VOL. 11, NO. 1, JANUARY 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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USING DISCONTINUOUS TRANSFORMATION IN NUMERICAL SIMULATION OF VEHICLE AERODYNAMIC INTERACTION

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ABSTRACT

Vehicle aerodynamic interaction results in short-term but significantly large forces, which can affect steerability and therefore safety, so the numerical study of these processes is a relevant problem. This article presents a mathematical formulation of the discontinuous coordinate transformation for the aerodynamic simulation of multiband vehicle traffic and demonstrates the results of a numerical simulation using this model.

Keywords: aerodynamics, aerodynamic interaction, numerical simulation.

INTRODUCTION

Vehicle aerodynamic parameters such as aerodynamic drag factor, lift force, overturning moment, etc., affect the fuel efficiency, maximum speed and yawing stability, especially while maneuvering [1] - [12]. When moving in the pack, airflows occur between the objects; these airflows influence each other and other objects. In this paper, we consider a number of cases of the vehicle moving in the pack.

Aerodynamic simulation of a group of moving bodies

Aerodynamic simulation of a group of vehicles is rather relevant for racecars. The article [13] examined the effect of air resistance depending on the relative position of several motor vehicles. The authors found that when driving in the group one by one, cars run faster together than either car by itself. This effect is actively used by NASCAR pilots. When moving in parallel in one direction, the speed of two cars drops significantly. This is due to partial blocking of the flow, which forces the air to flow around both vehicles at once.

The article [14] deals with experimental research of aerodynamic interaction of passenger vehicles. It considers movement in the pack, overtaking, driving in the tunnel, as well as vehicular movement drawing a trailer. The author studies influence of Reynolds number on the aerodynamic characteristics of the car.

During vehicular movement, as a rule, the airflow is directed parallel to the movement, thereby creating aerodynamic drag, and at high speeds by the lateral velocity component exerts considerable impact. When overtaking, these forces are large enough and they can affect the vehicle stability and steerability. These issues are discussed in [15]. The study also demonstrated a two-dimensional simulation of collision of airflows for the motionless and overtaking vehicles. The calculation results are in good agreement with the experimental data.

Experimental study of the effect of the aerodynamic characteristics on the vehicle steerability and stability is possible only after creating a fully operating prototype. It should also be noted that vehicle contamination essentially depends on the flow characteristics of motor vehicles (for example, distance, speed) and the contamination sources.

Aerodynamic simulation is connected with the severe restrictions on the ratio of steps in space and time due to the speed of sound. In this regard, it is important to use the available techniques to minimize the computation time. Another aspect that requires attention when choosing a method for vehicle gas dynamics simulation is the necessity of taking into account its complex shape, which results in a significant number of corner points when using a grid with rectangular cells.

Numerical simulation of vehicular aerodynamic interaction is even more complex, non-trivial task, since there is the possibility of building a single coordinate system providing the immobility of all bodies contained therein.

Overset Mesh is used in Star CCM+ to simulate relative motion of bodies [16]. This functionality allows solving the relative motion problems, but imposes strict limitations on the grid characteristics in the intersection area.

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That is why the development of new methods for simulation of aerodynamics and contamination of a group of periodically moving bodies under dynamic conditions is so significant.

Thus, for example, Harlow's method [17] as opposed to Patankar method [18], is more stable, though less precise.

As a part of this work, the authors use their own software system, built on the basis of Euler equations without turbulence model and discontinuous coordinate transformation, which is described in the paper. The first order accurate scheme, based on Harlow's method, is used for computational solution of equations [17].

METHOD

In the classical formulation of the vehicle airflow problem, it is a common practice to consider uniform rectilinear motion of the vehicle in the coordinate system attached to it [19]. The calculation is usually done in the area, having the shape of a rectangular parallelepiped, large enough to allow neglecting the vehicle impact on the airflow dynamics at the area boundaries.

To simulate aerodynamics in more complicated cases (for example, opposing traffic, overtaking or moving in the pack) specific boundary conditions should be considered. Thus, for pack of similar vehicles moving with constant distance, cyclic boundary conditions should be considered on the parallelepiped faces perpendicular to the pack driving direction. Length of the parallelepiped rib, which is parallel to the pack driving direction, is determined as a sum of vehicle length and a distance between the vehicles. Other dimensions of the parallelepiped are determined similarly as when solving classical airflow problem. Using this approach it is possible to generate some errors for the first and last vehicles in the pack.

Simulation of vehicular group motion presents some difficulties. Thus, one of the problems is the impossibility of introducing a coordinate system in which all the bodies would be motionless and the points would move at a constant speed relative to the observer. The problem is that during introduction of such a coordinate system, in general, the points do not always have similar speed relative to a stationary observer. To solve this problem it is necessary to use a transformation having discontinuities or deformations. From a practical point of view, the most convenient is to use the discontinuities at the boundaries of traffic lanes. Assume the bodies move in parallel and periodically. A group of similar vehicles can move along each lane at a predetermined speed. For each lane, the sum of the length of the body and the distance remains constant. The movement scheme for the case of two lanes is shown in Figure-1.

Peculiarities of simulation for periodically moving vehicles with multi-lane traffic

The inability to enter the coordinate system, which would move at a constant speed, and all the bodies would stay motionless in is a key problem in this formulation. As a consequence, the question arises how to simulate motion of vehicles driving at different speeds. As described above, for solving this problem it is necessary to use a deformation or discontinuities. From a practical viewpoint, it is convenient to use discontinuous transformation.

Let us introduce a new coordinate system associated with a stationary observer xOy. Subarea Γi, that is repeated periodically, is considered in each lane. The length of each subarea is the sum of the length and the distance between the vehicles in the group moving along the lane, and is designated L. It seems impossible to use subareas Γi with different lengths because it would violate the periodicity of the solution relative to the observer. Let us introduce a local coordinate system $x_i Oy$ in each lane of the area connected with the bodies moving along the i-*th* lane. Such coordinate transformation is determined by the correlations $x_i = x - U_i t$, where U_i is the driving speed of a group of bodies along the i-*th* lane, which in its turn is determined as $y_{i-1} \le y \le y_i$. So, this transformation will enable to simulate vehicular aerodynamics separately in each subarea Γi.

Let us introduce a coordinate system $\tilde{x}Oy$, where the point ordinate corresponds to its value in the global coordinate system xOy, while the point abscissa corresponds to the value in the local coordinate system acting in the considered subarea.

$$\tilde{x} = \begin{cases} x_1, npu\ y_0 < y < y_1 \\ x_2, npu\ y_1 < y < y_2 \\ \ldots \\ x_n, npu\ y_{n-1} < y < y_n \end{cases} \quad (1)$$

Thus, during simulation in the coordinate system $\tilde{x}Oy$ all bodies are motionless, whereas the system itself is bound to each of traffic lanes. However, there is a discontinuity in the transition from xOy to $\tilde{x}Oy$ on the boundaries y_i, therefore values of the initial magnitudes on the boundaries are recovered from the values in the system xOy. At the points $y = y_i$ discontinuities of the functions in the system $\tilde{x}Oy$ are recoverable, since the boundaries y_i are not bound with the sources of physically substantiated solution jumps (in Figure 1 subareas where computational solution is performed are designated as Γi).

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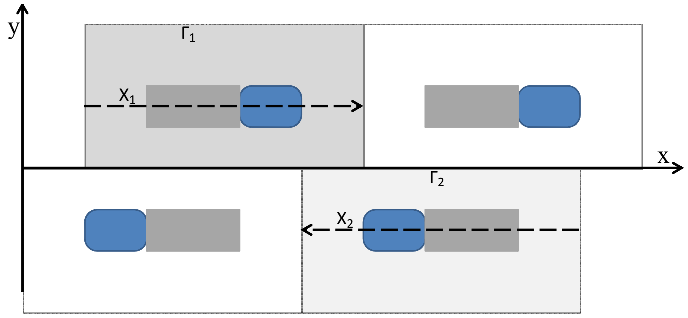

Figure-1. Traffic diagram.

Let us describe value search algorithm on the boundary of lane Γ_i. There is a point A with coordinates (a, y_{i-1}) on the boundary of the i-*th* subarea in the local systeme $x_i Oy$. To find values therein, it is necessary to know the values of the point B with coordinates $(a, y_{i-1} - \Delta y)$ in the local coordinate system situated the subarea Γ_{i-1}. Point B in the system xOy has coordinates $(a + U_i t, y_{i-1} - \Delta y)$. In the local coordinate system $x_{i-1}Oy$ this point has coordinates $(a + (U_i - U_{i-1})t, y_{i-1} - \Delta y)$.

The solution is periodic with period L along the coordinate x, whereby point B0 with coordinates $(\mathrm{mod}\,(a + (U_i - U_{i-1})t, L), y_{i-1} - \Delta y)$, belonging to the subarea Гi-1 corresponds to point B. Interpolation should be used in the computational solution to search for values in point B0, since this point does not correspond to any grid point. We used linear interpolation in our calculations. It should be noted that during the use of the numerical schemes applying several neighboring points at once, for example, FCT [20], the subarea-stitching algorithm undergoes significant changes.

When simulating single bodies application of the above approach is associated with some difficulties. Figure-1 shows that the boundaries of subareas Γ_1 и Γ_2, which correspond to the boundary between the lanes, intersect incompletely; due to this one should use boundary conditions of the "permeable boundary" type in the non-contiguous parts of the boundaries. If such boundary is located close to the streamlines body, this may have an adverse effect on the quality of the results. It is possible to avoid this by increasing the width of the calculated area, thereby moving the boundary aside from the body, however, in this case the required computational effort increases. This issue may also be addressed by means of using more complicated discontinuity line having dynamically changing structure.

RESULTS AND DISCUSSIONS

This section presents the solution of a model problem demonstrating possibilities of applying coordinate transformation described above. In all calculations, simulation is carried out in two-dimensional position (top view), vehicles are presented in the form of rectangles with sides of 2 m to 3 m. We consider two subareas; each is 17 m wide along the axis y. This value significantly exceeds the lane width of a real road, but is justified by the need for remote location of the boundaries free of cars for a reduction of error introduced by them. Free boundaries easily pass elastic waves and gas flows, but influence vortex flows. The solvable problem is periodic, so it can be viewed in one period. Dimensions of the period of 52 and 97 m along the coordinate x are considered, they correspond to the dimensions of the subareas represented in the figures. The color scale on the right corresponds to the ratio of the pressure at the point to the ambient pressure of 101,325 Pa. The distance between the packs is 7 m. The vector velocity field is presented in the form of arrows, the scale of which is observed within each figure. Coordinates are given in meters on the scales of the axes of abscissas and ordinates.

Results of numerical simulation of one vehicular pack movement without consideration of external effects

To test the operability of the algorithm that implements communication between the lanes and to test the numerical scheme a vehicular pack driving at a speed of 30 m/s along x-axis was simulated. However, according to the boundary-processing algorithm the second traffic lane moves in the opposite direction at a speed of 30 m/s. As a result, when the vehicle starts driving, a change of pressure and the formation of an elastic wave are observed (Figure-2). This effect does not significantly affect the further dynamics of vehicle flow, but it clearly demonstrates the operability of this algorithm. The figure also shows the beginning of airflow formation ahead of the vehicle and behind it.

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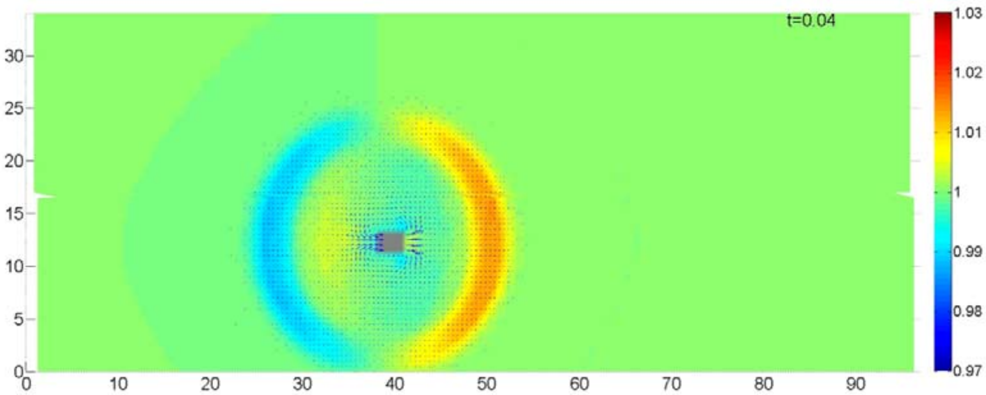

Figure-2. Pack movement aerodynamics at the moment of 0.04 sec.

Vortices occurring to the right and left of the vehicle are formed at the moment of 0.08 s. Figure-3 shows that the circular wave continues spreading. The left part of a negative pressure wave moves to the left side without distortion, passing through the right periodic boundary, as can be seen in the right part of the Figure. The waves pass through the free upper and lower boundaries of the computational area without substantial error.

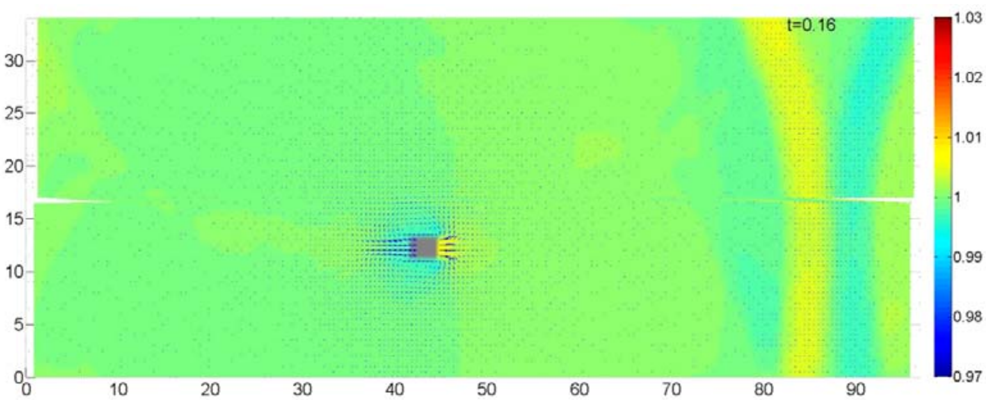

Figure-3. Pack movement aerodynamics at the moment of 0.08 seconds.

Figure-4 illustrates formation of Karman vortex street [21], occurring due to increase of the flow length behind the vehicle while it is driving. Despite the considerable distance between adjacent vehicles in the pack, flows generated by the vehicle, reach even vehicle driving behind. Figure-4.1.4(a) shows the vehicular aerodynamic simulation result obtained applying coordinate-stitching algorithm by means of Matlab environment. The result of similar numerical computation by means of Star CCM+ environment using k-ω turbulence model is shown in Figure-4(b). Grids used in both calculations are identical in structure and dimensions. As seen in Figure-4, the aerodynamic pattern of the flow formed behind the streamlined body has similar structure and characteristic dimensions.

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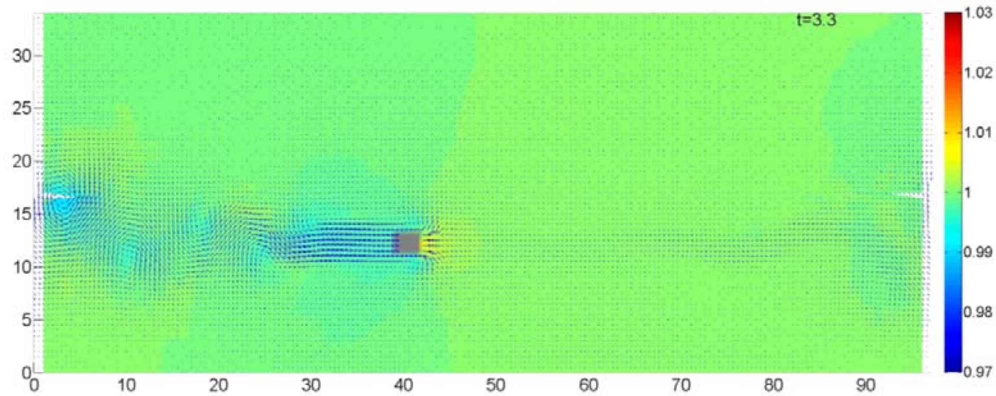

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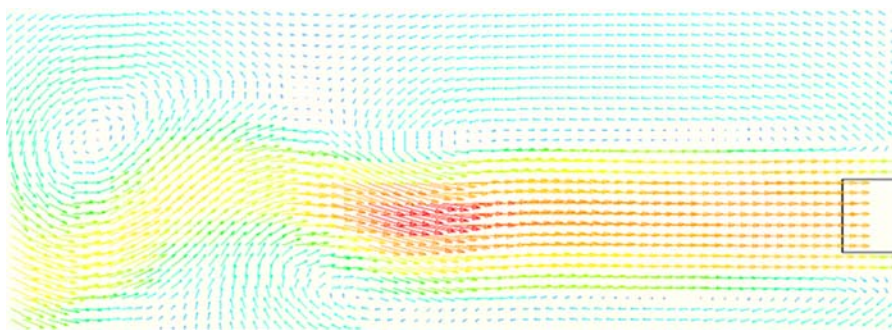

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Figure-4. Pack movement aerodynamics at the moment of 3.3 seconds.

As it was noted previously, there was no need to apply discontinuous coordinate transformation in solving the problem with c one vehicular pack driving. However, the calculations carried out when solving the model problem showed that discontinuous transformation does not have a significant impact on the configuration of the flows in the solution found.

Results of numerical aerodynamic simulation for vehicular pack driving in one direction at different speeds

As stated before, it is reasonable to apply discontinuous coordinate transformation in case of need to simulate pack driving at different speeds. The analysis of the obtained results of simulated vehicular pack driving in one direction is given below.

The distance between the vehicles in the pack, as opposed to the previous problem, makes 49 m and the distance between the packs is 7 m. in the upper computational subarea the vehicular pack drives at a speed of 20 m/s and in the lower subarea speed is 30 m/s.

Aerodynamics of one vehicular pack overtaken by another pack is shown in Figures 5-7.

Figure-5 demonstrates the result of interaction two circular waves formed at the moment of motion onset. A high-pressure area arises for a short time around the vehicle moving faster than the other one, and a low-pressure area occurs around the vehicle, driving behind it. The amplitude is increased due to the waves reflected from the vehicles, and also due to the interference of the elastic waves. Later waves of pressure drop gradually fade without having a significant impact on the formation of vortex flows that occur during the motion of the vehicular pack.

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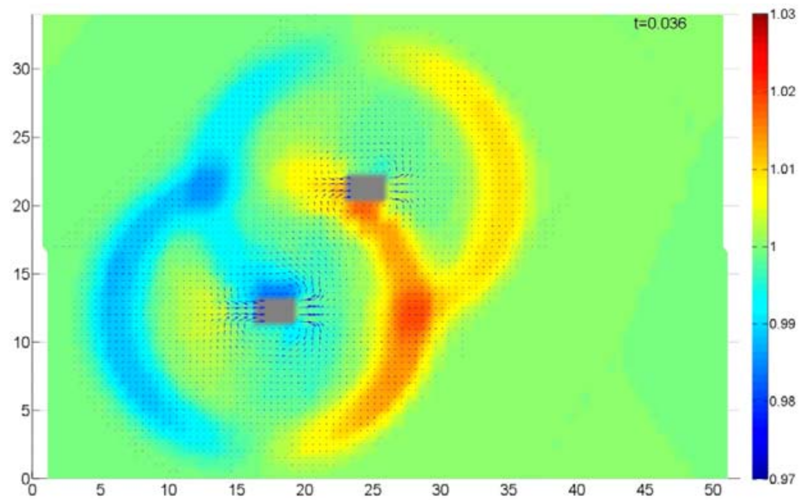

Figure-5. Pack overtaking aerodynamics at the moment of 0.036 seconds.

Figure-6 demonstrates aerodynamics of one vehicular pack overtaken by another pack driving at a high speed at the moment of 0.7 s. Flows occur behind the vehicles. It is worth mentioning that the speed of this flow at the vehicle driving at a faster speed is higher.

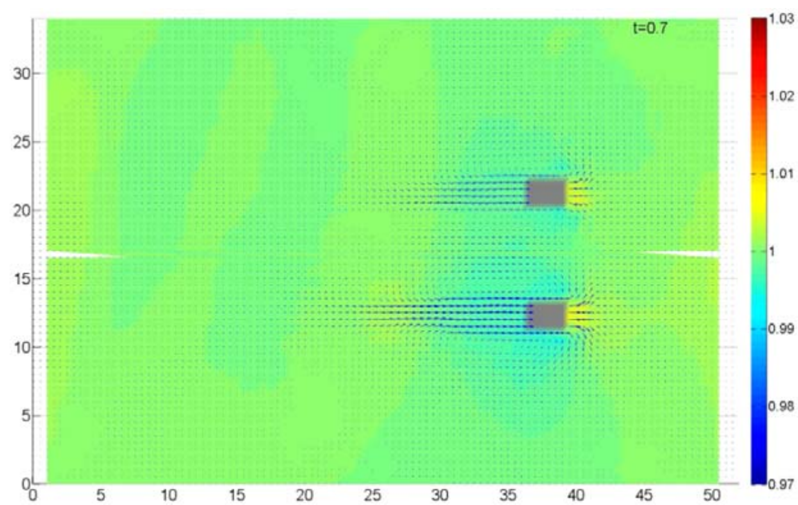

Figure-6. Pack overtaking aerodynamics at the moment of 0.7 seconds.

The direction of flow behind the vehicles gradually deviates from a straight line, as the process develops, resulting in the formation of Karman vortex street. Figure-7 shows that by the time moment of 2.7 s several vortices have already been formed. At that moment Karman vortex streets formed in the vehicular packs start slightly interacting with each other.

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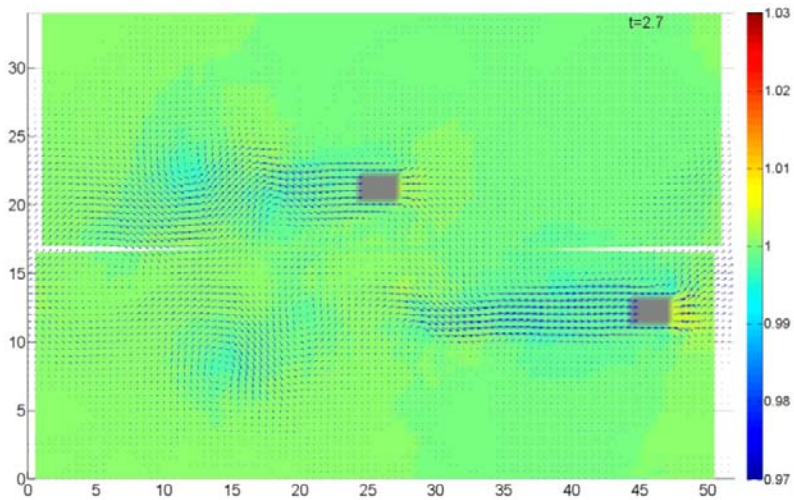

Figure-7. Pack overtaking aerodynamics at the moment of 2.7 seconds.

Over time, the vortices grow and shift across the vehicular traffic, causing vortex flow occurrence between the packs of the vehicles under consideration.

The collision of the vortex with the vehicle occurs by the time moment of 4.4 s with respect to the coordinates x = 10 m, y = 20 m (Figure-8). The figure shows that a significant distortion of the velocity field is not observed, even despite the fact that the vortex is on the lane boundary.

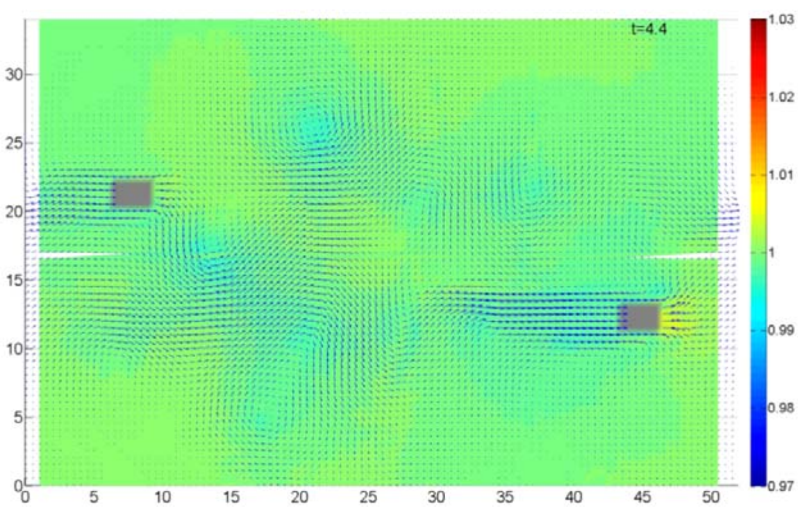

Figure-8. Pack overtaking aerodynamics at the moment of 4.4 seconds.

The computational results show that during the vehicular pack motion vortices are formed between the vehicles, which are able to retain their stability and move rather slowly to interact with a vehicle driving behind. As the process develops, vortices take the larger part of space between the vehicular packs. Vortex flow intensity depends both on the distance between the packs, and on the distance between the vehicles inside the pack.

Results of numerical aerodynamic simulation for opposing traffic of vehicular packs

This part below demonstrates the computational results of aerodynamics for packs driving towards each other with different distance of vehicles therein (49 and 94 m), and analyzes the obtained results. Pack opposing traffic aerodynamics is shown in Figures 9-10. Until 1.8 s the process dynamics is not demonstrated since initially flows formed by each vehicle do not exert significant impact on each other. Onset of the Karman street

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formation process is illustrated in Figure-9. It is important to note that flows forming behind each vehicle have similar configuration due to the similarой speed of the driving vehicles, their geometry and distance.

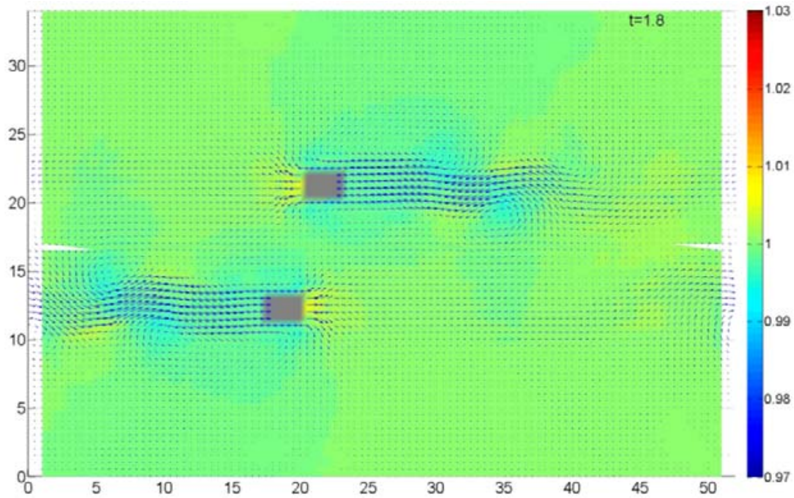

Figure-9. Pack opposing traffic aerodynamics with a distance of 49 m at the moment of 1.8 seconds.

As the process develops, vortices interact with each other, which results in their distortion. It is seen in Figure-10 that vortex flow is formed between the vehicular packs and vortex flows occupy the entire interval.

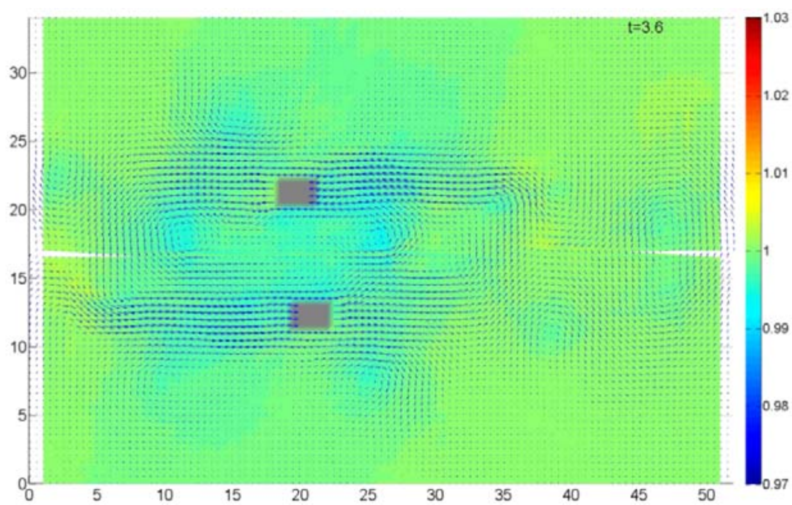

Figure-10. Pack opposing traffic aerodynamics with a distance of 49 m at the moment of 3.6 seconds.

To evaluate the impact of the distance between the vehicles in the pack on the aerodynamic pattern, the calculation with the distance between the vehicles increased to 94 m was performed. In this case the vehicle location at the initial moment in the lower lane was determined by the following coordinateгs: 37-40 m on the x-axis and 12-14 m on the y-axis. Vehicle located in the upper lane has the coordinates: 49-52 m on the x-axis and 21-23 on the y-axis.

In the course of calculations it was revealed that when the pack of vehicles is driving with the increased distance, vehicles running ahead slightly influence the vehicles running behinds. However, the vehicles still exert some impact on each other since the distance between the packs themselves remains the same.

VOL. 11, NO. 1, JANUARY 2016 ISSN 1819-6608
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Aerodynamic pattern of moving opposing pack at the moment of time 0.05 s and 0.4 s is shown in Figures 11-12. Elastic waves come out of the computational area through the upper free boundary, while the waves passing through the cyclic side boundaries can propagate longer as they decay. At the time of 0.4 seconds, the flow pattern is symmetric.

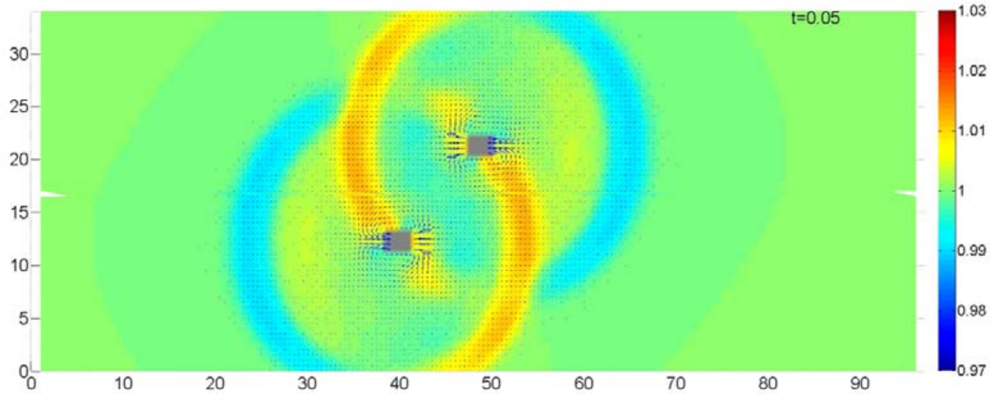

Figure-11. Pack opposing traffic aerodynamics with a distance of 94 m at the moment of 0.05 seconds.

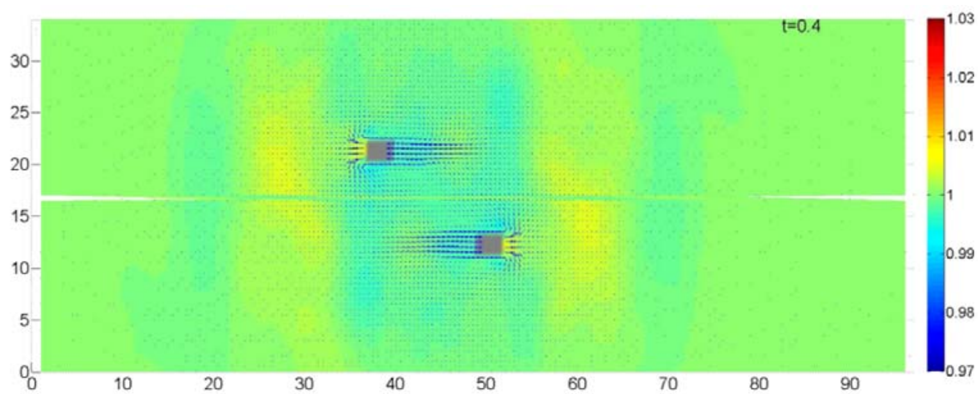

Figure-12. Pack opposing traffic aerodynamics with a distance of 94 m at the moment of 0.4 seconds.

Similar to calculations presented above, flows occur behind the vehicles, but in Figure-13, these flows hardly interact, since vehicles are at a great distance from each other. но на р

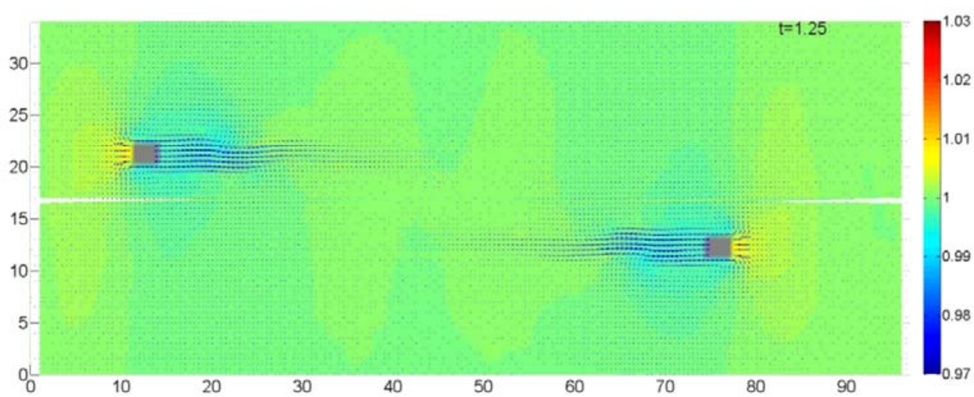

Figure-13. Pack opposing traffic aerodynamics with a distance of 94 m at the moment of 1.25 seconds.

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Figure-14 demonstrates formation of Karman Vortex Street by the crossed cyclic boundary. The calculation results show that the flow pattern is not distorted, and accordingly, the boundaries between the areas do not introduce any errors.

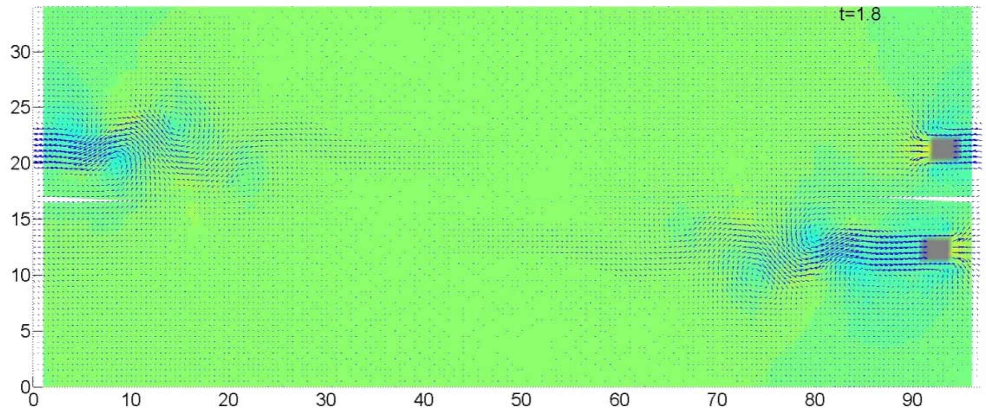

Figure-14. Pack opposing traffic aerodynamics with a distance of 94 m at the moment of 1.25 seconds.

It is clear from Figure-15 that a significant portion of the estimated area is occupied by the vortex flow formed between the lanes due to the interaction of two vehicles.

The distance between the vehicular packs exerts an impact on the velocity field in front of the vehicle, as well as on the lateral force occurring due to the interaction of the vortex flow formed as a result of interaction between the pack and the vehicle.

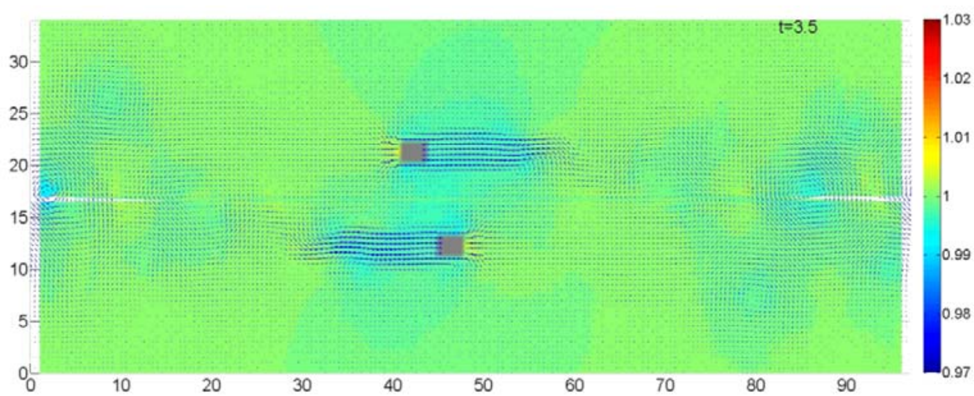

Figure-15. Pack opposing traffic aerodynamics with a distance of 94 m at the moment of 3.5 seconds.

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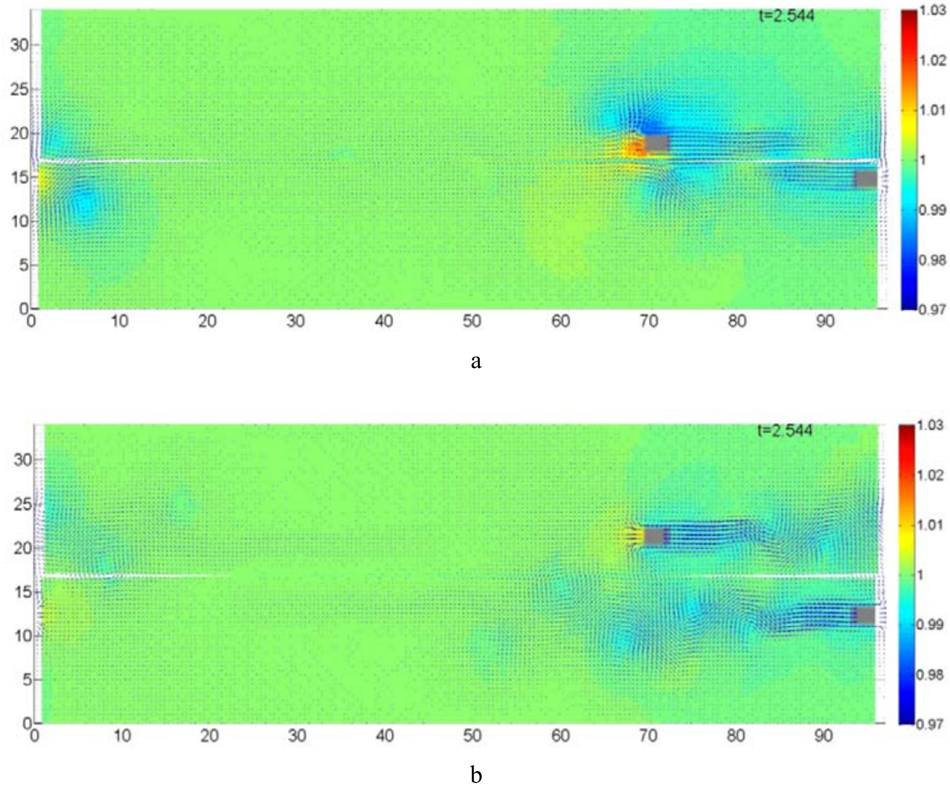

Figure-16. Pack opposing traffic aerodynamics with a distance of 94 m at the moment of 2.544 sec: a) distance between the packs being 2 m, b) distance between the packs being 7 m.

Impact of the lateral force increases with the distance decrease, which results in the increased yawing moment (Figure-16(a)), however with larger distance between the vehicular packs, being equal to 7 m, this effect is absent (Figure-16(b)).

CONCLUSIONS

Use of discontinuous coordinate transformation allows for computational algorithms, позволяет алгоритмы вычислений, it imposes no stringent requirements on the grid structure and, consequently, the computation time increases only slightly. It is shown that this technique is well applicable in the case of pack motion in various configurations, provided that the movement in all packs is periodic with the similar period. Although the results are related to the motion simulation of two packs, this technique can be easily applicable for a greater number of packs driving in different directions at different speeds.

According to our computational results, vortex flow is formed between the packs both in the opposing traffic and when overtaking, creating dynamically changing yawing force, however, in case of opposing traffic the intensity of vortices is somewhat higher, which is consistent with the results of [15]. At the same time, both the pack driving speed and the distances between the vehicles in the pack and between the packs also affect the formation of this vortex flow. Computational results demonstrate that the greater the length of the computational area (which corresponds to the period in the pack), the more slowly vortex flows are formed therein

The article provides the method for calculating aerodynamics of packs driving at different speeds and in different directions, and gives numerical results. As indicated by the results, subarea-stitching boundary introduces some error related to the interpolation inaccuracy; however, it does not lead to distortion of the results. This error may be decreased by using more accurate interpolation methods than the linear one. Nevertheless, in sections with rapidly changing solution, use of the interpolation of a higher order does not provide improved accuracy. The proposed approach may provide more accurate results in case of applying irregular grids; however, due to the movement of the subareas, it will be necessary to update information nearby the boundaries with regard to which cells are adjacent.

ACKNOWLEDGMENTS

The research was funded by the Ministry of Education and Science of the Russian Federation in the framework of the contract No. 02.G25.31.0006 of 02.12.2013 (Resolution of the Government of the Russian Federation dated April 9, 2010 No.218).

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