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# SIMULTANEOUS COMPUTATION OF MODEL ORDER AND PARAMETER ESTIMATION FOR ARX MODEL BASED ON MULTI-SWARM PARTICLE SWARM OPTIMIZATION

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**ABSTRACT**

Simultaneous Model Order and Parameter Estimation (SMOPE) is a method of utilizing meta-heuristic algorithm to iteratively determine an optimal model order and parameters simultaneously for an unknown system. SMOPE was originally introduced using Particle Swarm Optimization (PSO). However, the performance was worse than conventional ARX. Hence, the objective of this paper is to introduce a new computational model of the SMOPE which employs multi-swarm strategy in original SMOPE to diversify the search moves of meta-heuristic algorithm when searching for the best mathematical model. Experiments are performed on six system identification problems. The obtained results prove that incorporating the multi-swarm approach is a good idea to improve original SMOPE.

**Keywords**: particle swarm optimization, multi-swarm, system identification, model order selection and parameter estimation.

## INTRODUCTION

System identification is a method employed to obtain a mathematical model of a system by performing analysis on input-output behaviour of the system. Fundamental steps of system identification procedure are generally summarized into four main stages. The primary stage is collection of experimental data. Following that, the model order is selected. The next stage is to approximate the parameters of the model and lastly, the mathematical model is validated.

Auto-Regressive Model with Exogenous Inputs (ARX) is the most basic model in linear black box identification [1]. Conventionally, in addressing the system identification problem of ARX model, the model order selection and parameter estimation are done separately.

There are some techniques reported in literature in solving system identification problems. Hansson *et al.* presented a subspace system identification method based on weighted nuclear norm approximation [2,3]. Moreover, there are some methods proposed to address system identification problem based on meta-heuristic algorithm but it mainly focus on parameter estimation only [4,5].

SMOPE was proposed to address system identification problem efficiently using meta-heuristics algorithms [6]. The technique enabled the computation of model order and parameters values to be done concurrently. This is achievable through the way the problem is encoded in the search agents. Furthermore, SMOPE could also successfully be adapted to fit with other meta-heuristic algorithm like Gravitational Search Algorithm (GSA) [7].

However, SMOPE method unable to offer the best mathematical model and its performance is significantly worse than conventional ARX due to limitation of its computational model. To overcome this defect, inspired by the concept of multi-swarm technique, a new computation model termed as Simultaneous Model Order and Parameter Estimation based on Multi-Swarm approach (SMOPE-MS) is proposed. The strategy is assigning each swarm of meta-heuristic algorithm to each model order of ARX mathematical equation. Outcomes show that the proposed method is valid and can attain better solution quality.

PSO is a bio-inspired optimization algorithm introduced by Kennedy and Eberhart [8]. The search is based on the concept that particles move through the search space from their current positions with velocities dynamically modified depending on their current velocities, best self-experienced position and best global-experienced position with some influence of randomness.

In this paper, the implementation of SMOPE-MS using PSO is studied and compared with original SMOPE as well as Conventional ARX. Six ARX system identification problems are used for verification. The results show that SMOPE-MS is better than original SMOPE and has comparable performance when compared to conventional ARX.

The remainder of this paper is organized as follows: Section 2 briefly reviews the SMOPE technique. Section 3 explains the proposed SMOPE-MS technique. Section 4 provides the experimental settings and discusses the experimental results. Section 5 concludes the paper.

## SIMULTANEOUS COMPUTATION OF MODEL ORDER AND PARAMETER ESTIMATION (SMOPE)

Contrary to other system identification techniques, SMOPE obtain the optimal system order and the parameters values simultaneously. The key of SMOPE is the encoding of the search agents. For that reason, by applying same encoding, SMOPE can simply be integrated to other meta-heuristic algorithms. The agent's encoding employed in SMOPE is shown in Table-1. The transfer function of ARX model used in SMOPE is as follow:

VOL. 10, NO. 22, DECEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences



$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} \quad (1)$$

where $m$ and $n$ are the number of numerator and denominator orders of the transfer function respectively and $a_n$ and $b_m$ are the pole and zero parameters that will be tuned by optimization algorithm as well as model order.

**Table-1.** Particle representation.

| Dimension | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Variable in ARX | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ |

**Table-2.** ARX parameters selected for the calculation of best fit (n=1,2,3,4,5,6).

| Order, $n$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | X | | | | | | | | | X | | | | | | | | |
| 2 | X | X | | | | | | | | X | | | | | | | | |
| 2 | X | X | | | | | | | | X | X | | | | | | | |
| 3 | X | X | X | | | | | | | X | | | | | | | | |
| 3 | X | X | X | | | | | | | X | X | | | | | | | |
| 3 | X | X | X | | | | | | | X | X | X | | | | | | |
| 4 | X | X | X | X | | | | | | X | | | | | | | | |
| 4 | X | X | X | X | | | | | | X | X | | | | | | | |
| 4 | X | X | X | X | | | | | | X | X | X | | | | | | |
| 4 | X | X | X | X | | | | | | X | X | X | X | | | | | |
| 5 | X | X | X | X | X | | | | | X | | | | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | | | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | X | | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | X | X | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | X | X | X | | | | |
| 6 | X | X | X | X | X | X | | | | X | | | | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | | | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | X | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | X | X | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | X | X | X | | | |

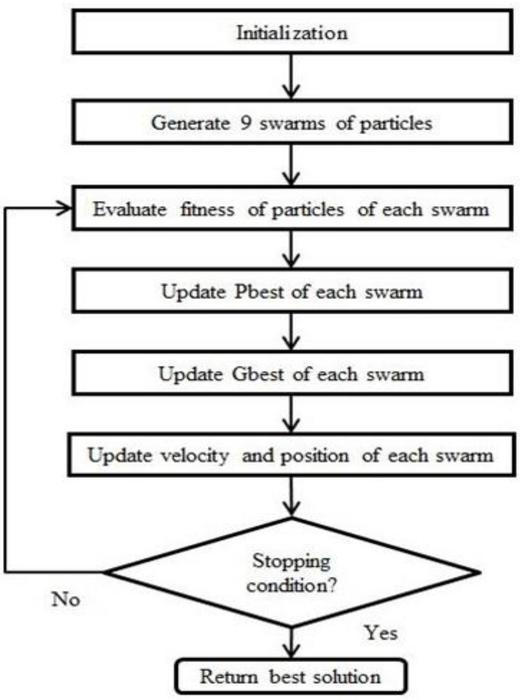

**Figure-1.** Flowchart of PSO for SMOPE-MS.

In SMOPE, maximum order of 9th is taken into account. To determine the parameter ‘a’ and ‘b’, the constraint n ≥ m is considered. This is based on the transfer function form which the order value of poles (n value) must be the same or greater than the order of zeroes (m value).

## SIMULTANEOUS COMPUTATION OF MODEL ORDER AND PARAMETER ESTIMATION BASED ON MULTI-SWARM APPROACH (SMOPE-MS)

In this part, the new computational model of SMOPE is discussed. SMOPE-MS computes both model order and parameter in identification problem simultaneously by using multi-swarm PSO.

The function of multi-swarm PSO, which is presented in Figure-1, is to broaden the search ability of PSO in SMOPE when searching for the best mathematical model. Each swarm will be assigned to tune the parameters of ARX mathematical equations for particular model order. Table-2 and Table-3 specify which ARX equation parameters should be considered for any assigned number of order, n. Thus, a set of 45 mathematical models are tested according to n value and 9 swarm of PSO will be employed to search for the best mathematical model.

VOL. 10, NO. 22, DECEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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As an example, if the model order is 2, therefore the second swarm of PSO is assigned and all possible mathematical models are subjected to fitness calculation. In that case, the computations focus on two mathematical models, which are

$$\frac{b_1 z^{-1}}{1+a_1 z^{-1}+a_2 z^{-2}} \text{ and } \frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

**Table-3.** ARX parameters selected for the calculation of *best fit* (*n*=7,8,9).

| Order, *n* | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | X | X | X | X | X | X | X | | | X | | | | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | | | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | X | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | X | X | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | X | X | X | | |
| 8 | X | X | X | X | X | X | X | X | | X | | | | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | | | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | X | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | X | X | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | X | X | X | |
| 9 | X | X | X | X | X | X | X | X | X | X | | | | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | | | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X |

**Table-4.** PSO parameters value.

| Parameters | Value |
|---|---|
| Population size | 100 |
| Decrease inertia weight | 0.9~0.4 |
| Cognitive component | 2 |
| Social component | 2 |
| Number of iterations | 2000 |
| Number of run | 50 |

Another example, if the model order is 3, then third swarm of PSO is assigned and the computations involve three mathematical models, which are $\frac{b_1 z^{-1}}{1+a_1 z^{-1}+a_2 z^{-2}+a_3 z^{-3}}$, $\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}+a_3 z^{-3}}$ and $\frac{b_1 z^{-1}+b_2 z^{-2}+b_3 z^{-3}}{1+a_1 z^{-1}+a_2 z^{-2}+a_3 z^{-3}}$.

Note that up to ninth order mathematical model of ARX is considered for the purpose of this research. The PSO parameter values used in this study is shown in Table-4. The PSO parameter comprises the number of particles, inertia weight, cognitive component $c_1$ and social component $c_2$. The initial placement of particles is randomly located in a search space. After the initialization stage is complete, the fitness function is evaluated as follows:

$$best\ fit = 100\left[1-\frac{norm(y_{(actual)}-y_{(estimated)})}{norm(y_{(actual)}-y_{(mean)})}\right]\% \qquad (2)$$

The personal best or *pbest* is the best solution discovered by each particle and *gbest* is defined as the best *pbest*. Both *pbest* and *gbest* are updated once the best solution is found at every iteration. Note that, there are 9 swarm used in this algorithm, thus there will be 9 *pbest* and 9 *gbest* are going to be updated. The velocity of a particle in each swarm is updated using Equation. (3).

$$\begin{aligned} v_i^n(t+1) &= \omega v_i^n(t) + c_1 rand(p_{best}^n - x_i^k(t)) + \\ & c_2 rand(g_{best}^n - x_i^n(t)) \end{aligned} \qquad (3)$$

where $v_i^n(t)$ is the velocity particle $i$ in swarm $n$ at iteration $t$, $p_{best}^n$ and $g_{best}^n$ are personal best and global best in swarm $n$ respectively, *rand* is random numbers [0,1], $\omega$ is inertia weight, and $c_1$ and $c_2$ indicate the cognitive and social coefficients, respectively. The

VOL. 10, NO. 22, DECEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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particle's new velocity is then used to update the particle's position in each swarm according to Equation. (4).

$$x_i^n(t+1) = x_i^n(t) + v_i^n(t+1) \quad (4)$$

where $x_i^n(t)$ is the position of particle $i$ in swarm $n$ at iteration $t$. In this research, the linear dynamic inertia weight is used and calculated according to Equation. (5) as follows:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{t_{\max}} \times t \quad (5)$$

where $\omega_{max}$ and $\omega_{min}$ denote the maximum and minimum values of inertia weight, respectively, and $k_{max}$ is the maximum iteration.

The algorithm terminates when the stopping condition is satisfied. When the algorithm process ends, the final optimum solution which is the best solution among 9 *gbest* of each swarm is reported as shown in Equation. (6).

$$optimum\_solution = \min(g_{best}^n) \quad (6)$$

**Table-5.** Summary result for six dataset.

| Data Set | Best Fit (Training) (%) | | Average Best Fit (Training) (%) | STDEV (Training) | Best Fit (Testing) (%) | | Average Best Fit (Testing) (%) | STDEV (Testing) |
|---|---|---|---|---|---|---|---|---|
| | Min | Max | | | Min | Max | | |
| Heating System | 98.98 | 99.14 | 99.04 | 0.03 | 98.50 | 98.90 | 98.71 | 0.07 |
| Hair Dryer System | 94.00 | 95.44 | 95.08 | 0.33 | 93.92 | 95.31 | 94.98 | 0.36 |
| Ball Beam System | 97.41 | 97.47 | 97.46 | 0.01 | 97.75 | 97.86 | 97.80 | 0.02 |
| Robot Arm System | 96.63 | 99.39 | 97.74 | 0.80 | 96.58 | 99.33 | 97.67 | 0.81 |
| Wing Flutter System | 98.68 | 99.47 | 99.08 | 0.14 | 93.56 | 97.62 | 96.13 | 0.74 |
| Exchanger System | 80.86 | 81.23 | 80.99 | 0.09 | 50.30 | 52.06 | 50.73 | 0.49 |

## EXPERIMENTS

The experimental data used for the benchmarking, is Database for Identification of Systems, (DaISy), which is taken from http://www.esat.kuleuven.ac.be/sista/daisy. The data for heating system, hair dryer system, ball beam system, robot arm system and exchanger system are produced from laboratory works while the data of wing flutter system is obtained from industry.

The data is equally separated for training and testing. The training data is used to find the best mathematical model based on ARX model while the testing data is used to assess the quality of mathematical model offered by SMOPE-MS.

For each system, the numbers of data points are separated equally into the proportions of 50% for training samples and 50% for testing samples from the entire dataset. As an example, for heating system, 400 number of samples are used for training and another 400 number of samples are used for testing. The similar procedure has been employed by L. Ljung in conventional ARX [1].

In the SMOPE-MS based on PSO, each particle determines a suitable model order and parameters of the ARX model from first order up to ninth order. The algorithm will stop when the iteration count exceeds 2000. Each of the experiment is repeated 50 times and the results are averaged.

## RESULT AND DISCUSSION

Using MATLAB for simulation and based on PSO parameter in Table-4, the results obtained from the experiment are shown in Table-5. Based on the results, out of the six systems used, SMOPE-MS outperforms the original SMOPE in all cases as shown in Table-6. All the comparison is based on the best fit value acquired at the testing stage. Table-9 shows the mathematical equation offered by SMOPE-MS (PSO) and Figure-2 shows examples of convergence performance of the proposed technique for heating system and hair dryer system.

To analyse the results of these techniques statistically, the best fit value for each benchmark case study were used. Based on the data, the performances of all techniques were ranked by using Friedman statistical test for non-parametric data. Based on Friedman Test, the average rankings of these techniques are shown in Table-7.

These techniques can be sorted by average ranking into the following order: Conventional ARX, SMOPE-MS (PSO), SMOPE (PSO) and SMOPE (GSA). The Friedman statistic for this experiment is 18.000. The performance difference is significant if this value is greater than 7.815 (based on 3 degree of freedom at a 0.05 level of significance according to Chi-square table). Since this value is greater than 7.815, hence, significant different exist in term of performance among these techniques.

Thus, to compare the performance differences significantly between these techniques, the Friedman Post Hoc Test was executed. Post Hoc Test using Holm's procedure is chosen to evaluate the significance difference between the techniques' performance. Table-8 shows the resultant p-values when comparing between SMOPE-MS and the other three techniques. Holm's procedure rejects those hypotheses that have an unadjusted p-value lower than 0.0167. The rejection of the hypotheses indicates a significant difference exists between the performances of two techniques. The p-values below 0.0167 are shown in bold.

VOL. 10, NO. 22, DECEMBER 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences



Based on the result, it shows that original SMOPE method is significantly worse than conventional ARX, while our proposed SMOPE-MS has comparable performance with conventional ARX. To put it simply, our technique can become an alternative to solve system identification problem, instead of using conventional method.

**Table-6.** Best fit value comparison of SMOPE-MS with other techniques.

| DATASET | Conventional ARX | SMOPE-MS (PSO) | SMOPE (PSO) | SMOPE (GSA) |
|---|---|---|---|---|
| Heating System | 98.94 | 98.71 | 98.08 | 96.54 |
| Hair dryer System | 95.51 | 94.98 | 88.23 | 83.61 |
| Ball beam System | 97.82 | 97.80 | 97.35 | 95.88 |
| Robot arm System | 99.82 | 97.67 | 95.08 | 93.69 |
| Wing flutter System | 99.46 | 96.13 | 93.31 | 88.89 |
| Exchanger System | 51.68 | 50.73 | 49.92 | 44.99 |

**Table-7.** Average rankings of the techniques.

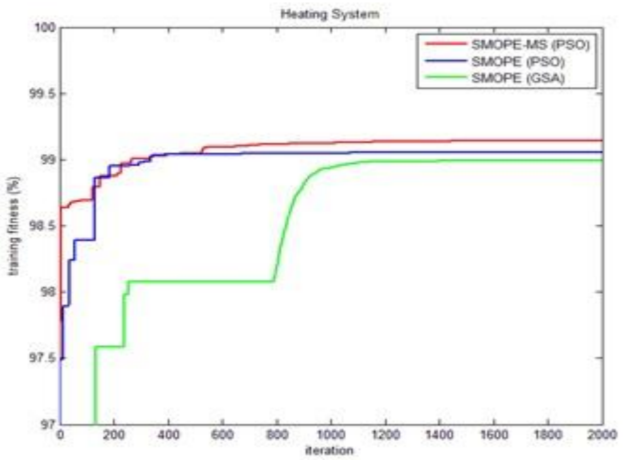
| Techniques | Ranking |
|---|---|
| Conventional ARX | 1 |
| SMOPE-MS (PSO) | 2 |
| SMOPE (PSO) | 3 |
| SMOPE (GSA) | 4 |

**Table-8.** P-values table for α=0.05.

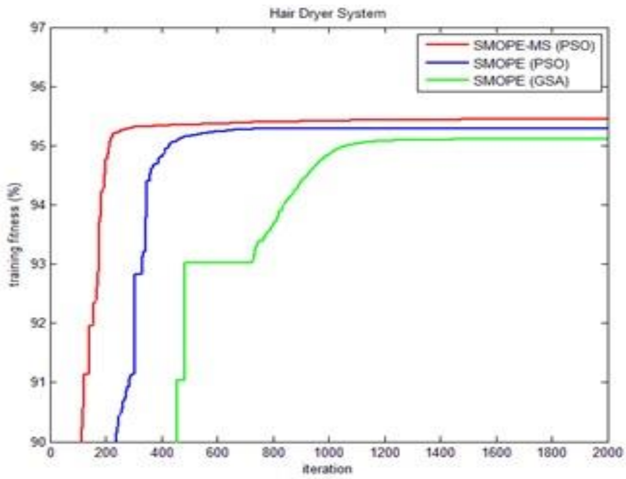
| Techniques | z | p | Holm |
|---|---|---|---|
| Conventional ARX vs. SMOPE (GSA) | 4.0249 | **0.0001** | 0.0083 |
| Conventional ARX vs. SMOPE (PSO) | 2.6833 | **0.0073** | 0.0100 |
| SMOPE-MS (PSO) vs. SMOPE (GSA) | 2.6833 | **0.0073** | 0.0125 |
| SMOPE-MS (PSO) vs. SMOPE (PSO) | 1.3416 | 0.1797 | 0.0167 |
| Conventional ARX vs. SMOPE-MS (PSO) | 1.3416 | 0.1797 | 0.0250 |
| SMOPE (PSO) vs. SMOPE (GSA) | 1.3416 | 0.1797 | 0.0500 |

**Table-9.** Mathematical model.

| | |
|---|---|
| Heating System | $G(z)=\dfrac{-0.0005z^{-1}+0.4808z^{-2}+0.1311z^{-3}+0.0690z^{-4}-0.1463z^{-5}-0.1596z^{-6}+0.0042z^{-7}-0.2298z^{-8}}{1-1.2113z^{-1}+0.0836z^{-2}+0.0348z^{-3}+0.1160z^{-4}-0.2929z^{-5}+0.3976z^{-6}-0.2510z^{-7}+0.2325z^{-8}-0.1024z^{-9}}$ |
| Hair Dryer System | $G(z)=\dfrac{0.0013z^{-1}+0.0037z^{-2}+0.0649z^{-3}+0.0659z^{-4}+0.0215z^{-5}}{1-0.9091z^{-1}-0.0605z^{-2}+0.0927z^{-3}+0.0827z^{-4}-0.0434z^{-5}}$ |
| Ball Beam System | $G(z)=\dfrac{-0.2015z^{-1}+0.3150z^{-2}-0.2408z^{-3}+0.3258z^{-4}}{1-0.8758z^{-1}-0.3297z^{-2}+0.1050z^{-3}-0.1338z^{-4}-0.0146z^{-5}+0.0351z^{-6}+0.1600z^{-7}+0.0465z^{-8}+0.0046z^{-9}}$ |
| Robot Arm System | $G(z)=\dfrac{-0.4368z^{-1}+0.6524z^{-2}-0.8483z^{-3}+1.1425z^{-4}-0.5533z^{-5}}{1-1.3849z^{-1}+0.3952z^{-2}+0.1516z^{-3}+0.6102z^{-4}-0.4014z^{-5}-0.4310z^{-6}+0.3623z^{-7}+0.1793z^{-8}}$ |
| Wing Flutter System | $G(z)=\dfrac{-0.0120z^{-1}}{1-2.7286z^{-1}+2.5742z^{-2}-0.9038z^{-3}+0.6270z^{-4}-1.0478z^{-5}+0.5797z^{-6}-0.0612z^{-7}}$ |
| Exchanger System | $G(z)=\dfrac{0.0351z^{-1}-0.3797z^{-2}+0.0399z^{-3}-0.3511z^{-4}+0.7647z^{-5}}{1-1.1395z^{-1}+0.2493z^{-2}-0.1812z^{-3}+0.2446z^{-4}-0.3078z^{-5}+0.2063z^{-6}+0.0422z^{-7}-0.0872z^{-8}-0.0264z^{-9}}$ |



(a)



(b)

**Figure-2.** Convergence curve for (a) heating system, (b) hair dryer system.

## CONCLUSIONS

This paper proposed an enhanced version of SMOPE in order to combine model order selection and parameter estimation of ARX model based on PSO algorithm. The overall performance is evaluated based on six case studies. According to the experimental results, it was observed that the SMOPE-MS approach outperforms the basic SMOPE and has comparable performance with conventional ARX. More importantly, the significant advantage of this method is the ability to skip common approaches for model order selection such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) by simultaneously computed it with the parameters of a system [9,10]. For future research, different optimization algorithm shall be considered to improve further the capability of the proposed technique.

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