



AN IMPROVED ENERGY BASED LOAD-DISPLACEMENT PREDICTION FOR SLOTTED DISC SPRING

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ABSTRACT

An improved calculation of the load-displacement prediction for a slotted disc spring is proposed. There are two types of slotted disc spring; a straight slotted disc spring and a bended slotted disc spring. By considering these two types of slotted disc spring, this work attempts to extend the previous work on load-displacement prediction for the slotted disc spring using the energy method. To show the validity of the improved method, the load-displacement results using the improved computation, the previous proposed method, and the finite element analyses are compared. In this work, the improved computation and the finite element analyses have accounted for the radial deflection of the disc spring which is different in the previous work. The radial deflection of the disc spring was ignored in the previous work and the comparison to the finite element analyses without accounting the radial deflection is not practical. The present work is compared to the finite element analyses which make the comparison to be more reasonable and more practical.

Keywords: straight slotted disc spring, bended slotted disc spring, energy method, load-displacement, radial stresses, tangential stresses.

INTRODUCTION

A slotted disc spring is designed to increase the allowable deflection range of the coned disc spring at smaller loads [5]. The theory of Almen was extended by Schremmer [5] to predict the load-displacement for the slotted disc spring. However, this extended work ignored the radial stresses at the coned segment [4, 7]. The extended equations were also only for a straight slotted disc spring [1, 5]. To solve this issue, the prediction equations for a non-straight slotted disc spring were proposed in Ref. [1] in the form of energy method. However, the radial deflection was also ignored in this work. A comparison to the finite element analyses was not practical since the deflection in the radial direction was allowed in the finite element analyses.

By adopting the Curti's computation [4, 8, 10], the improved calculation is employed to a single unit coned disc spring and extended to either the straight slotted disc or bended slotted disc spring.

IMPROVED LOAD-DISPLACEMENT FOR CONED DISC SPRING IN THE FORM OF ENERGY METHOD

The all notations for the present improved computation are consistent with Ref. [1]. Only the central lines within the cross-sections are considered. Figure- 5 and Figure- 6 describe the notations of the improved proposed energy method for all referred disk springs. For coned disc spring the $F_3=F_1$, $l_3=l_1$, and $\delta_3=\delta_1$. In this paper, the F_3 adopt the Curti's equation to describe the load-displacement of the coned segment as in equation (2). The term γ in Ref. [1] is equal to θ for a straight slotted

disc and non-zero for a bended slotted disc spring. Similar assumptions as in the Ref. 1 are used. For the coned disk spring, the whole body rigidly rotates through rotation θ [1]. For the slotted disk spring, it rigidly undergoes similar rotation and at the same time is loaded by the force F_1 [1]. By accounting the radial deflection, the load-displacement for all types of disc springs in Ref.1 can be modified as follows;

$$F_1 = F_3(\bar{C}, \bar{D}) \frac{d\delta_3}{d\delta_1} \quad (1)$$

$$F_1 = \frac{4Es}{d_e^2} \left[(h_0 - s) \left(h_0 - \frac{s}{2} \right) \bar{C}t + \bar{D}t \right] \frac{l_3}{l_1} \left[\frac{1}{\cos\gamma - [\tan(\alpha - \theta)\sin\gamma]} \right] \quad (2)$$

where θ and δ_3 can be calculated using [1];

$$\theta = \alpha - \sin^{-1} \left[\sin\alpha \frac{\delta_3}{l_3} \right] \quad (3)$$

$$\delta_3 = l_3 \left[\sin\alpha - \sin \left(-\gamma + \sin^{-1} \left(\sin(\alpha + \gamma) \frac{\delta_1}{l_1} \right) \right) \right] \quad (4)$$

In equation (1), the \bar{C} and \bar{D} are used. Similar to Ref. [1], the term F_3 is the force acts at the coned segment. This is defined from the Curti's work to predict the force-displacement for the coned segment.

A bending deflection is considered for disc spring that has slots segments [1, 5, 6, 7]. This computation is



done by additional calculations to displacement δ_l . This bending deflection can be expressed as follows [1, 5, 6, 7]:

$$\delta_b = K \frac{4P(d_i/2 - d_0/2)^3}{Eb_0t^3m} (1 - \mu^2) \tag{5}$$

where K is defined as

$$K = \frac{3}{(1-b/b_0)^3} \left[\frac{1}{2} - 2\left(\frac{b}{b_0}\right) + \left(\frac{b}{b_0}\right)^2 \left(\frac{3}{2} - \ln \frac{b}{b_0} \right) \right] \tag{6}$$

The plotted graph of $F_l(\delta_l + \delta_b)$ is the final load-displacement for slotted disc spring. The schematic diagram for the straight slotted disc spring and the bended slotted disc spring are shown in Figure-1 and Figure-2 respectively.

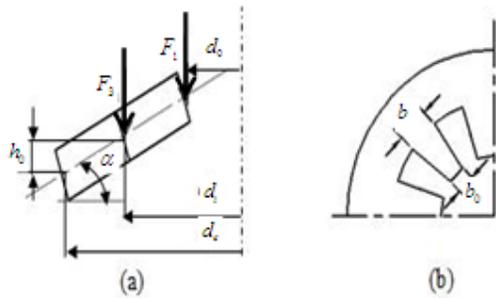


Figure-1. (a) Straight slotted disc (b) quarter top view.

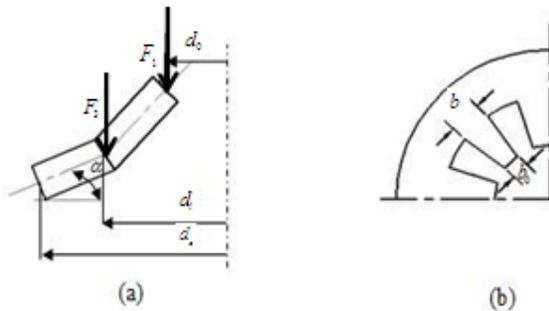


Figure-2. (a) Bended slotted disc (b) quarter top view.

COMPUTATION RESULT FOR A STRAIGHT SLOTTED DISC SPRING

Figure-3 shows the computation result for the straight slotted disk spring. The error between finite element analyses and the improved energy method computation seems to be approximately lesser in comparison to the previous work [1]. Even though the load-displacement using the improved energy based-method is not exactly similar to the finite element analyses, the reduction of error in comparison to the Ref.

[1] shows the effectiveness of the improved computation. Not only that, the comparison is more reasonable since both the analytical improved energy method and the finite element analyses account for the radial deflection.

Slotted disc spring has a more complex stress distribution than the regular coned disc spring [5]. To extend the present improved energy-based method, stresses analyses at the inner diameter are recommended. These include the compressive tangential stress, the radial tensile stress due to bending, the stress concentration influence at the root of the tongues, and the residual stresses from the presetting operation. Since the present study is limited to the extension of the Curti's work, the mentioned stresses analyses are not considered. Further improvement is deferred for future work.

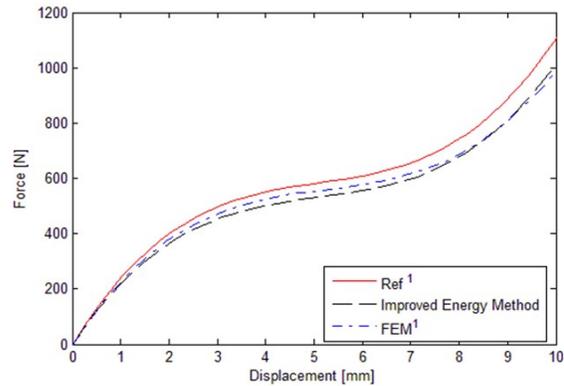


Figure-3. Load displacement prediction for a straight slotted disc spring.

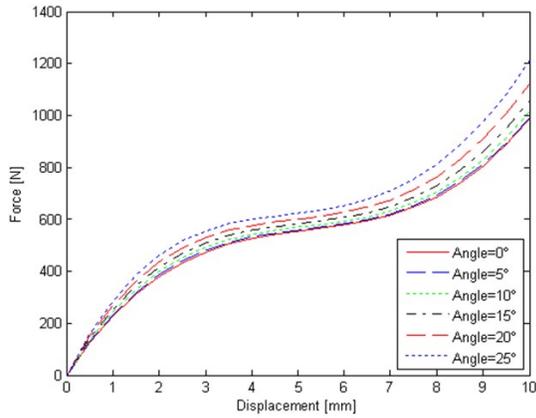
COMPUTATION RESULT FOR A BENDED SLOTTED DISC SPRING

The improved computation method was then extended to bended slotted disc springs. Similar geometric profiles and material properties were used as in Ref [1]. By employing the improved energy method, the calculated results are compared to the finite element analyses results and the previous proposed energy method1 as shown in Figure- 4 and Figure- 5. The error between the calculated load-displacements using the improved energy method and the finite element analyses result for both cases (angle and height variations) seems to be smaller than the previous work [1]. The improved calculations approximately approach the finite element analyses mostly within the all displacement regions in comparison to the previous computations. These computations improve the previous works.

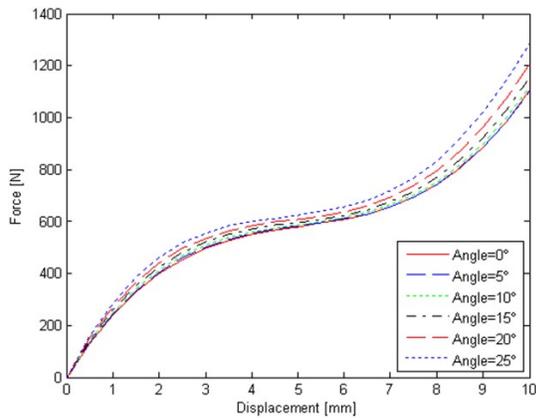
Even though there is still a slight difference between the computations using the improved energy method and the finite element analyses, the computation using the improved energy method is more reasonable. The radial deflection is accounted in the improved energy method which is similar to the finite element analyses. The



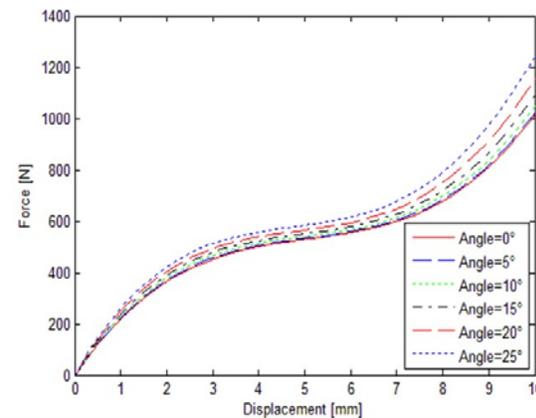
deflection in the radial direction is not considered in the previous proposed energy method.



(a)

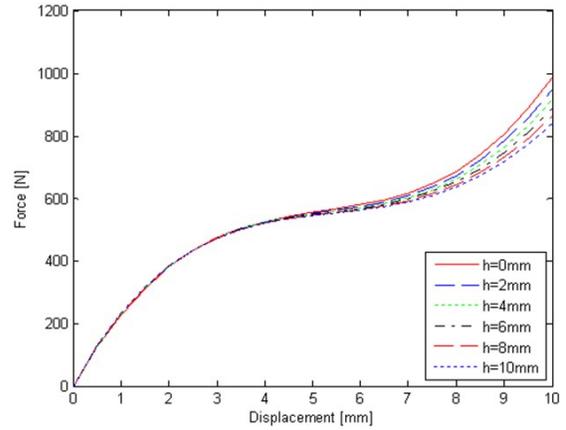


(b)

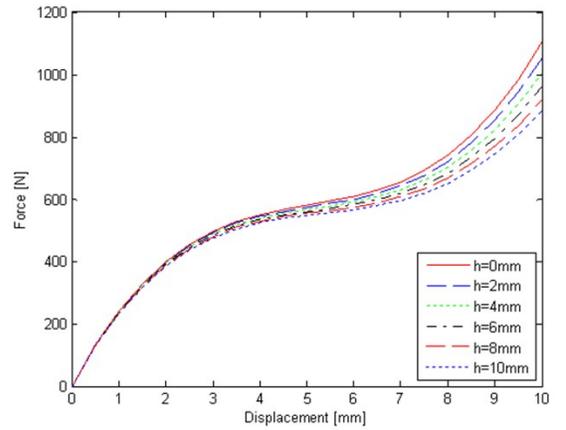


(c)

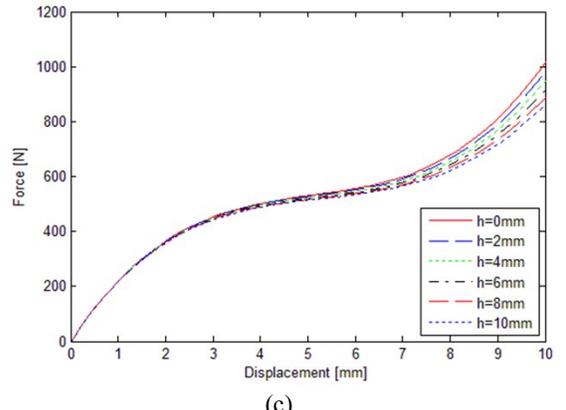
Figure-4. Comparison of load-displacement curves with angle variation using (a) Improved method (b) FEM (c) Ref [1].



(a)



(b)



(c)

Figure-5. Comparison of load-displacement curves with height variation using (a) Improved method (b) FEM (c) Ref. [1]



CONCLUSIONS

An improved analytical load-displacement for both slotted disc springs are presented by considering the radial deflection. This extended analytical prediction is proposed to match with the three-dimensional finite element analyses results besides accounting for the radial deflection which is similar to the boundary conditions of the finite element analyses.

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