ x_2(t) &= x_2(0)e^{-kt} \end{aligned} \tag{10}$$

Therefore, $\dot{x}_3(t)$ is given by

$$\dot{x}_{3}(t) = x_{1}(0)e^{-2kt}u_{2} - x_{2}(0)e^{-2kt}u_{1}$$

= $-kx_{1}(0)x_{2}(0)e^{-2kt} + kx_{1}(0)x_{2}(0)e^{-2kt}$ (11)
= 0

it follows that

$$x_3(t) = x_3(0) \tag{12}$$

From the constant term of this $x_3(t)$, it is possible to select

$$S(\boldsymbol{x}) = \boldsymbol{x}_{3}(t) \tag{13}$$

as one candidate of an invariant manifold.

Under the above conditions, let the feedback law given by

$$u_1 = -kx_1$$

$$u_2 = -kx_2, \qquad k > 0$$
(14)

be applied to Equation. (8). Differentiating $S(\mathbf{x})$ with respect to time yields

$$\dot{S}(\mathbf{x}) = \dot{x}_{3}(t)
= x_{1}u_{2} - x_{2}u_{1}
= -kx_{1}x_{2} + kx_{1}x_{2}
\equiv 0$$
(15)

and under the feedback law given by Equation. (14), it holds that

 $S(\boldsymbol{x}) = \text{Const.} \tag{16}$

so it is seen that $S(\mathbf{x})$ is reduced to one invariant manifold.

From these, since if $S(\mathbf{x}) = 0$ can be assured at time t = T, then it can hold that $S(\mathbf{x}) = 0$ for $t \ge T$, applying Equation. (14) to Equation. (8) gives

 $\dot{x}_1(t) = -kx_1, \qquad \dot{x}_2(t) = -kx_2$

so that $x_1(t)$ and $x_2(t)$ are asymptotically stable, i.e., $x_1 \rightarrow 0$ and $x_2 \rightarrow 0$ as $t \rightarrow \infty$. Of course, it is seen from Equation. (13) that $x_3 \rightarrow 0$, because it has been already assured that $S(\mathbf{x}) = 0$.

INVARIANT MANIFOLD FOR ENDI SYSTEMS Case 1: 2-inputs system

Let the controlled object be represented by the following extended nonholonomic double integrator system:

$$\dot{x}_{1} - y_{1} \\
\dot{x}_{2} = y_{2} \\
\dot{x}_{3} = x_{1}y_{2} - x_{2}y_{1} \\
\dot{y}_{1} = u_{1} \\
\dot{y}_{2} = u_{2}$$
(17)

and consider a stabilizing problem such that $\mathbf{x}(t) = [x_1 \ x_2 \ x_3]^T$ is converged to zero as $t \to \infty$. Here, all the states are assumed to be measurable.

To derive an invariant manifold for this system, assume that the following state feedback law

$$u_{1} = -2ky_{1} - k^{2}x_{1}$$

$$u_{2} = -2ky_{1} - k^{2}x_{2}$$
(18)

is applied to Equation. (17).

Now, defining the state vector of the linear partial system in (17) as

$$\boldsymbol{x}_{s}(t)\underline{\Delta}\left[x_{1}\,x_{2}\,y_{1}\,y_{2}\,\right]^{T}\tag{19}$$

Its closed-loop linear partial system becomes

$$\mathbf{x}_{s}(t) = A\mathbf{x}_{s}(t),$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k^{2} & 0 & -2k & 0 \\ 0 & k^{2} & 0 & -2k \end{bmatrix}$$
(20)

so that its time response is written by

$$\boldsymbol{x}_s(t) = e^{At} \boldsymbol{x}_s(0) \tag{21}$$

where

$$e^{At} = \begin{bmatrix} e^{-kt} & 0 & te^{-kt} & 0\\ 0 & e^{-kt} + kte^{-kt} & 0 & te^{-kt} \\ -k^{2}te^{-kt} & 0 & e^{-kt} - k^{2}te^{-kt} & 0\\ 0 & -k^{2}te^{-kt} & 0 & e^{-kt} - k^{2}te^{-kt} \end{bmatrix}$$

Therefore, the closed-loop linear partial system is reduced to

$$\begin{aligned} x_{1}(t) &= x_{1}(0) \left[e^{-kt} + kte^{-kt} \right] + y_{1}(0)te^{-kt} \\ x_{2}(t) &= x_{2}(0) \left[e^{-kt} + kte^{-kt} \right] + y_{2}(0)te^{-kt} \\ y_{1}(t) &= x_{1}(0) \left[-k^{2}te^{-kt} \right] + y_{1}(0) \left[e^{-kt} - kte^{-kt} \right] \\ y_{2}(t) &= x_{2}(0) \left[-k^{2}te^{-kt} \right] + y_{1}(0) \left[e^{-kt} - kte^{-kt} \right] \end{aligned}$$
(22)

Furthermore, it is easy to derive that

$$x_{3}(t) = x_{3}(0) - \frac{x_{1}(0)y_{2}(0)}{2k} \left[e^{-2kt} - 1\right] + \frac{y_{1}(0)x_{2}(0)}{2k} \left[e^{-2kt} - 1\right]$$
(23)

From this constant term, it is found that



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$$S(\mathbf{x}) = x_3(t) + \frac{1}{2k}x_1(t)y_2(t) - \frac{1}{2k}x_2(t)y_1(t)$$
(24)

can be derived as one candidate for the invariant manifold. Applying the following feedback law

$$u_{1} = -k^{2}x_{1} - 2ky_{1}$$

$$u_{2} = k^{2}x_{2} - 2ky_{2}, \qquad k > 0$$
(25)

to the original system (8) and then taking its time derivative to examine the behavior of $S(\mathbf{x})$, it follows that

$$\dot{S}(\mathbf{x}) = \dot{x}_{3}(t) + \frac{1}{2k} [\dot{x}_{1}(t)y_{2}(t) + x_{1}(t)\dot{y}_{2}(t)] - \frac{1}{2k} [\dot{x}_{2}(t)y_{1}(t) + x_{2}(t)\dot{y}_{1}(t)] = x_{1}y_{2} - x_{2}y_{1} + \frac{1}{2k} [-k^{2}x_{1}x_{2} - 2kx_{1}y_{2} + k^{2}x_{1}x_{2} + 2kx_{2}y_{1}] = 0$$
(26)

and moreover it holds under the above feedback control that

$$S(\boldsymbol{x}) = \text{Const.}$$
(27)

subsequently, S(x) becomes one invariant manifold.

Thus, for a switching method in two-step control, if it holds that $S(\mathbf{x}) = 0$ at any time t = T, then it keeps that $S(\mathbf{x}) = 0$ for $t \ge T$. On the other hand, when the above feedback law is adopted at $t \ge T$, it is easily found that $x_1(t), x_2(t), y_1(t)$ and $y_2(t)$ are all asymptotically stable, i.e., $x_1 \to 0, x_2 \to 0, y_1 \to 0, y_2 \to 0$ as $t \to \infty$. Then, it can be also seen that $x_3 \to 0$ because $S(\mathbf{x}) = 0$ has been already satisfied.

Case 2: 4-inputs system

Let the controlled object be represented by the following extended nonholonomic double integrator system:

$$\begin{split} \dot{x}_{1} &= y_{1} \\ \dot{x}_{2} &= y_{2} \\ \dot{x}_{3} &= y_{3} \\ \dot{x}_{4} &= y_{4} \\ \dot{x}_{5} &= x_{1}y_{2} - x_{2}y_{1} \\ \dot{x}_{6} &= x_{1}y_{3} - x_{3}y_{1} \\ \dot{x}_{7} &= x_{1}y_{4} - x_{4}y_{1} \\ \dot{x}_{7} &= x_{2}y_{3} - x_{3}y_{2} \\ \dot{x}_{9} &= x_{2}y_{4} - x_{4}y_{2} \\ \dot{x}_{10} &= x_{3}y_{4} - x_{4}y_{3} \\ \dot{y}_{1} &= u_{1} \\ \dot{y}_{2} &= u_{2} \\ \dot{y}_{3} &= u_{3} \\ \dot{y}_{4} &= u_{4} \end{split}$$

$$\end{split}$$

$$(28)$$

and consider a stabilizing problem such that $\mathbf{x}(t) = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10} \ y_1 \ y_2 \ y_3 \ y_4]^T$ is converged to zero as $t \to \infty$.

To derive an invariant manifold for this system, assume that the following state feedback law

$$u_{1}(t) = -2ky_{1}(t) - k^{2}x_{1}(t)$$

$$u_{2}(t) = -2ky_{2}(t) - k^{2}x_{2}(t)$$

$$u_{3}(t) = -2ky_{3}(t) - k^{2}x_{3}(t)$$

$$u_{4}(t) = -2ky_{4}(t) - k^{2}x_{4}(t)$$
(29)

is applied to Equation. (28).

Now, defining the state vector of the linear partial system in (29) as $\mathbf{x}_s(t) \Delta [x_1 x_2 x_3 x_4 y_1 y_2 y_3 y_4]^T$ its closed loop linear partial becomes

$$\dot{\boldsymbol{x}}_{s}(t) = A\boldsymbol{x}_{s}(t) \tag{30}$$

Where

<i>A</i> =	0	0	0	0	1	0	0	0]
	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	1
	$-k^{2}$	0	0	0	-2k	0	0	0
	0	$-k^{2}$	0	0	0	-2k	0	0
	0	0	$-k^{2}$	0	0	0	-2k	0
	0	0	0	$-k^{2}$	0	0	0	-2k

Then, the time response of Equation. (30) is described by

$$\boldsymbol{x}_{s}(t) = e^{At} \boldsymbol{x}_{s}(t) \tag{31}$$

where e^{At} is

$$e^{-kt} \begin{bmatrix} 1+kt & 0 & 0 & 0 & t & 0 & 0 & 0 \\ 0 & 1+kt & 0 & 0 & 0 & t & 0 & 0 \\ 0 & 0 & 1+kt & 0 & 0 & 0 & t & 0 \\ 0 & 0 & 0 & 1+kt & 0 & 0 & 0 & t \\ -k^2t & 0 & 0 & 0 & 1-kt & 0 & 0 \\ 0 & -k^2t & 0 & 0 & 0 & 1-kt & 0 \\ 0 & 0 & -k^2t & 0 & 0 & 0 & 1-kt \\ 0 & 0 & 0 & -k^2t & 0 & 0 & 0 & 1-kt \end{bmatrix}$$

Therefore, the closed-loop linear partial system is reduced to

$$\begin{aligned} x_{1}(t) &= x_{1}(0) \left[e^{-kt} + kte^{-kt} \right] + y_{1}(0)te^{-kt} \\ x_{2}(t) &= x_{2}(0) \left[e^{-kt} + kte^{-kt} \right] + y_{2}(0)te^{-kt} \\ x_{3}(t) &= x_{3}(0) \left[e^{-kt} + kte^{-kt} \right] + y_{3}(0)te^{-kt} \\ x_{4}(t) &= x_{4}(0) \left[e^{-kt} + kte^{-kt} \right] + y_{4}(0)te^{-kt} \\ y_{1}(t) &= x_{1}(0) \left[-k^{2}te^{-kt} \right] + y_{1}(0) \left[e^{-kt} - kte^{-kt} \right] \\ y_{2}(t) &= x_{2}(0) \left[-k^{2}te^{-kt} \right] + y_{1}(0) \left[e^{-kt} - kte^{-kt} \right] \\ y_{3}(t) &= x_{3}(0) \left[-k^{2}te^{-kt} \right] + y_{1}(0) \left[e^{-kt} - kte^{-kt} \right] \\ y_{4}(t) &= x_{4}(0) \left[-k^{2}te^{-kt} \right] + y_{1}(0) \left[e^{-kt} - kte^{-kt} \right] \end{aligned}$$
(32)

The time response of nonlinear term becomes

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$$x_{5}(t) = x_{5}(0) + \left(x_{1}(0)y_{2}(0) - y_{1}(0)x_{2}(0)\int_{0}^{t} e^{-2k\tau}d\tau\right)$$

$$= x_{5}(0) - \left(\frac{x_{1}(0)y_{2}(0)}{2k} - \frac{y_{1}(0)x_{2}(0)}{2k}\right)\left(e^{-2k\tau} - 1\right)$$
(33)

Utilizing constant term of Equation. (33),

$$S_{1}(t) = x_{5}(t) + \frac{1}{2k}x_{1}(t)y_{2}(t) - \frac{1}{2k}x_{2}(t)y_{1}(t)$$
(34)

are selected to the candidate of invariant manifold. The differentiation of Equation. (34) is

$$\dot{S}_{1}(t) = \dot{x}_{5}(t) + \frac{1}{2k} \left[\dot{x}_{1}(t) y_{2}(t) + x_{2}(t) \dot{y}_{2}(t) \right] - \frac{1}{2k} \left[\dot{x}_{2}(t) y_{1}(t) + x_{2}(t) \dot{y}_{1}(t) \right] = 0$$
(35)

Then it is found that $S_1(t)$ is an invariant manifold. From other nonlinear terms, invariant manifolds can be selected as

$$S_{2}(t) = x_{6}(t) + \frac{1}{2k}x_{1}(t)y_{3}(t) - \frac{1}{2k}x_{3}(t)y_{1}(t)$$

$$S_{3}(t) = x_{7}(t) + \frac{1}{2k}x_{1}(t)y_{4}(t) - \frac{1}{2k}x_{4}(t)y_{1}(t)$$

$$S_{4}(t) = x_{8}(t) + \frac{1}{2k}x_{2}(t)y_{3}(t) - \frac{1}{2k}x_{3}(t)y_{2}(t)$$

$$S_{5}(t) = x_{9}(t) + \frac{1}{2k}x_{2}(t)y_{4}(t) - \frac{1}{2k}x_{4}(t)y_{2}(t)$$

$$S_{6}(t) = x_{10}(t) + \frac{1}{2k}x_{3}(t)y_{4}(t) - \frac{1}{2k}x_{4}(t)y_{2}(t)$$
(36)

The differentiation of Equation. (36) is

$$\begin{split} \dot{S}_{2}(t) &= \dot{x}_{6}(t) + \frac{1}{2k} \Big[\dot{x}_{1}(t) y_{3}(t) + x_{1}(t) \dot{y}_{3}(t) \Big] \\ &- \frac{1}{2k} \Big[\dot{x}_{3}(t) y_{1}(t) + x_{3}(t) \dot{y}_{1}(t) \Big] \equiv 0 \\ \dot{S}_{3}(t) &= \dot{x}_{7}(t) + \frac{1}{2k} \Big[\dot{x}_{1}(t) y_{4}(t) + x_{1}(t) \dot{y}_{4}(t) \Big] \\ &- \frac{1}{2k} \Big[\dot{x}_{4}(t) y_{1}(t) + x_{4}(t) \dot{y}_{1}(t) \Big] \equiv 0 \\ \dot{S}_{4}(t) &= \dot{x}_{8}(t) + \frac{1}{2k} \Big[\dot{x}_{2}(t) y_{3}(t) + x_{2}(t) \dot{y}_{3}(t) \Big] \\ &- \frac{1}{2k} \Big[\dot{x}_{3}(t) y_{2}(t) + x_{3}(t) \dot{y}_{2}(t) \Big] \equiv 0 \\ \dot{S}_{5}(t) &= \dot{x}_{9}(t) + \frac{1}{2k} \Big[\dot{x}_{2}(t) y_{4}(t) + x_{2}(t) \dot{y}_{4}(t) \Big] \\ &- \frac{1}{2k} \Big[\dot{x}_{4}(t) y_{2}(t) + x_{4}(t) \dot{y}_{2}(t) \Big] \equiv 0 \\ \dot{S}_{6}(t) &= \dot{x}_{10}(t) + \frac{1}{2k} \Big[\dot{x}_{3}(t) y_{4}(t) + x_{3}(t) \dot{y}_{4}(t) \Big] \\ &- \frac{1}{2k} \Big[\dot{x}_{4}(t) y_{3}(t) + x_{4}(t) \dot{y}_{3}(t) \Big] \equiv 0 \end{split}$$

(37)

Then, $S_i(t)$ is invariant manifold because $S_i(t)$ converges to zero.

CONCLUSIONS

In this paper, we study an invariant manifold for double integrator and extended double integrator nonholonomic systems with two and more control inputs. In future, we will develop a switching control technique based on the invariant manifold theory to stabilize nonholonomic underactuated systems.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge all the people who contributed directly or indirectly and this work is supported in part by the Research Acculturation Collaborative Effort (RACE) Grant Scheme under grant no. RDU141301.

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