



ATTITUDE CONTROL OF QUADROTOR

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ABSTRACT

This paper presents the attitude behaviour of quad-rotor by using an LQR controller. The performance of the LQR controller is compared to a PD controller. The controller is designed by using mathematical approaches and the results are obtained through the MATLAB simulations in the term of settling time and overshoot percentage.

Keywords: quad-rotor, control technique, LQR, PD controller.

INTRODUCTION

Quad-rotor is an object that can be categorized as Unmanned Air Vehicle (UAV). It uses a similar concept with helicopters but propelled by using four motors and four propellers. Its lift and motion are supported by all four propellers and motors. Quad rotor does not have any tail to support the pitch like a helicopter. In case of stability during hovering, the direction of the propellers' rotation plays the important role. Classifying quad-rotor as a Vertical Take-Off and Landing (VTOL) UAV shows more advantages compared to air-planes that use fixed wings. Capable to move in any direction, hovering and fly at low speed and so on make quad rotor a special VTOL vehicle. Its complex dynamic and characteristic leads on higher amounts of control technique research. Non-linear structure of quad-rotor is currently being researched to be applied with the linear controller. LQR controller is chosen in this project due to its unlimited range and it is a good comparative controller.

Previous researches on quad-rotor control provide various types of controller such as PID, SMC, Fuzzy, Neural Network and so on [1]. Most of the controllers are higher in complexity and do not have manual tuning. Thus, the model should be improved in order to obtain a precise value. To achieve stability in quad-rotor, an optimal control algorithm is suggested to be applied. This type of control can be found in LQR, LQG, L_i , H_∞ , Fuzzy, Neural network and genetic algorithm. Thus, LQR controller is chosen due to its simplicity and its fast convergence response.

This paper is organized as follows; Part 2 will discuss the system and modelling of the quad-rotor. Part 3 is for the design of linear quadratic regulator. Part 4 will be the results and discussions. And last, part 5 is the conclusion of the research.

QUAD-ROTOR SYSTEM AND MODELLING

Quad-rotor basic structure

A quad rotor consists of translational and rotational axes where translational is on x,y and z axes and rotational is on roll, ϕ , pitch, θ and yaw, ψ . This is due to the state of quad rotor that being rigid and not grounded.

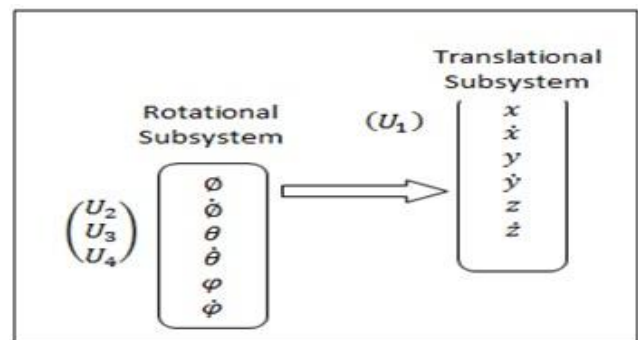


Figure-1. Quad-rotor systems.

Rotors aerodynamics

A quad-rotor consists of four propellers that will rotate in clockwise direction for the front and back rotors and anticlockwise direction for the left and right rotors as shown in Figure-2. The directions are fixed to maintain the drag created by the cancellation of induced moments due to opposite movements. [2][3].

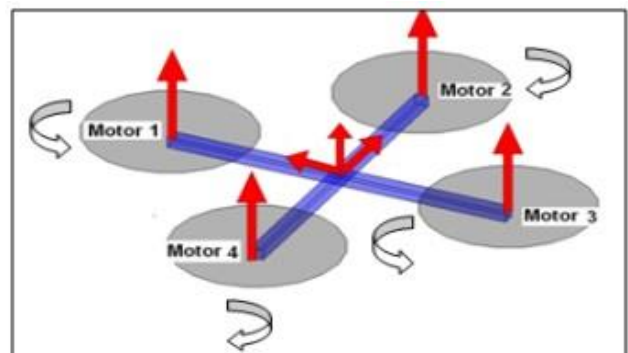


Figure-2. Quad-rotor motor rotation.

Each rotor changes its speed to satisfy the desired position. Different motion requires different action on each rotor.

i. Roll rotation

Rotation on x-axis is called roll. The speed of rotor 2 and 4 are manipulated. To make a roll movement



to the right, the speed of rotor 2 is decreased and rotor 4 is increased. The reverse strategy, increasing speed on rotor 2 and decreasing speed on rotor 4 is applied when the quad rotor roll to the left side. During the roll movement, rotor 1 and 3 are maintained in speed. Roll rotation gives an effect to the y-axis angular velocity.

$$\begin{aligned} \text{Rotor1} &= \text{Rotor3} = \text{pwm} \\ \text{Rotor2} &= \text{pwm} + \Delta\text{pwm} \\ \text{Rotor4} &= \text{pwm} - \Delta\text{pwm} \end{aligned} \quad (1)$$

ESCs controls the rotors speed using *pwm* signal. The *pwm* is calculated because of its duty cycle. Its power loss when switching the devices is very low that leads to higher system inficiency.

ii. Pitch rotation

Increasing motor 1 and decreasing the motor 3 while maintaining the speed of motor 2 and 4 will lead to positive pitch rotations. Additionally, for the negative pitch rotation the reverse operation can be done where speed of motor 1 is decreased and motor 3 is increased.

$$\begin{aligned} \text{Rotor2} &= \text{Rotor4} = \text{pwm} \\ \text{Rotor1} &= \text{pwm} + \Delta\text{pwm} \\ \text{Rotor3} &= \text{pwm} - \Delta\text{pwm} \end{aligned} \quad (2)$$

iii. Yaw

Increasing the speed of rotor on y-axis and decreasing the rotor speeds on x-axis will perform the positive yaw rotation. Reversely, negative yaw rotation can be obtain by decreasing the speed of rotors on y-axis and increasing the rotors speeds on x-axis.

$$\begin{aligned} \text{Rotor1} &= \text{Rotor3} = \text{pwm} - \Delta\text{pwm} \\ \text{Rotor2} &= \text{Rotor4} = \text{pwm} + \Delta\text{pwm} \end{aligned} \quad (3)$$

By performing the *pwm* as shown above, positive yaw rotation will be performed.

Dynamic modelling

Quad rotor dynamic can be described by 12 states which consist of position (*P*) in *x, y, z* axes, rotational angles (Ω) of roll, pitch, yaw, linear velocity (*V*) of *u, v, w*, and angular velocities (ω) of *p, q, r* [4]. Linear motion, rotational angles, linear velocity and angular velocities are identified as the outputs. The inputs applied are torque and force that had been generated based on the all of motor rotations. This system state can be represented as shown in Figure-3 below.

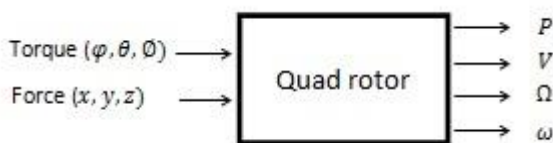


Figure-3. Quad-rotor input/ output

Attitude model dynamic can be derived from Newton's Law [5]. Each rotor rotation can be cancelled out due to vice versa rotating pairs which automatically

removes coupling between dynamic of pitch and roll. The usage of a small propeller can reduce the torque and increase the motor efficiency. The blades can drive higher velocity without using any additional of mechanical vibration. Stress on mechanical parts of the craft is automatically reduces if the vibration and torque decreased [6]. To simplify the dynamics of the system, some assumptions are assigned where the quad rotor and propeller are rigid and symmetric systems, and the ground effect is neglected.

Simplifying this project, quad-rotor system dynamics are taken from Samir Bouabdallah's paper [7] as shown in Equation.4.

$$f(X, U) = \begin{bmatrix} x_2 \\ (\cos x_7 \sin x_9 \cos x_{11} + \sin x_7 \sin x_{11}) \left(\frac{U_1}{m} \right) \\ x_4 \\ (\cos x_7 \sin x_9 \sin x_{11} - \sin x_7 \cos x_{11}) \left(\frac{U_1}{m} \right) \\ x_6 \\ -g + (\cos x_7 \cos x_9) \left(\frac{1}{m} \right) U_1 \\ x_8 \\ x_{12} x_{10} \left(\frac{I_y - I_z}{I_x} \right) - \left(\frac{J_r}{I_x} \right) x_{10} \Omega + \left(\frac{1}{I_x} \right) U_2 \\ x_{10} \\ x_{12} x_8 \left((I_z - I_x) I_y \right) - \left(\frac{J_r}{I_y} \right) x_8 \Omega + \left(\frac{1}{I_y} \right) U_3 \\ x_{12} \\ x_{10} x_8 \left(\frac{I_z - I_y}{I_x} \right) + \left(\frac{1}{I_z} \right) U_4 \end{bmatrix} \quad (4)$$

Model implemented is a non-linear system. It is necessary to linearize the equation obtained before using the linear controller to have a stable hovering condition. Including mass of the quad rotor and assuming that lever, *b* and drag factor, *d* have a same value for each rotor, the Torque applied is,

$$T_{\text{rotor}} = \begin{bmatrix} lb (\Omega_4 - \Omega_2) \\ lb (\Omega_3 - \Omega_1) \\ d (\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \end{bmatrix} \quad (5)$$

The system input is represented using U_1, U_2, U_3 and U_4 and Ω as the disturbance. Speed of rotor 1, 2, 3 and 4 can be clearly seen as the main components in the system inputs as shown in Eq. 6.

$$\left. \begin{aligned} U_1 &= b(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4) \\ U_2 &= b(\Omega_4 - \Omega_2) \\ U_3 &= b(\Omega_3 - \Omega_1) \\ U_4 &= d(\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \\ \Omega &= \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3 \end{aligned} \right\} \quad (6)$$

Found that to generate thrust motion required $-z$ direction [8], thus



$$F_{rotors} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -U1 \end{bmatrix} \quad (7)$$

$$T_{rotor} = \begin{bmatrix} U2 \\ U3 \\ U4 \end{bmatrix} \quad (8)$$

$$U = \begin{bmatrix} U1 \\ U2 \\ U3 \\ U4 \end{bmatrix} = \begin{bmatrix} b(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4) \\ lb(\Omega_4 - \Omega_2) \\ lb(\Omega_3 - \Omega_1) \\ d(\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \end{bmatrix} \quad (9)$$

In order to find the linearization part, Equation.9 is inverted by cancelling out each function of the system inputs in Equation.6. Thus the rotor speed equation in terms of system inputs $U1$, $U2$, $U3$ and $U4$ are simplified through the simultaneous equation.

$$\begin{aligned} \Omega_1 &= \frac{U1}{4b} - \frac{U4}{4d} - \frac{U3}{2lb} \\ \Omega_2 &= \frac{U4}{4d} - \frac{U2}{2lb} + \frac{U1}{4b} \\ \Omega_3 &= \frac{U3}{2lb} + \frac{U1}{4b} - \frac{U4}{4d} \\ \Omega_4 &= \frac{U2}{2lb} + \frac{U4}{4d} + \frac{U1}{4b} \end{aligned} \quad (10)$$

This equation acts as an element to decompose the actuating force and torque into four virtual inputs.

LQR CONTROL TECHNIQUES

Control algorithm

Controller is used to minimize the system settling time. LQR algorithm used an optimal control approach where it minimizes the cost functions.

Dynamic model is basically formed in,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (11)$$

J , the cost function of Linear Quadratic Regulator as shown in Equation.12 will be minimized. The value will be used as the linear control law as shown in Equation.13.

$$J = \frac{1}{2} \int_{t_0}^{t_1} (x^T Q x + u^T R u + 2x^T N u) dt + \frac{1}{2} x^T(t_1) F(t_1) x(t_1) \quad (12)$$

$$u = -Kx \quad (13)$$

In order to simplify the minimization of the cost function, LQR command shown in Eq. 14 is used.

$$[k, p, e] = lqr(A, B, Q, R) \quad (14)$$

To obtain linear state space representation, non-linear system is derived as follows based on assumptions that flight is stationary [9].

$$A = \begin{bmatrix} \frac{df_1}{dx_1} & \cdot & \cdot & \cdot & \frac{df_1}{dx_{12}} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{df_{12}}{dx_1} & \cdot & \cdot & \cdot & \frac{df_{12}}{dx_{12}} \end{bmatrix} \quad (15)$$

$$B = \begin{bmatrix} \frac{df_1}{dU_1} & \cdot & \cdot & \cdot & \frac{df_1}{dU_4} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{df_{12}}{dU_1} & \cdot & \cdot & \cdot & \frac{df_{12}}{dU_4} \end{bmatrix} \quad (16)$$

Thus, linear system state space that can be applied in LQR function is obtained as shown in Equation. 17 and Eq. 18

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{Ix} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{Iy} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{Iz} \end{bmatrix} \quad (18)$$

Based on Bryson's Rule, matrices Q and R can be obtained in a reasonable way [10].



$$Q_{ii} = \frac{1}{\text{maximum acceptable value of } z^2} \quad (19)$$

$$R_{jj} = \frac{1}{\text{maximum acceptable value of } u^2} \quad (20)$$

However, trial and error design procedures will lead to the most desired output. Thus, Q and R matrices can be tuned to obtain the desired output.

$$Q = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1M & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1M & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

$$R = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \quad (22)$$

Simulink model

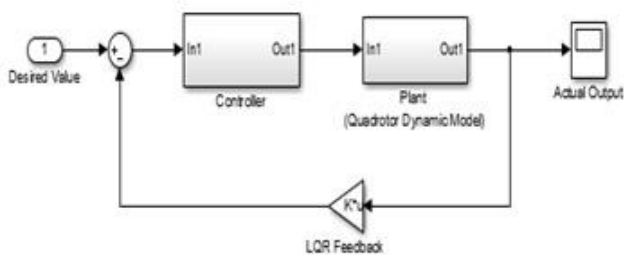


Figure-4. LQR feedback Simulink.

LQR controller is connected as a feedback. It provides the gain from output to be compared with the desired input. Thus, error is able to be reduced and controlled.

RESULTS AND ANALYSIS

The simulation is conducted with a PD and LQR controllers. Step function is set at value 1 and is given as a reference of each input for both systems. Output of the system is compared to the desired input to observe the performance.

LQR control performance

1. Desired attitude is given and the output of the system is connected as a feedback through the LQR gain feedback.
2. LQR selects closed-loop poles that balancing the state error with control effort.

3. Over the time, the magnitude of the control action itself is included to remain the stability.

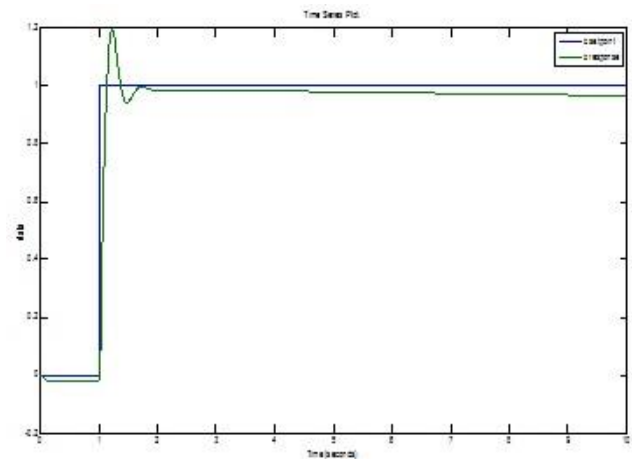


Figure-5. Thrust motion.

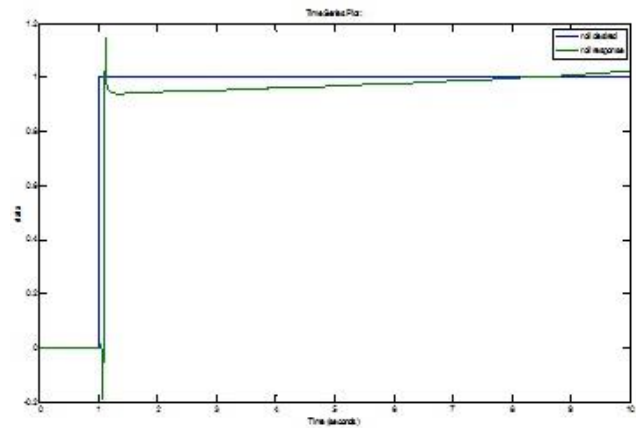


Figure-6. Roll motion.

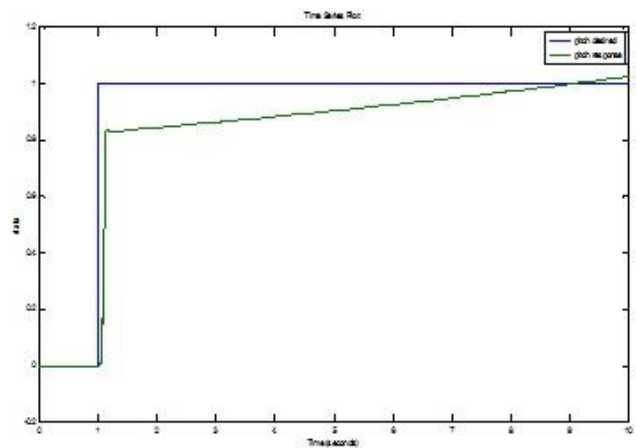


Figure-7. Pitch motion.

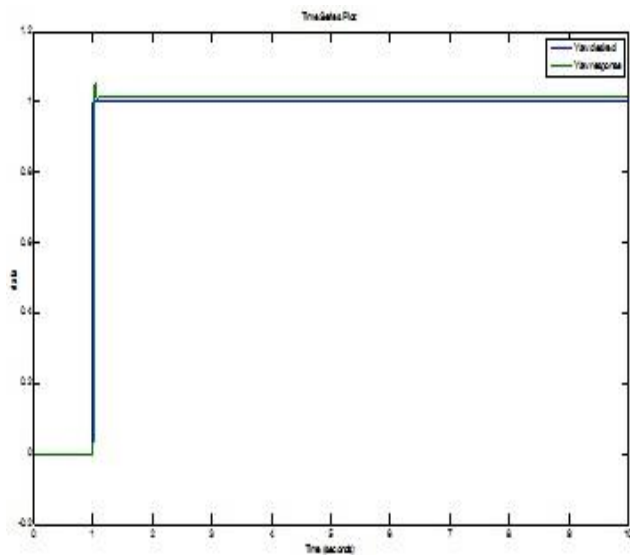


Figure-8. Yaw motion.

PD control performance

1. Output of the system is connected as a unity feedback into the controller.
2. Proportional terms produce output value that proportional to error value.
3. Derivatives function predicts the system performance and improve the system stability.

Table-1. PD controller gain.

System Inputs	K_P	K_D
U_1	0.8	0.4
U_2	1.2	0.4
U_3	1.0	0.4
U_4	100	20

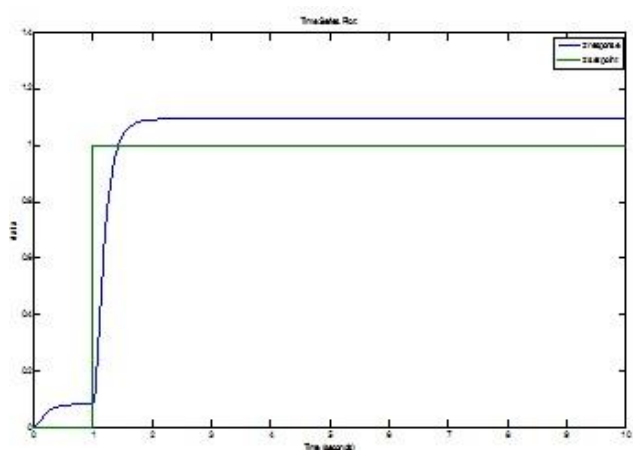


Figure-9. Thrust motion.

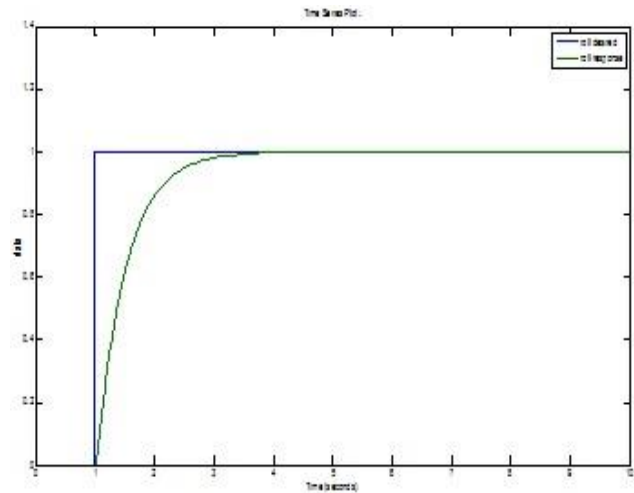


Figure-10. Roll motion.

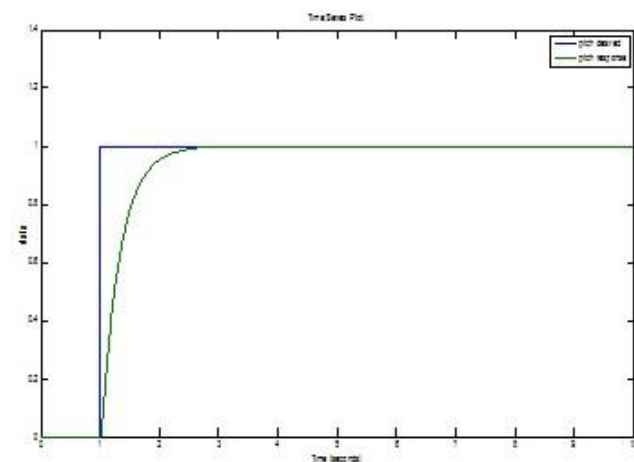


Figure-11. Pitch motion.

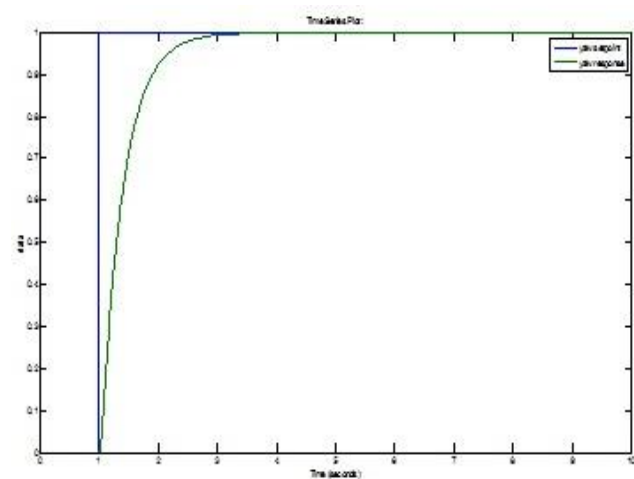


Figure-12. Yaw motion.

The results obtained from both simulations are concluded in terms of settling time and overshoot condition. Table-2 and Table-3 below show the performance of both controllers.

**Table-2.** Attitude response.

	Controller	LQR	PD
Settling Time	Roll	0.25 sec	2.6sec
	Pitch	$\leq 5\text{sec}$	1.7sec
	Yaw	$\leq 0.05\text{sec}$	2.3sec
Overshoot Percentage	Roll	$< 18\%$	0%
	Pitch	0%	0%
	Yaw	$< 5\%$	0%

Table-3. Thrust response.

Controller	LQR	PD
Settling Time	0.7sec	1.3sec
Overshoot Percentage	$< 20\%$	0%

Based on Table-2 and Table-3, LQR is the fastest but PD is the most stable during initiation. Both controllers are seen providing different advantage as concluded in Table-4.

Table-4. Advantages of LQR versus PD control.

Controller	Advantages
LQR	Provide Faster Response
PD control	Overcome overshoot condition

It can be concluded that, LQR controller and PD controller, both can still fulfil the objective to control the attitude of quad-rotor even though they produces an overshoot condition and slows settling time respectively. Tuning parameter give a high impact to the system performance. Thus, trial and error method need to be done carefully. An optimization method such as Genetic Algorithm or PSO can be used as an alternative to get the precise value.

CONCLUSIONS

Simulation of quad-rotor needs knowledge on both system dynamics and control systems. Mathematical model of linear control system for non-linear systems is developed and verified through MATLAB simulations. The performance is evaluated and compared to the other controller systems. Derivation is based on the rotor dynamics and aerodynamics systems that found in literature review. Using LQR control technique, the attitude performance of quad-rotor is successfully controlled. Derivation of linear controller LQR and the gain tuning of proportional gain and LQR gain are done for better response. Other differential and integral terms can provide higher stability of quad-rotor performance. This showed that it is possible to control non-linear system

using a linear controller and angular movements of quad-rotor is possible to be controlled in optimal ways.

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REFERENCES

- [1] C. Algorithms. 2009. A Review of Control Algorithms, Library (Lond). pp. 547–556.
- [2] S. A. Raza and W. Gueaieb. 2010. Intelligent Flight Control of an Autonomous Quadrotor. no. 1.
- [3] Claudia Mary, L.C Totu and S.K Koldbaek. 2010. S. Project, Modelling and Control of Autonomous Quad-Rotor. Intelligent Autonomous Systems Master Programme, Denmark.
- [4] D. Shatat and T. A. Tutunji. 2014. UAV Quadrotor Implementation : A Case Study. Multi-conference on Systems, Signals & Devices (SSD), 11th International.
- [5] W. M. Hussein. 2009. Quad Rotor Design , Simulation And Implementation. International Conference on Computer Science from Algorithms to Applications, (CSAA09) JW Marriott, Mirage City, Cairo, Egypt
- [6] A. Y. Elruby. 2012. Dynamic Modeling And Control Of Quadrotor Vehicle. Arab Academy for Science and Technology, Egypt, pp. 13.
- [7] S. Bouabdallah and R. Siegwart. 2004. Design and Control of an Indoor Micro Quadrotor. pp. 4393–4398.
- [8] T. Bretl. 2012. Quad-Rotor Dynamics (and a little control), pp. 1–5 (AE483).
- [9] R.M. Murray. 2006. 1 Linear Quadratic Regulator, Control and Dynamic System, California Institute of Technology, pp. 1–14.
- [10] P. Hespanha. 2005. Lecture notes on LQR / LQG controller design.