



## HF-FUZZY LOGIC BASED MOBILE ROBOT NAVIGATION: A SOLUTION TO FINITE ESCAPE TIME

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### ABSTRACT

This paper deals with  $H_\infty$  Filter (HF)-Fuzzy logic based mobile robot localization and mapping as an approach to prevent the Finite Escape Time (FET) problem in HF. The FET problem has been limiting the HF capabilities in estimation for decades and has been one of the important aspects to be considered to ensure HF performs well during mobile robot observations. The proposed technique focusses on the HF innovation stage by including very few Fuzzy Logic rules, and fuzzy sets. The design is generally divided into two stages; firstly, the analysis of HF innovation characteristics and then the implementation of Fuzzy Logic technique into the system. The analysis also presents the preliminary study on different membership functions to discover the best possible technique to combine with HF based mobile robot localization. The simulation results proved that Fuzzy Logic can be used to avoid the FET from occurred while at the same time improving the estimation of both mobile robot and landmarks.

**Keywords:** mobile robot, navigation,  $h_\infty$  filter, fuzzy logic ,finite escape time.

### INTRODUCTION

Development of mobile robot in navigation has gained much interest for decades. One of the main reasons is because the research attempts to build a truly autonomous mobile robot that able to explore and identify the surrounding area especially in hazardous conditions. One of the famous problem in navigation is the Simultaneous Localization and Mapping (SLAM) which requires the mobile robot to observe the area of interest and concurrently build a map while at the same time localize itself in the map [1]-[2]. The study has covered varieties of issues such as the inconsistencies of estimation, existent of uncertainties, data association problem, and computational complexity [3]-[4].

During mobile robot observations, there are a lot of uncertainties need to be taken into account. To tolerate this condition, Extended Kalman Filter (EKF) is aggressively used to infer the mobile robot locations and any identified landmarks. Unfortunately, EKF cannot work well in an environment that has non-Gaussian noise. Owing to this limitation, its other family known as  $H_\infty$  Filter (HF) offers a better solution [5]-[6]. In these reports, certain conditions are satisfied, HF surpassed the Kalman Filter performance. HF is employed when the noise energy is bounded and in unknown noise characteristics. The technique attempts to minimize the estimation error in the form of energy gain.

Several papers have been investigated the performance of HF regarding its capability and performances. To list few of them are as analyzed by Ahmad, H *et al.* about the filter theoretical behavior [6],[7], the marine applications [8], and sensor fusion [9]. Even the performance is promising, the filter may exhibit Finite Escape Time (FET) problem during mobile robot navigation. This is a case where the estimation is diverging and consequently leads to erroneous results. Unfortunately, FET only occurs in HF and do not exist in

EKF. To avoid this, a switching strategy or the suboptimal condition has been proposed.

The switching strategy only modifies the value of  $\gamma$  to make HF similar to EKF and the suboptimal technique adds some pseudo state covariance which finally increases the state covariance. However, none of the research was found to design and combine the HF with Fuzzy Logic in mobile robot navigation even though a number of research have successfully implemented the hybrid approach through EKF-Fuzzy combination [10]-[15].

Motivated by this reason, this paper proposed a HF-Fuzzy Logic technique to refrain the FET from occurring during mobile robot observations. This is done by modifying the innovation error exhibits during measurements to pose appropriate information. Innovation error which contains the information of error generated by the actual measurement and measurement estimation of the relative angle and distance errors defines how the state covariance characteristics. This will be shown later in the next section. It is found that by reducing this error through observation of the innovation characteristics, the mobile robot preserve better estimation and at the same time is able to avoid the FET from happening.

This paper is organized as follows. In section 2, the proposed technique is presented. This is followed by section 3 which demonstrates the simulation results and the comparison with the original HF approach. Finally, our work of this paper is summarized in the last section.

### PROPOSED TECHNIQUE

Our approach attempts to control the information gained from the mobile robot measurement. In this paper the measurement information of HF is applied as a comparison to the estimated measurement through the measurement innovation. The measurement innovation inherently describes the level of uncertainties observed by the mobile robot such that if it has large error, then the estimation becomes erroneous. Therefore, if the



measurement innovation can be preserved to be small at all times then the estimation result is improved. Next section described about the measurement innovation in details.

FET occurs when the state covariance suddenly become positive or negative infinite during mobile robot observations. As a result, the mobile robot loses its confidence about its location even though multiple measurements have been recorded about its surroundings. This phenomenon is unavoidable for a normal  $H_\infty$  Filter, thus making the filter less favorite than the Kalman Filter. Even though several technique has been proposed as mentioned on the earlier section, the study needs to be organized to overcome the FET problem.

Another important aspect to be recognized that can contribute to the FET problem is the behavior of the updated state covariance equation. Notice that the equation can become negative positive semi definite if  $\Psi$  becomes negative. This is possible especially when  $\gamma$  is not selected properly according to the noise characteristics and the environment conditions.

**$H_\infty$  FILTER ALGORITHM**

The  $H_\infty$  Filter is introduced in this subsection. For mobile robot localization and mapping, there are two processes involved which are the process and measurement models. The process model calculates the kinematic of mobile robot and the measurement model defines the observations made by the mobile robot. The process model is presented as follow.

$$X_{k+1} = f(X_k, \omega_k, v_k, \delta\omega, \delta v) \tag{1}$$

where  $X_k \in R^{n \times 1}$  is the  $x, y$  positions and heading angle  $\theta_k$  of the mobile robot and landmarks  $x_i, y_i$  positions that is represented by mobile robot.  $\omega_k, v_k$  are the angular acceleration and velocity respectively with its associated noise of  $\delta\omega, \delta v$ .

The measurement model consists of the following information.

$$z_k = \begin{bmatrix} r_k \\ \varphi_k \end{bmatrix} = H_k(X_k) + v_{r,k} \tag{2}$$

$$r_k = \sqrt{(y_i - y_k)^2 + (x_i - x_k)^2} + v_{r,k} \tag{3}$$

$$\varphi_k = \arctan \left[ \frac{y_i - y_k}{x_i - x_k} \right] - \theta_k + v_\varphi \tag{4}$$

$r_k, \varphi_k$  are the relative distance and angle measurements between mobile robot and a landmark. The measurement is affected by measurement noise  $v_{r,k}$ .

For  $H_\infty$  Filter, there are two recursive stages i.e the prediction and update stage which is similar to the Kalman Filter. The prediction stage is shown as follow.

$$\hat{X}_{k+1}^- = f_k(\hat{X}_k, \omega_k, v_k, 0, 0) \tag{5}$$

$$P_{k+1}^- = f_k P_k [I_n - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k]^{-1} f_k^T + g_{\omega v} \Sigma g_{\omega v}^T \tag{6}$$

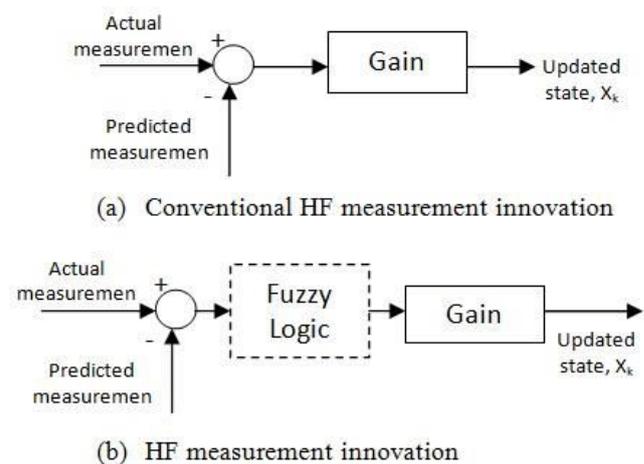
Above two Equations.(5) and (6) defines the predicted states and its state covariance  $P_k$  respectively.  $R$  is the measurement covariance and  $g_{\omega v}, \Sigma$  are the jacobian of the noise covariance and the noise covariance respectively.

The updated part is described as follow for the mobile robot states.

$$\hat{X}_{k+1}^+ = f_r \hat{X}_k + K_k (z_{i,k} - H_i \hat{X}_k) \tag{7}$$

where  $K_k = P_k H_i^T (H_i P_k H_i^T + R_k)^{-1}$  is the gain of the system.

From Equation(7), the second right hand term shows the measurement innovation. It indicates the error of the system and most of the time, the error is preferably small. If else, then the estimation becomes erroneous and subsequently results in divergence of state estimation. Looking into this aspect, there is a possibility of controlling the innovation error as the Finite Escape Time could also due to this innovation characteristics. The research attempts to reduce the error generated by the second right term of Equation(7) if its suddenly becomes bigger.



**Figure-1.** Proposed method of FHF for mobile robot localization and mapping.

**FUZZY LOGIC DESIGN**

As been stated, the measurement innovation will be referred as the main reference in designing the Fuzzy logic. Fuzzy logic is designed such that it received the angle and distance errors as its inputs. Our objective is to decrease those errors by configuring the fuzzy sets to produce smaller errors. By choosing the output appropriately, the effects or measurement error due to sensor inaccuracies can be minimized further.

Previously it has been proven that the mobile robot has a level of confidence about its location if

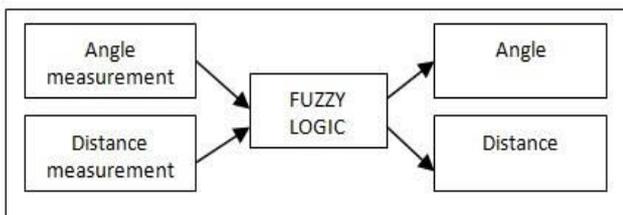


multiple measurements are recorded about its surroundings [3]. For non-Gaussian noise characteristic, the sensors reading might be interfered and exhibits bigger error, and hence result in bigger measurement noise covariance,  $R$ . If the gain  $K$  is small at all time during observations, then it is possible to have smaller measurement errors. Inspired by this fact, fuzzy logic is proposed to find the best value of measurement innovation to pursue lower error. Kobayashi *et al.* [14] have selected the  $P$ ,  $Q$ , and  $R$  from Fuzzy logic to gain smaller uncertainties. Work by Wang *et al.* [8] have recognized this as one of the way to realize smaller measurement noise even it was been applied to the other HF family, the Kalman Filter.

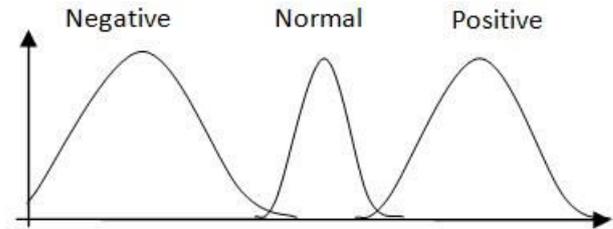
The proposed design used the Mamdani technique for analysis purposes. The technique is proposed as it calculates the output by considering and utilizing the maximum information gained from measurement compared to the Sugeno method.

The general design is illustrated in Figures 2-4 that consists of the input and output and their respective fuzzy sets. The following describes the rules of Fuzzy logic that are used to define the output of the measurement innovation.

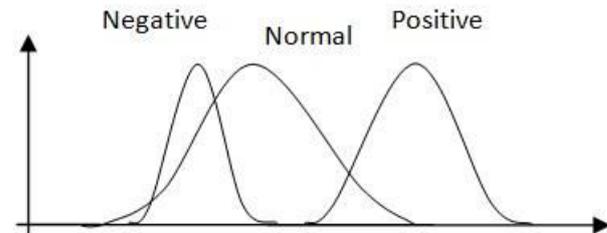
- IF angle error is negative and distance error is negative, THEN angle is negative
- IF angle error is negative and distance error is normal, THEN angle is normal
- IF angle error is negative and distance error is positive, THEN angle is negative, distance is normal
- IF angle error is positive and distance error is normal, THEN angle is negative
- IF angle error is positive and distance error is negative, THEN distance is normal
- IF angle error is positive and distance error is positive, THEN angle is negative, distance is normal



**Figure-2.** Fuzzy logic with inputs and outputs.

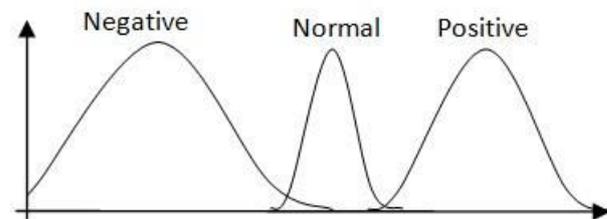


(a)

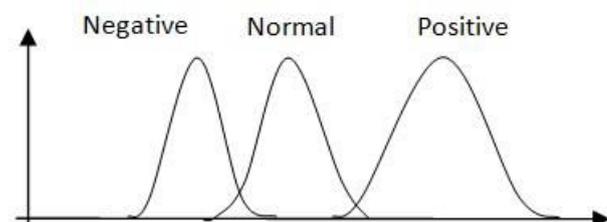


(b)

**Figure-3.** (a) Angle measurement (b) Distance measurement



(a)



(b)

**Figure-4.** (a) Fuzzified angle (b) Fuzzified distance measurement.

Gaussian membership function is considered in this paper for evaluation purposes. Only three fuzzy sets is defined which are divided into three different categories; the negative, normal and positive regions. The scale of each of the fuzzy sets is selected based on the normal condition that has high errors. The value differs to each of the fuzzy sets and it has been tuned several times to obtain the best estimation results. The tuning is taking into account the uncertainties behavior throughout the simulation. Other than that, both angle and distance measurement characteristics are observed prior to the tuning process. Wang *et al.*, [8] designed the membership function of the angle error to be positive at all the time.



However, random mobile robots movements may also show negative angle especially when a global coordinate system is being considered. This is one of the major differences between our approach and what they have been investigated.

## SIMULATION RESULTS AND DISCUSSION

This section demonstrates the simulation results and its related discussions. Some assumptions about the system have been made as follows.

- Data association is expected to be available at all time
- Landmarks are point landmark and stationary
- Both of process and measurement covariances are not correlated

The first assumptions are made to exclude a situation where mobile robot can be accidentally lost about its location in a similar or look-alike environment. This is also called as kidnapped robot problem. In this research condition, the mobile robot has some prior information about the environment with certain level of certainties. This is defined through the initial state covariance,  $P$  as calculated in Equation (7).

The latter assumption defines that the landmarks are not dynamic and holds the same properties. All landmarks are defined as a point landmark and located in specified location. This assumption is made as the main purpose is to recognize either the proposed technique is able to overcome the FET problem. FET as mentioned earlier can exist either in a case of defined conditions or in an unknown landmarks.

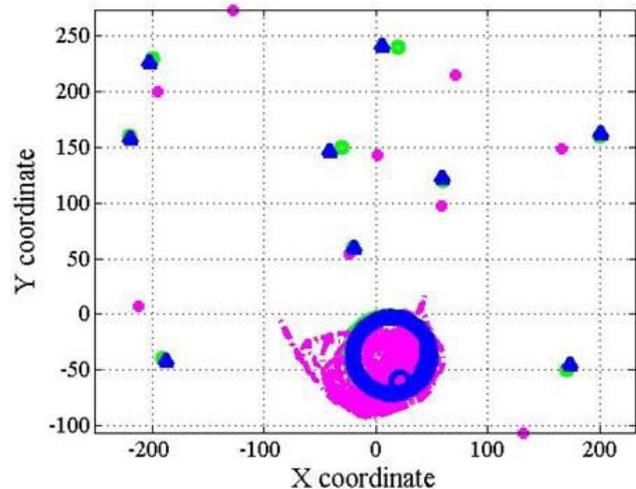
Mobile robot in this simulation is a two wheeled mobile robot with a castor that continuously and consistently measures about its surroundings for a specific time. The mobile robot moves randomly into the environment and recognized any landmarks near to it. The measurement data is then fed into the HF for analysis and update process.

One of the important settings need to be adjusted or selected is  $\gamma$  as it defines the performance of the estimator and consequently leads the estimation. The selection of  $\gamma$  is based on [5] where it is calculated to suit

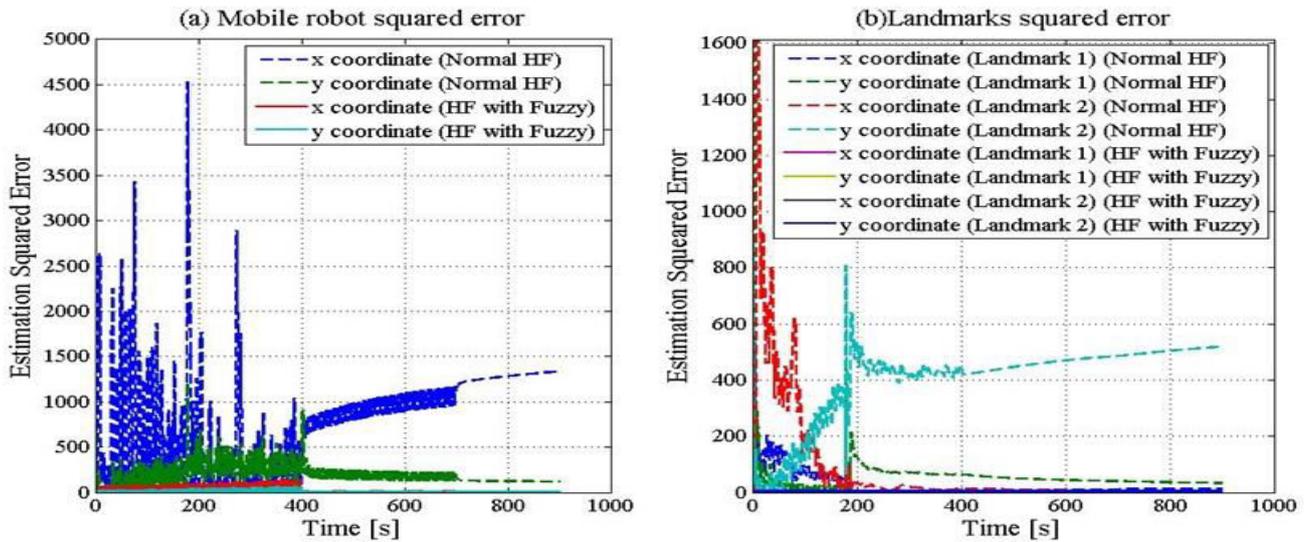
the prescribed condition of the environment; especially the measurement noise covariance. The initial covariance defines the prior information that the mobile robot has in order to explore the environment. The simulation considered non-gaussian noise conditions that meet the purpose why HF is suggested for SLAM problem.

**Table-1.** Simulation parameters.

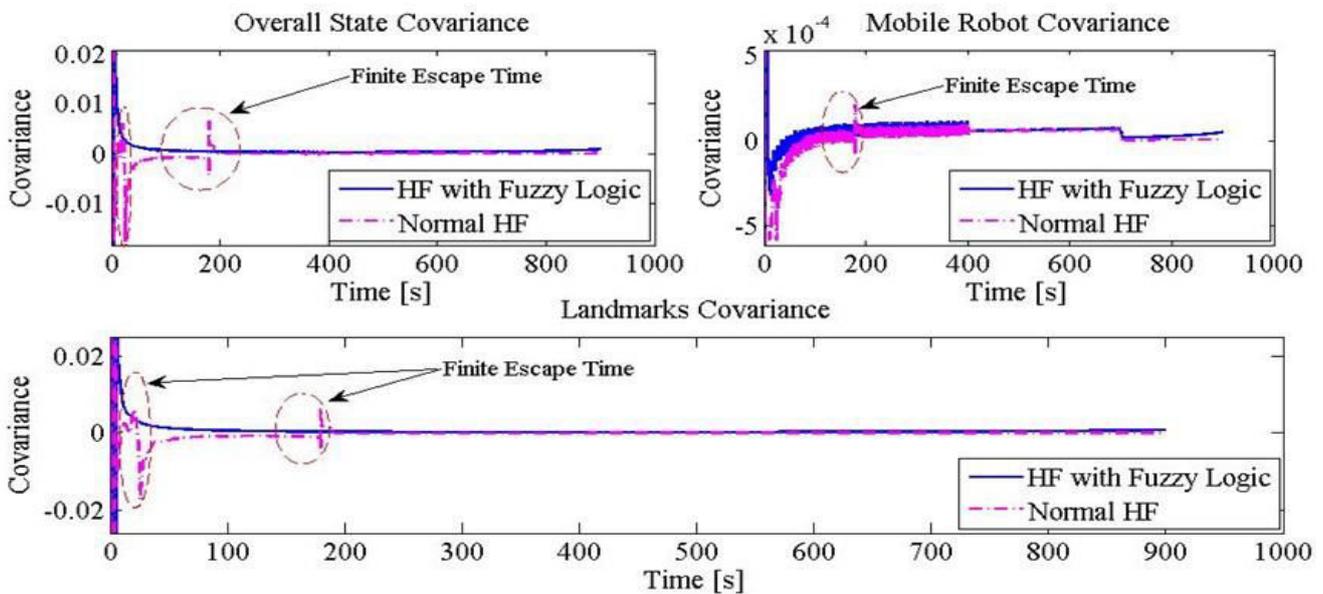
Variables	Parameter values
Process noise; $Q_{min}, Q_{max}$	-0.002, 0.001
Measurement noise; $R_{\theta_{min}}, R_{\theta_{max}}$ $R_{dist-min}, R_{dist-max}$	-0.04, 0.01 -0.15, 0.3
Initial covariance; $P_{robot}, P_{landmark}$ Simulation time	0.001, 100 1000[s]



**Figure-5.** The mobile robot movements through the environment. Green colour defines the actual positions while the blue and magenta colour presents the FHF and normal HF estimation performance.



**Figure-6.** A performance comparison between FHF and normal HF estimations for both mobile robot and landmarks estimations about the errors.



**Figure-7.** The state covariance conditions between normal HF and FHF. Normal HF exhibits frequent FET compared to the FHF.

Simulations are carried in MATLAB Simulink about 5000[s] to observe the consistency of estimation results. Parameters used for evaluations are included in Table 1 which described the noises parameters,  $\gamma$ , initial state covariance and others. The initial state covariance defines the prior information that the system has about the environment. These parameters are selected to model the real mobile robot which equipped with at least one sensor for measurement.

Figures 5-6 illustrate the performance comparison between normal HF and FHF in non-Gaussian noise environment. Figure 5 determines that the estimation of mobile robot for normal HF has higher errors than FHF. Consistent results have been also observed about the landmarks estimation. The squared error analysis sketches

in details of the error exhibit by normal HF for both mobile robot and landmarks estimations. Looking on the state covariance update described in Figure 7, the FHF produced lower uncertainties than the normal HF. Moreover, there are multiple FET occurred during the estimations. These characteristics have shown that the role of Fuzzy Logic has improved the performance of HF especially when the FET is being considered.

The outcomes presented and discussed above have proven that FHF performs better than the normal HF. The method is able to preserve the FET from happening during estimation. Take note, in some of the reviewed papers, the normal HF estimation seems to be good but FET is happening several time during measurement. It is difficult to design the HF due to the presence of  $\gamma$  and



other parameters. It needs some designed rules such as stated in Ahmad, H. *et al* [5]. Due to this disadvantage, HF has not been one of the solutions for navigation. As an alternative approach, the proposed technique has been presented to take care of the issue. The method is also generate fewer computational time as only a number of rules has been designed in correspond to the defined two inputs. However, if the design is being compared to the normal HF, it consumes slightly higher time of information processing. Nevertheless, the application of Fuzzy Logic in mobile robot can be found available in various reports. Therefore the proposed technique is relevant and can be applied for navigation purposes. The other membership types such as the triangular and the trapezoid memberships are not examined at this time as the main objective is only to avoid the FET phenomena. It will be the next research objective to identify the best membership types for estimation purposes.

## CONCLUSIONS

FHF is one of the possible solutions for mobile robot localization and mapping especially when the mobile robot motions are uncertain, and the environment conditions is partly understood. For a non-Gaussian noise environment, HF has been proven to perform better than the celebrated Kalman Filter. To overcome the FET issue, FHF has delivered good estimation results and then avoid the FET from exhibit during mobile robot observations. To apply Fuzzy Logic in HF, the measurement innovation is modified to produce smaller results by looking into its characteristics during measurement process. The computational cost and processing time are slightly higher than normal HF estimation due to addition of Fuzzy Logic technique. Even though this is the trade-off that the proposed technique has, as long as FET can be avoided, then the estimation can be assured to be effective.

## ACKNOWLEDGEMENTS

The author would like to thanks Ministry of Higher Education to support this work under FRGS grant RDU130106. Thanks to UMP for continuous support in realizing this research.

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