



APPLICATION OF LQR CONTROL TECHNIQUE TO OFFSHORE JACKET PLATFORM SUBJECTED TO EARTHQUAKE

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ABSTRACT

Offshore structures are different than onshore structures in several sense. Very importantly they differ in Natural frequency and Mass distribution. Natural frequency of onshore structures is much higher than offshore structures. Generally, onshore structures have distributed mass, while offshore structure has the mass concentrated on the top of the platform. Such offshore structures are prone to extreme environmental conditions. Under extreme environmental conditions, offshore structures are vulnerable to damages because of higher response. Structural workability conditions get affected due to damage. Hence the vibratory behaviour needs to be tackled with available damping techniques. Among the available techniques Semi-active control insures that structure should not exceed the response limit for a wider frequency range. This can be achieved by properly modelling the control parameters. Efficiently chosen parameter guaranties the minimization of responses for all types of environmental forces. In this study a 50 mts offshore structure is considered. Linear control of the structure has been modelled for the earthquake force. The structure is subjected to El-Centro earthquake force. The response of the structure was well controlled by the Linear Quadratic Regulator (LQR) Methodology.

Keywords: tuned mass damper, offshore structure control, earthquake, single degree of freedom, semi-active control, LQR control.

INTRODUCTION

Important structures have to be check for the all conditions of loadings. The structural vibration due to dynamic environmental loads makes the structure unsafe. Extreme loads damage the structure and so life time period of the structure gets reduced. The protective measure of structural control is studied by several researchers for onshore and offshore structures. For onshore structures plenty of work has been done (Dyke *et al.*, [6]; & Spencer *et al.*, [7]). For offshore structure mainly the control is carried out on jacket platform. Floating structure control is not studied because of its high structural time period.

Model is being tested for the already recorded Earthquake force. But in reality the sensors mounted on the structure, sense the force and provide the data to the computer to evaluate the counter force to be provided for the active part of control.

In this single degree of freedom (SDOF) structure has been modelled with Tuned Mass Damper (TMD) for an offshore jacket structure. The structure is subjected to the el-centro earthquake loadings. The response of the structure is reduced by applying the semi-active control strategy. LQR Methodology is utilized to obtain the gain values. The obtained gain value is used to minimize the response of the structure.

FORCE CALCULATIONS

Recorded acceleration data were obtained from the USGS for the elcentro earthquake for the time period of approximately 53 seconds. The accelerations are multiplied by the mass to get the inertial force acting on the modal mass. Figure-1 represents the inertial force acting on the structure. Wave condition is assumed to be calm sea state. Hence no force is added by the wave. Higher accelerations were only up to 30 seconds. The

forces acting on the TMD will be lower since the mass is just 2% of the main structure.

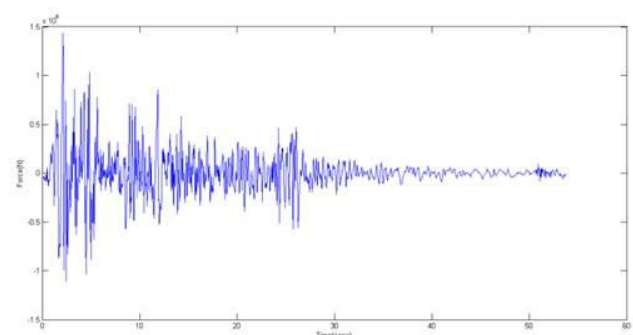


Figure-1. Force acting on the structure.

EQUATION OF MOTION

The general equation of motion is given by

$$m\ddot{x} + c\dot{x} + kx = F(t) \quad (1)$$

m - Modal Mass of the structure

c - Damping co-efficient

k - Stiffness co-efficient

$F(t)$ - Inertial force on the structure

\ddot{x}, \dot{x}, x - Acceleration, Velocity & Displacement

The internal damping forces of the structure are neglected as they are very less relative to the semi-active control forces.

The mode shapes are obtained from the SACS software. Modal masses are evaluated by using normalized



mode shape. Modal mass can be evaluated by following formula (Rahul Rana., [2])

$$m = \Phi_{1n}^T M \Phi_{1n} \quad (2)$$

m - Modal mass and Actual mass
 Φ_{1n}^T, Φ_{1n} - Normalised Mode shape transpose and normalised mode.

The Figure-2 shows the behaviour of the structure due to inertial force generated by the earthquake acceleration. The figure represents single degree of freedom.

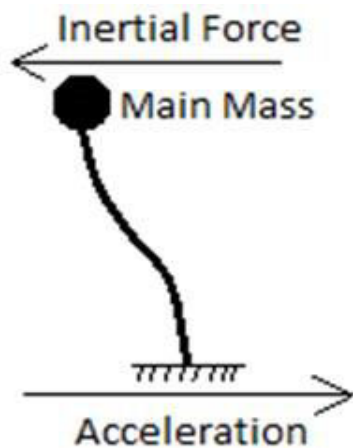


Figure-2. Structure behaviour under inertial force.

STATE-SPACE FORMULATION

The general form of representation of the state space is given by following equations.

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \quad (3)$$

$$y(t) = C(t)x(t) + D(t)u(t) \quad (4)$$

$A(t)$ - State matrix

$B(t)$ - Input matrix

$C(t)$ - Output matrix

$D(t)$ - Direct transmission matrix

The equation of motion of the single degree of freedom without damping is given in equation 5 and TMD is given in equation 6

$$m\ddot{y} = -ky - k(y - y_T) + \text{External Force} \quad (5)$$

$$m_T \ddot{y}_T = -k_T(y_T - y) - c_T(\dot{y}_T - \dot{y}) \quad (6)$$

m_T - Modal Mass of tuned mass damper

k - Stiffness of SDOF

y - Displacement of main mass

\dot{y} - Velocity of main mass

\dot{y}_T - Velocity of TMD

\ddot{y} - Acceleration of main mass

\ddot{y}_T - Acceleration of TMD

The chosen state variables are

$$x_1 = y; x_2 = \dot{y}; x_3 = y_T; x_4 = \dot{y}_T \quad (7)$$

The state space matrices obtained are given below

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -(k - k_T)/m & 0 & k_T/m & 0 \\ 0 & 0 & 0 & 1 \\ k_T/m_T & c_T/m_T & -k_T/m_T & -c_T/m_T \end{pmatrix} \quad (8a)$$

$$B = \begin{pmatrix} 0 \\ 1/m \\ 0 \\ 0 \end{pmatrix} * m * a \quad (8b)$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad (8c)$$

$$D = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (8d)$$

MODEL PARAMETERS

The parameters are obtained from SACS model. State values are evaluated accordingly.

$m = 5611200$ kg

$m_T = 112224$ kg

$k_1 = 210000$ N/mt

$c_T = 33600$ N-S/mt

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -0.0749 & 0 & 0.0374 & 0 \\ 0 & 0 & 0 & 1 \\ 25 & 4 & -25 & -4 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 1.8e-7 \\ 0 \\ 0 \end{pmatrix} * m * a$$



The TMD is placed where the maximum displacement occurs. This helps in effective reduction of the responses. The Figure-3 represents the general setup with TMD placed at top of the structure.

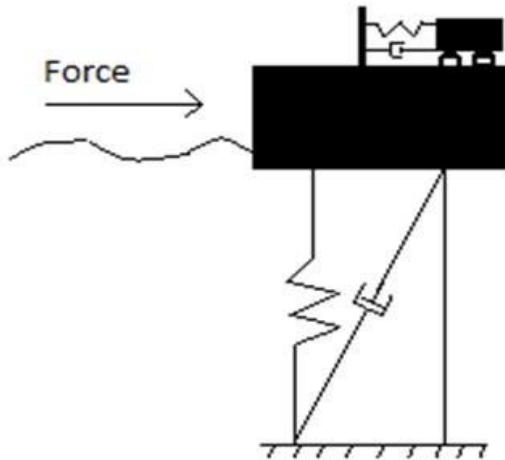


Figure-3. Structure behaviour under inertial force.

SEMI-ACTIVE CONTROL METHODOLOGY

The control algorithms adopted for active control part of the structure is LQR technique. The control has to be designed for linear and non-linear values of the structure. The multiloop feedback gains for linear and non-linear will reduce the vibrational responses.

LQR methods general form for a finite time (N) is given as

$$J_{0,N} = \mathbf{x}_N^T \mathbf{S}_L \mathbf{x}_N + \sum_{k=0}^{N-1} \mathbf{x}_k^T \mathbf{Q}_L \mathbf{x}_k + \mathbf{u}_k^T \mathbf{R}_L \mathbf{u}_k \quad (9)$$

The state feedback law is given by the following equations for the linear part

$$\mathbf{u}_i = -\mathbf{k} \mathbf{x}_i \quad (10a)$$

And

$$\mathbf{k}_L = (\mathbf{R} + \mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A} \quad (10b)$$

Where S and Q are symmetric and non-negative definite matrix and R is a symmetric and positive definite

The riccati equation provides the solution to obtain the value of \mathbf{P}

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = 0 \quad (11)$$

Linear gain is mentioned as \mathbf{k}_L and the Non-linear gain (\mathbf{k}_N) is given as

$$\mathbf{k}_N = \rho \mathbf{R}_N^{-1} \mathbf{B}^T \mathbf{S} \quad (12)$$

The riccati equation for the non-linear compensation is given as

$$\mathbf{S}(\mathbf{A} - \mathbf{B} \mathbf{k}_L^T) + (\mathbf{A} - \mathbf{B} \mathbf{k}_L^T)^T \mathbf{S} - \mathbf{S} \mathbf{B} (\rho \mathbf{R}_N^{-1}) \mathbf{B}^T \mathbf{S} + \mathbf{Q}_N = 0 \quad (13)$$

The final compensatory control force is given as

$$\mathbf{u} = -(\mathbf{R}_L^{-1} \mathbf{B}^T \mathbf{P} + \rho \mathbf{R}_N^{-1} \mathbf{B}^T \mathbf{S}) \mathbf{X}_i \quad (14)$$

In this case since the model is considered to be linear. So only linear values have been obtained for control. The following gain values have been obtained for $\mathbf{Q}_L = 1000 * (\mathbf{c}^T \mathbf{c})$ & $\mathbf{R} = 1e-5$.

$$\mathbf{K}_L = 1.0e+4 \begin{bmatrix} -0.4391 & 7.2596 & 0.4866 & 0.0012 \end{bmatrix}$$

$$\mathbf{P}_L = 1.0e+6 \begin{bmatrix} 0.5862 & -0.2464 & -0.3015 & -0.0008 \\ -0.2464 & 4.0735 & 0.2731 & 0.0007 \\ -0.3015 & 0.2731 & 0.1696 & 0.0004 \\ -0.0008 & 0.0007 & 0.0004 & 0.0000 \end{bmatrix}$$

SIMULINK MODEL

The simulation has been carried out with the Simulink model shown below. The feedback is shows the evaluated control gains to the system which reduces the responses of the structure. The gain can also be included in state-space function block in which parameter can be taken as $\mathbf{A} - \mathbf{B} \mathbf{k}_L$. The equation3 can also be solved by integrating (Mohamed zribi.,[3]).

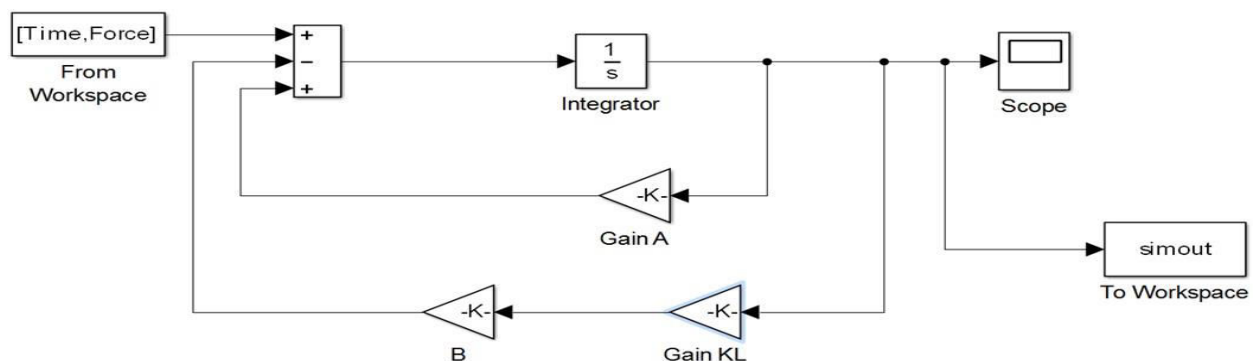


Figure-4. Simulink model.



RESULTS AND DISCUSSION

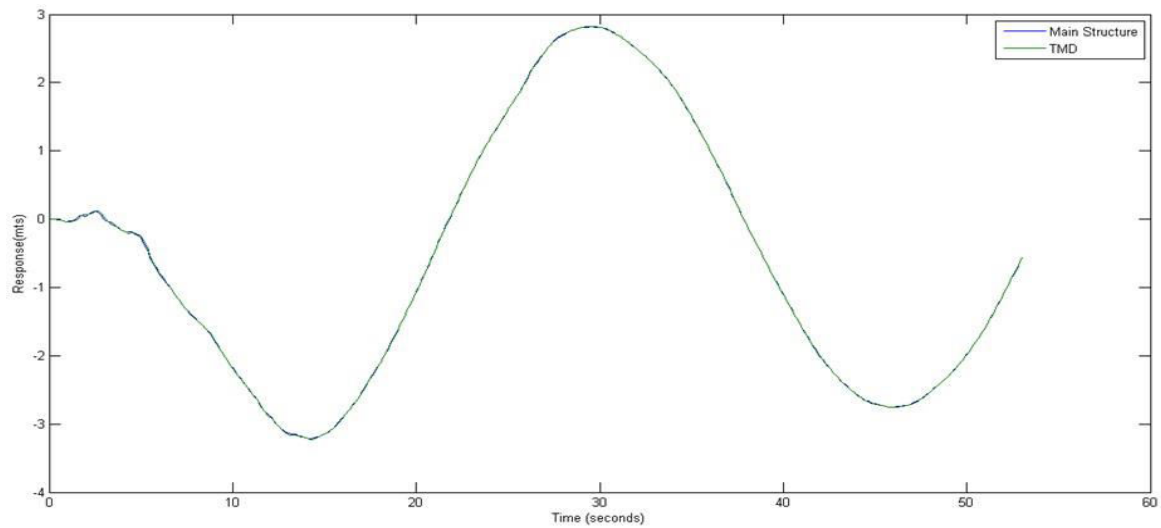


Figure-5. Passive control of the structure with TMD.

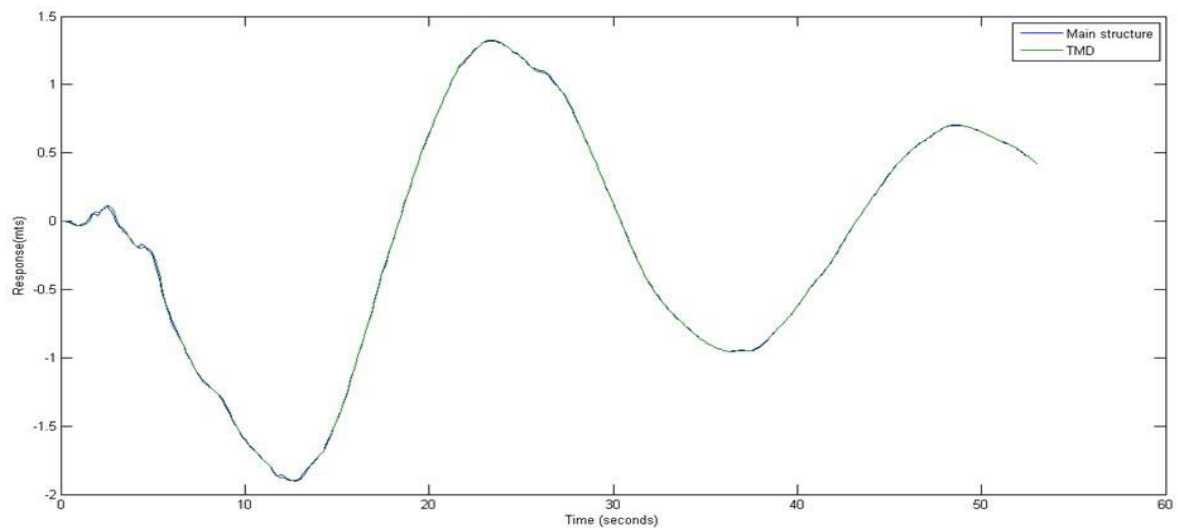


Figure-6. Semi-Active control of the structure with TMD.

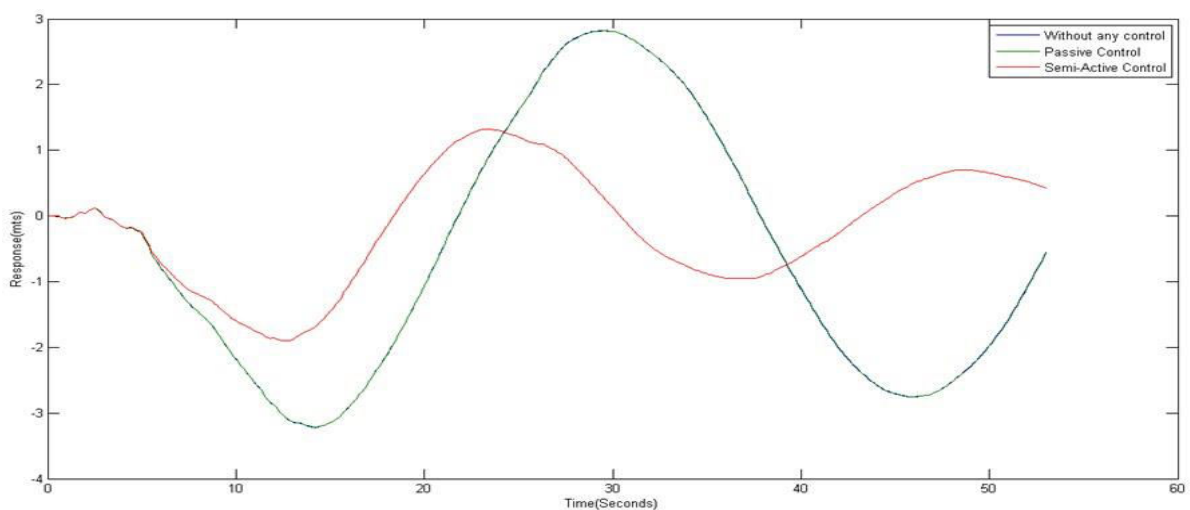


Figure-7. Without control, passive and semi-active control.



Percentage control in passive control depends on mass of control device. In this case mass ratio of the TMD is taken as 2%. For the same mass ratio, semi-active control (LQR) is used to obtain the control response. Uncontrolled and controlled response using semi-active control is shown in Figure-7. The percentage control obtained is 53% in comparison with passive control. Clearly there is increase in percentage control in semi-active control over passive control. The obtained simulation results in the above Figures-5 & 6 show the passive and semi-active control of the structure. The passive control shows a maximum response of 2.82mts at 29.5secs. Semi-Active control gives a maximum response of 1.325mts at 23.4secs.

CONCLUSIONS

The effect of earthquake response is studied on jacket platform of 50mts. State-space Single Degree of Freedom (SDOF) Model has been developed with TMD. Simulation has been carried out for passive and semi-active control of the system subjected to elcentro earthquake force. Only linear part of the control was studied with Linear Quadratic Regulator (LQR). Displacement response of jacket deck in surge direction is obtained using passive control, semi-active control and without any control. The control was effectively applied and responses were reduced as desired. Percentage reduction in the response between passive damping and semi-active damping was 53.05%.

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