VOL. 11, NO. 3, FEBRUARY 2016 SSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

FAST PHASE UNWRAPPING METHOD BASED ON G-PUMA AND SPA TECHNIQUES: G-PUMA-SPA

Sharoze Ali [1], Habibulla Khan[1], Idris Shaik[2] and Firoz Ali[3]
[1]Department of Electronics and Communication Engineering, KL University, Vaddeswaram, Guntur, Andhra Pradesh, India
[2]Department of Electronics and Communication Engineering, Bapatla Engineering College,Bapatla, Andhra Pradesh, India
[3]Department of Electrical and Electronics Engineering, Nimra College of Engineering and Technology, Vijayawada, Andhra Pradesh, India
E-Mail:Sharu786786@gmail.com

ABSTRACT

The problem of Phase unwrapping (PU) is solved by many Phase unwrapping algorithms. Thus far, many PU methods with high accuracy have been achieved. However, the memory utilization and CPU limitations are ignored during designing such PU algorithms. To effectively solve this problem, a fast PU method is proposed in this method. The proposed algorithm consists of two steps firstly the phase is unwrapped by using the cache efficient G-PUMA algorithm and later on, the unwrapped phase is further denoised by Second order polynomial approximation. The proposed algorithm smartly selects the window according to the smoothness and shows greater attenuation to noise. G-PUMA-SPA algorithm not only unwraps phase faster but also robust to noise. Experiments show that the proposed method can achieve better results than the method PUMA-SPA, Congruence Operation and Least Squares Fitting (CO-LSF) proposed recently.

Keywords: G-PUMA, fast phase unwrapping method, local polynomial approximation (LPA), 2-D phase unwrapping.

1. INTRODUCTION

Estimation of an absolute (true, ϕ) phase from the measured phase (wrapped, principle, ψ) is a key problem for many imaging techniques. For instance, in remote sensing applications [1] like Synthetic aperture radar (SAR) or Sonar (SAS), phase difference between the terrain and the radar is captured by two or more antennas. The measured phase by SAR or SAS are wrapped in the interval of [-π, π].

Similarly for MRI (Magnetic Resonance imaging), PU technique is used to determine magnetic field deviation maps, chemical shift based thermometry, and to implement BOLD contrast based venography. PU also acts as a necessary tool for the three-point Dixon water and fat separation. In optical interferometry, phase measurements are used to detect objects shape, deformation, and vibration.

In all these applications, there are two problems associated with the wrapped phases. Observation mechanism in these applications is a 2π-periodic function of the true phase or absolute phase. The mapping of this function in the interval [−π; π] yields the so-called wrapped phases, or interferogram. Even if the true phase is outside the interval [-π; π] the associated observed value is wrapped into it. It is thus highly impossible to unambiguously reconstruct the absolute phase. Similarly the phase images acquired are interrupted by noise. The former issue can be resolved by using the Phase unwrapping technique, where we can obtain the absolute phases from the measured one (ψ).The later one by properly designing the filters which can eliminate noise only rather than sensitive data.

During the last three decades, redundant of algorithms were proposed on the above two issues. We can summarize them into two types: 1) two step method including denoising and unwrapping, and 2) unwrapping along with denoising.

According to the first method, the phase is firstly filtered and later on it is unwrapped by the PU algorithms. The key advantage of this method is the denoising step. If the high quality of wrapped phase map can be achieved, then the phase unwrapping can be accomplished by a simple unwrapping algorithm [2]. So the main problem turns to the denoising of the wrapped phase images. It has been tackled by lots of researchers and many wrapped phase denoising methods were proposed. Some methods propose the window filtering techniques [3, 4, 5] to denoised the phase. Some other uses the local filter techniques [6]-[8] to denoise the phase. This algorithm [5] combines a particle filter, a matrix-pencil (MP) local slope estimator, and an optimized region-growing technique is proposed to do the unwrapping and denoising simultaneously. But PEARLS [9] is a wrapped phase denoising method which smartly selects the fitting window size adaptively according to the smoothness of the profile. A set of the unwrapping methods were proposed during the past ten years [10], [11-14] to do unwrapping along with denoising. A large amount of them do the denoising and unwrapping simultaneously, and there are also algorithms doing the unwrapping firstly and post processed by a denoising method (e.g., [13]). A fourth-order polynomial approximation is proposed to do denoising in [13], but it is more sensitive to phase discontinuities because of the fixed window size. Similarly, in this method [15] second order local polynomial approximation is proposed to suppress the noise of unwrapped phase. PUMA [10] is a generic form for the optimization based phase unwrapping method to let the user to select the potential function.

All the above stated algorithms are focused on the problems of Phase unwrapping and denoising the wrapped phase. However, the limitation of computation time and computer's memory requirement are ignored in the design of most of these methods. So, there is a need of designing the PU algorithm in a smarter way, so that it unwraps the phase faster, as well as it should eliminate the errors of wide range. This challenging task of designing PU is an interest area of research and draws attention of many researchers.

A section of faster algorithms are dividing the large images into blocks and processing it [16, 17, 18]. In Literature [16], the wrapped phase image is tessellated into small blocks. Each block is unwrapped separately by minimum discontinuity phase unwrapping algorithm, and then the unwrapped blocks are merged together.

Similarly, a large-scale interferogram is first partitioned into small tiles according to a strategy based on the residue clustering characteristics [17].Once partitioned; each tile will be independently unwrapped by minimum-spanning-tree-based PU method either in parallel or in series.

In another literature [18], partitioning the wrapped phase map into high quality (HQ) pixels and Low Quality Pixels by utilizing the edge pixels on object image and processed.

As phase unwrapping errors are often caused by shadows on fringe patterns images and object surface discontinuities. Once these areas are identified then these areas are unwrapped by highly efficient technique and the rest of the image is unwrapped by efficient phase unwrapping technique as in [19].

There are also some other algorithms which achieve faster runtimes by reducing the complexity or applying new techniques to the existing algorithms. A new and fast algorithm proposed in [20], which does only one-dimensional searching for SAR Images and therefore reduces the complexity significantly of Old Chinese remainder theorem (CRT). Similarly, I-PUMA [21] is a technique which reduces the runtimes of the PUMA algorithm by reducing the complexity of PUMA to$O(n^2m)$. In [22], Cache efficient optimization techniques are used to achieve faster runtimes than PUMA.

In this literature [23], Total Variation (TV) technique is used for prior estimation and graph cut as an optimization technique to achieve faster run times than MAP-MRF Phase estimation technique. A new implementation of the minimum spanning tree (MST) phase unwrapping method is presented in [24]. The time complexity of the MST method is reduced from $O(n^2)$ to $O(n \log_2 n)$ where n is the number of pixels in the phase map. For speed up the existing algorithm, new cost function is used for Unwrapping [25].

Another type of algorithms [26]-[28], uses FFT technique for unwrapping and achieves faster runtimes. In Literature [26], the use of the fast Fourier transform makes it possible to have a fast algorithm that is to process an image of N pixel in $O(N \log_2 N)$ elementary operations. In another literature [27], the execution time of algorithm is equivalent to the computation time required for performing eight fast Fourier transforms. In [28], achieves faster runtimes by executing the parallel implementation of a single-step Fourier-based phase unwrapping algorithm on the graphics processing unit (GPU) of a standard graphics card.

In this letter, we analyze the computational burden of PUMA –SPA due to additional step of denoising and then propose a fast G-PUMA-SPA method which not only unwraps phase faster but also smartly eliminate the noise by adaptive window. With this method, interferogram is firstly unwrapped using the smarter G-PUMA technique [22] and then do the denoising using the Second Polynomial Approximation as mentioned in [15]. The significant advantage of the proposed method (G-PUMA-SPA) is that we can unwrap the phase faster even for larger interferogram's and it consumes less memory than the existing PUMA-SPA. So the proposed method is more intelligent, faster and can compromise between protecting the details and preventing the noise. From the experiments in this letter our proposed method achieves faster runtimes than other unwrapping methods and gains the significant advantage of PUMA-SPA algorithm.

The remaining of the letter is organized as follows. Section II presents the PUMA-SPA algorithm; in Section III, we present the G-PUMA-SPA algorithm and Section IV presents a set of experiments and the results to compare with other algorithms and we conclude this letter in Section V.

2. PUMA-SPA:A PHASE UNWRAPPING METHOD BASED ON PUMA AND SPA APPROXIMATION

In general there may be two types of errors in the phase map. One type is caused by the noise itself and another type of error is caused by the noise plus wrapping operation. This second type of errors is called as impulse errors. Noise in Region 1 of Figure-1 is caused due to noise only where as noise in Region 2 of Figure-1 is caused by impulse error. The impulse errors caused by wrapping often have a jump of -2π or 2π because of the wrapping operation. By using the denoising algorithms it is easy to remove the other noises than the impulse errors. But, Phase unwrapping algorithms can easily remove the impulse errors. So it is better to remove the impulse errors first and the rest by using denoising algorithm. PUMA-SPA algorithm suggests this type of approach to reduce the errors. In literature [15], it is showed that more noise (RMSE) is eliminated by PUMA-SPA algorithm than PEARLS, PUMA and WFF-QG-CO-LSF algorithms.

The approach of PUMA –SPA is to first unwrap the phase by PUMA algorithm, later on the unwrapped phase is denoised by Second order polynomial algorithm. There are two advantages by doing so. The first one is the information related to the discontinuities is not erased

ARPN Journal of Engineering and Applied Sciences

during the denoising step and another advantage is the unwrapping phase reconstructed by PUMA do not have the impulse errors.

Let's have a closure look at the PUMA-SPA Algorithm. Assume that the noisy wrapped phase is unwrapped by PUMA and we denote the unwrapped phase as ϕ. For all the pixels around $\phi(i,j)$, we use a second-order polynomial to approximate this area and then the pixels are organized in the window w_h column wise into a vector and denote it as $\phi_h(i,j)$

$$\phi_h(i,j) = A_h \Theta_h(i,j) \quad (1)$$

In the above equation, $A_h \in \mathcal{R}^{(2h+1)(2h+1)\times 6}$ is just related to the size of the window and $\Theta_h(i,j)$ is the parameters of the second-order polynomial at point (i,j). We can estimate these parameters by

$$\widehat{\Theta}_h(i,j) = \Theta_h(i,j) + (A_h^T A_h)^{-1} A_H^T \phi_h^n(i,j) \quad (2)$$

If we substitute the equation 2 in equation 1 then the phase estimate by the second order polynomials are

$$\hat{\phi}_h(i,j) = A_h \Theta_h(i,j) + A_h (A_h^T A_h)^{-1} A_H^T \phi_h^n(i,j) \quad (3)$$

If the noise is zero mean with variance σ^2, the variance of the estimated phase is as below:

$$\sigma_h^2\left(\widehat{\phi}_h(i,j)\right) = \sigma^2 A_h \left(A_h^T A_h\right)^{-1} A_H^T \quad (4)$$

The denoised result of pixel $(i,j)$is $\hat{\phi}_h(i,j)|_{u=0,v=0}$, then the variance of the denoised result is the accordance entity in matrix $\sigma^2 A_h (A_h^T A_h)^{-1} A_H^T$ .Thus we propose the algorithm to denoised the unwrapped phase as follows.

a) For every window size $h \leq h_{max}$ compute the matrix $\left(A_h^T A_h\right)^{-1} A_H^T$ and$A_h \left(A_h^T A_h\right)^{-1} A_H^T$;

b) For every window size $h \leq h_{max}$ and every pixel, estimate the second-order polynomial parameters by (2) and get the denoised result by $\widehat{\phi}_h(i,j)|_{u=0,v=0}$;

c) According to the denoised result and the variance, select the best window size $h_{+(i,j)}$ of every pixel.

d) Output $\widehat{\phi}_{h+(i,j)}(i,j)|_{u=0,v=0}$ as the final denoised version of pixel (i,j)

Adaptive window size

For a large window size h, with the denoised result$\hat{\phi}_h(i,j)|_{u=0,v=0}$, it holds two hypotheses {H_0 : the estimate of denoised result is good; H_1: the estimate of denoised result is bad}. We accept the hypothesis H_0 for a window size h if two conditions are satisfied:

1) The result is in the interval $[\hat{\phi}_h(i,j)|_{u=0,v=0} - \Gamma\,\sigma_h, \hat{\phi}_h(i,j)|_{u=0,v=0} + \Gamma\,\sigma_h]$in which σ_h is the standard deviation of $\widehat{\phi}_h(i,j)|_{u=0,v=0}$, which we can get from (10). $\Gamma$is a parameter which is set to Γ=2;

2) For all the window sizes $h\_ < h$, the estimate result is in the interval of $\left[\widehat{\phi}_{h-}(i,j)|_{u=0,v=0} - \Gamma\sigma_{h-}, \widehat{\phi}_{h-}(i,j)|_{u=0,v=0} + \Gamma\sigma_{h\_}\right]$. It is equivalent to say the denoised result by the window size is not different too much from the denoised result get from smaller window sizes than it. Then the adaptive window size selection method is selecting the largest window size that we can accept the hypothesis H_0.

3. PUMA-SPA:A PHASE UNWRAPPING METHOD BASEDON PUMA AND SPA APPROXIMATION

At a high level, PUMA-SPA is a two-step process. As a First step, the noisy wrapped phase is blindly unwrapped by the PUMA algorithm and ensures that the impulse noise is removed in the unwrapped noise. As a second step, the unwrapped noise is further filtered by using the second order polynomial algorithm with an adaptive varying window. The only disadvantage of PUMA-SPA algorithm is it consumes a lot of time and memory to unwrap phase of larger images. This often time limits the utilization of PUMA-SPA algorithm.

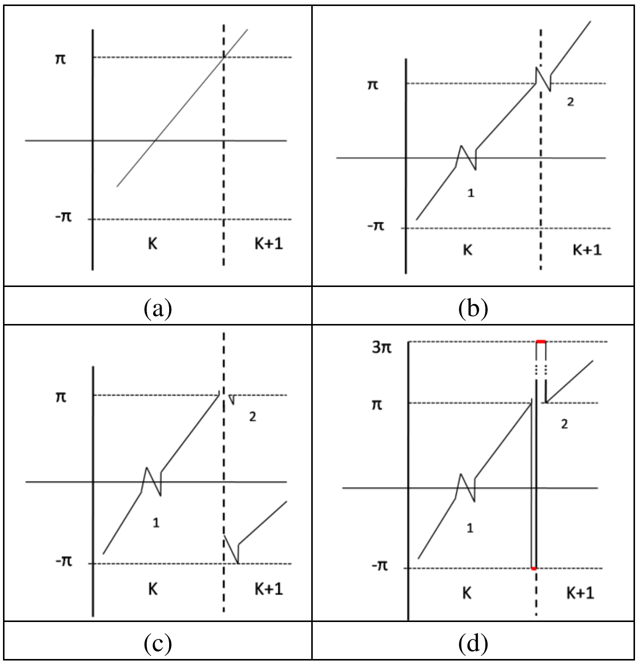


Figure-1. Example of error types in the phase map image in a one dimension case. a) Original unwrapped phase, with the multiples of integers k and k + 1; (b) noisy unwrapped phase, with two regions interrupt by noise, regions marked 1 and 2; (c) wrapped noisy phase; and (d) the unwrapping result by adding the true integers time 2π, the impulse errors are marked red.

In-order to overcome the difficulty and to increase the speed of the algorithm, we have to adopt either of the below two approaches.

ARPN Journal of Engineering and Applied Sciences

a) To reduce the complexity of the PUMA-SPA Algorithm
b) To use the cache efficient techniques while calculating the max flow.

At this point of time, we have to discuss the algorithm I-PUMA [21] which reduces the complexity of the PUMA algorithm and achieves the faster runtimes. But among the available methods, G-PUMA [22] is the best unwrapping method which uses the cache efficient techniques to unwrap the phase faster by reducing the stress on the memory bandwidth. G-PUMA algorithm achieves faster runtimes than I-PUMA, PUMA, CUNWRAP methods. When we analysed the PUMA-SPA algorithm for improvement, especially the optimization part, these below are the issues faced by the PUMA-SPA algorithm.

a) During the computation of the max flow algorithm, there will be frequent transfer of data between memory and CPU .Also constant updates are required, which causes heavy stress on the memory bandwidth.
b) More time consumed while finding the connectivity information using pointers on every time. The Pointers are generally used to represent the general graphs and it provides the connectivity information. These pointers often comprise the majority of the graph's memory footprint, in particular on 64-bit CPUs where single pointer occupies eight bytes.
c) Cache is poorly handled.

The run times will be very faster if we are able to solve the above three issues. In G-PUMA-SPA, we are able to solve the above three stated issues where we have employed the cache efficient techniques during the optimization step of PUMA. In this section, we introduce our new, fast and robust unwrapping algorithm "G-PUMA-SPA".

All the above three mentioned issues will play a crucial role in finding min cut of large graphs. The first issue (stress on memory bandwidth) is resolved by employing a compact graph representation with cache-friendly memory layout that exploits the regular structure of grid-like graphs which reduces the stress on the memory bandwidth. Secondly (calculation of connectivity information), by exploiting prior knowledge of the graph structure we can eliminate the need of pointers altogether by determining connectivity information on the fly. Thirdly, Cache can be efficiently utilized by segregating the memory fields as Hot and Cold depending upon their accessibility (Structure splitting) and by dividing the given graph into blocks (Blocked array layout).

Among the available, the best cache efficient unwrapping technique is G-PUMA. Like PUMA algorithm, G-PUMA algorithm also comprises of two steps. In G-PUMA the sequences of steps to unwrap the phase for both convex and non-convex will be remain same. As a First step, we have to construct the elementary graphs by using the energy equations. The main limitation of Gridcut optimization technique is hidden in the fact that it supports only graphs which has grid-like topology. In order to use the same graphs of PUMA on Gridcut, we have constructed the full grid by purging the missing Pixel interactions from its 1- offset neighbour's and pad the edges with Zeros. Once the elementary graph is constructed, the minimization of energy equation with respect to δ is now mapped onto a max-flow problem. By iterating these two steps of constructing of graphs and minimization of graph for 'k' iterations we can minimize the graph and get the unwrapped phase.

The unwrapped phase by G-PUMA is denoised by the Second order polynomial algorithm as specified in the literature [15]. We have also analysed the coding parts of PUMA-SPA and we have identified that there are some unnecessary calls between various programming parts of PUMA-SPA and by properly tunning it we can make the algorithm bit faster. As per [29] the run times will be faster if we use the optimization techniques while compiling the matlab mex's. So, we have also make use of full compiler optimizations technique (/Ox in Visual Studio) while compiling the Mex. We have in-cooperated all these above mentioned changes and named it as G-PUMA-SPA.

From the experiments in Section V, We have attained 40-75% faster in run times for both 32-bit and 64-bit machines. Also our new algorithm consumes less memory than PUMA-SPA algorithm of about 20-40% for 32-bit and 40-60% for 64-bit machines.

4. EXPERIMENTAL RESULTS

The effectiveness of PUMA-SPA algorithm is illustrated by conducting several experiments on continuous phase surfaces and discontinuous phase surfaces. To effectively compare our new algorithms G-PUMA-SPA with PUMA-SPA, we too conducted experiments on the same profiles used by PUMA-SPA. Once the phase is unwrapped by G-PUMA, then we use the local polynomial SPA as in literature [15].

Specification of computer in experimental is processor Intel Core Duo2 Processor, 32-bit, 4 GB RAM, 500 GB HDD, 2 MB Cache memory and LCD 15". Image Simulations are as below:

a. Gaussian Continuous Phase Surface

b. Gaussian Discontinuous phase surface

c. Interferometric Synthetic aperture Radar

i. Gaussian continuous phase surface

Gaussian surface of 220 × 220 with $\sigma_x = 75$ and $\sigma_y = 50$ is generated and then we add the Gaussian noise with $\sigma_n = 0.3$ to the Gaussian surface. The window size set in the window size selection method is $1-10$.The parameter in the window size selection method is set to Γ=2. The wrapped data is shown in Figure-2 (a). The unwrapped phase by G-PUMA algorithm is shown in

VOL. 11, NO. 3, FEBRUARY 2016 SSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Figure-2(b). The denoised unwrapped phase by G-PUMA-SPA is as shown in Figure-2(c). The Root mean square error (RMSE) of the two methods PUMA-SPA and G-PUMA-SPA is same whereas advantage of the proposed method is the phase is unwrapped quickly. The time taken to unwrap the Phase by PUMA is 4.1161 seconds where as the time taken by G- PUMA is only 2.6837 seconds. PUMA-SPA shows superior performance in removing the errors than WFF-QG-CO-LSF and PEARLS methods. Similarly the inference we can drawn that G-PUMA shows superior performance than these methods as the RMSE is same as PUMA-SPA.

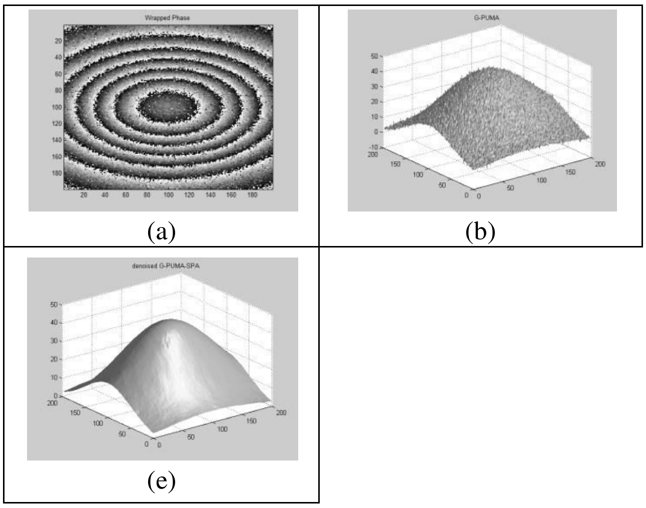


Figure-2. Comparison of denoised and unwrapping results. a) Wrapped data b) Unwrapped phase of G- PUMA c) Denoised by SPA of G-PUMA image.

ii. Gaussian discontinuous phase surface

In the second experiment, we consider the Gaussian surface with discontinuities. The parameters of the Gaussian surface are $\sigma_x = 30$ and $\sigma_y = 20$ and the standard deviation of the noise is $\sigma_n = 0.3$. The adaptive window size set which we select from is $1-20$. Since the surface is discontinuous, we use the nonconvex potential G-PUMA. The wrapped data is shown in Figure-3(a). The unwrapped phase by G-PUMA algorithm is shown in Figure-3(b). The denoised unwrapped phase by G-PUMA-SPA is as shown in Figure-3(c). The time taken to unwrap the Phase by PUMA is 4.1161 seconds where as the time taken by G-PUMA is only 2.6837 seconds. Another advantage of the proposed algorithm is memory improvement than PUMA-SPA.

iii. Interferometric synthetic aperture radar

In this experiment, the original data is the interferometric synthetic aperture radar phase map distributed by [30].Figure-4(a) is the wrapped phase where the presence of noise is clearly visible. Figure-4(b) is the estimated phase by G-PUMA algorithm.The unwrapped phase by G-PUMA is further denoised by SPA algorithm as shown in Figure-4(c). In order to compare the results, we have rewrapped the denoised image by G-PUMA as shown in Figure-4(d) and it shows great improvement than the noisy wrapped phase as shown in 4(a) but still some noise is still there. The rewrapped Phase of G-PUMA-SPA is shown in Figure-4(e) and it eliminates the rest of noise and shows greater robust to noise. The rewrapped phase of G-PUMA-SPA is very clear visible in Figure-4(e) where 4(a) and 4(d) are noisy.The interesting feature of the proposed algorithm is the phase is easily and faster unwrapped by the G-PUMA algorithm i.e. within 4.91 sec whereas the PUMA algorithm unwrapped slowly unwraps the phase i.e. 6.61 seconds. Proposed algorithm shows greater improvement in runtimes.

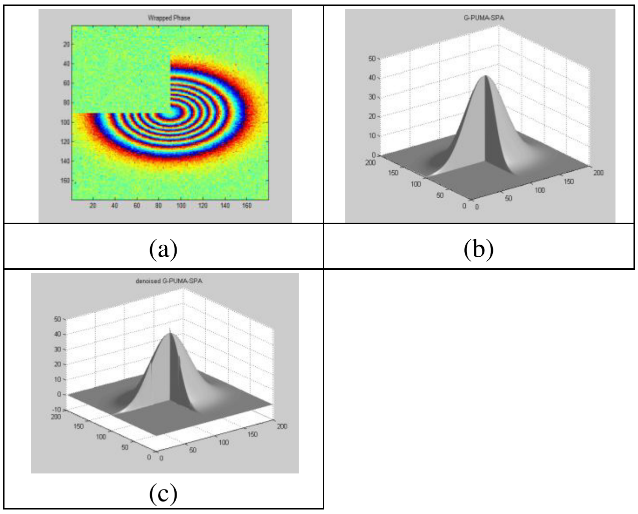
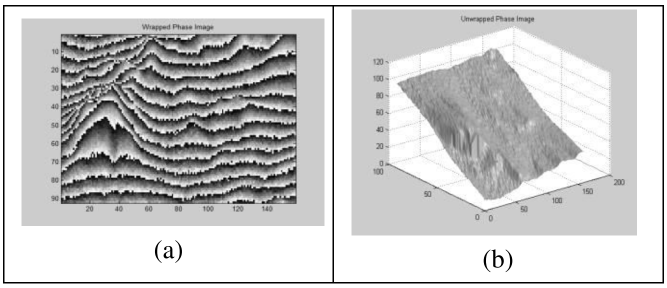


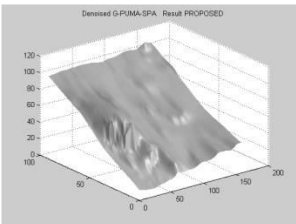
Figure-3. Denoised and unwrapping results of the Gaussian surface with discontinuity. a) Wrapped Data b) Unwrapped phase of G-PUMA c) Denoised phase by SPA further.

5. CONCLUSION

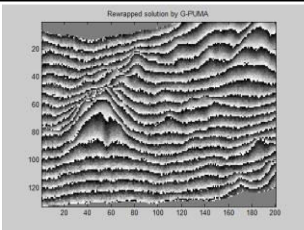
Even though PUMA-SPA is good at unwrapping the phase, but runtimes are affected due to the additional step of de-noising. The runtimes can be further reduced by utilizing the cache efficient graph cut techniques as an optimization technique. In order to reduce the runtimes, we have utilized the cache efficient algorithm "G-PUMA" as an unwrapping algorithm and Second order Polynomial approximation as an denoising algorithm.



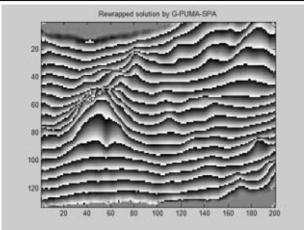
VOL. 11, NO. 3, FEBRUARY 2016 SSN 1819-6608
ARPN Journal of Engineering and Applied Sciences



(c)



(d)



(e)

Figure-4. Denoised and unwrapping results of the interferometric synthetic aperture radar phase. a) Wrapped Data b) Unwrapped phase of G-PUMA c) Denoised Phase further by SPA of G-PUMA image.(d) Rewrapped Solution of G-PUMA (e) Rewrapped solution of G-PUMA-SPA.

The combination of G-PUMA and Second order Polynomial (SPA) shows greater improvement in time, memory and also robust to noise than other algorithms. We abbreviated this new proposed algorithm as G-PUMA-SPA. The algorithms showed greater speed of about 10% - 30% for the same attenuation to noise and a ready. alternative to PUMA-SPA.

ACKNOWLEDGEMENT

The author would like to express great thanks for the help from Hao Hongxing and the other authors of [15] for the helpful discussions.

REFERENCES

[1] Graham L. C. 1974. Synthetic interferometer radar for topographic mapping. Proceedings of the IEEE. 62(6): 763-768.

[2] R. M. Goldstein, H. A. Zebker, and C. L.Werner. 1988. Satellite radar interferometry: Two-dimensional phase unwrapping. Radio Sci. 23(4): 713-720.

[3] Q. Kemao, L. T. H. Nam, L. Feng and S. H. Soon. 2007. Comparative analysis on some filters for wrapped phase maps. Appl. Opt. 46(30): 7412-7418.

[4] J. Lorenzo-Ginori, K. Plataniotis and A. Venetsanopoulos. 2002. Nonlinear filtering for phase image denoising.IEE Proc. Vis., Image Signal Process. 149(5): 290-296.

[5] J. Martinez-Espla, T. Martinez-Marin and J. Lopez-Sanchez. 2009. An optimized algorithm for insar phase unwrapping based on particle filtering, matrix pencil, and region-growing techniques. IEEE Geosci. Remote Sens. Lett. 6(4): 835-839.

[6] Villa J., Quiroga J. A. and De la Rosa I. 2009. Regularized quadratic cost function for oriented fringe-pattern filtering. Optics letters. 34(11): 1741-1743.

[7] Villa J., Rodríguez-Vera R., Quiroga J. A., de la RosaI.and González E. 2010. Anisotropic phase-map denoising using a regularized cost-function with complex-valued Markov-random-fields. Optics and Lasers in Engineering. 48(6): 650-656.

[8] Li Y. H., Qu S. L., Chen X. J.andLuo Z. Y. 2010. Phase pattern denoising using a regularized cost function with complex-valued Markov random fields based on a discrete model. Applied optics. 49(36): 6845-6849.

[9] Bioucas-Dias J., Katkovnik V., AstolaJ.andEgiazarian K. 2008. Absolute phase estimation: adaptive local denoising and global unwrapping. Applied optics. 47(29): 5358-5369.

[10] Bioucas-Dias J. M. andValadão G. 2007. Phase unwrapping via graph cuts. Image Processing, IEEE Transactions on. 16(3): 698-709.

[11] Dias J. M.andLeitao J. M. The ZπM Algorithm for Interferometric Image Reconstruction in SAR/SAS.

[12] Katkovnik V., AstolaJ.andEgiazarian K. 2008. Phase local approximation (PhaseLa) technique for phase unwraps from noisy data. Image Processing, IEEE Transactions on. 17(6): 833-846.

[13] Kemao Q., Gao W. and Wang H. 2010. Windowed Fourier filtered and quality guided phase unwrapping algorithm: on locally high-order polynomial phase. Applied optics. 49(7): 1075-1079.

[14] Nico G. 2013. An application of the principle of minimal frustration to phase unwrapping. Digital Signal Processing. 23(3): 791-796.

[15] HongxingH.andLingda W. 2014. PUMA-SPA: A Phase Unwrapping Method Based on PUMA and

ARPN Journal of Engineering and Applied Sciences

Second-Order Polynomial Approximation. Geoscience and Remote Sensing Letters, IEEE. 11(11): 1906-1910.

[16] Zhong H., TangJ.and Liu D. 2010, May. A fast phase unwrapping algorithm based on minimum discontinuity by blocking. In Future Computer and Communication (ICFCC), 2010 2nd International Conference on. 1: V1-717. IEEE.

[17] Yu H., Xing M. andBao Z. 2013. A fast phase unwrapping method for large-scale interferograms. Geoscience and Remote Sensing, IEEE Transactions on. 51(7): 4240-4248.

[18] Chen K., Xi J.and Yu Y. 2012, June. Fast quality-guided phase unwrapping algorithm for 3D profilometry based on object image edge detection. In Computer Vision and Pattern Recognition Workshops (CVPRW), 2012 IEEE Computer Society Conference on (pp. 64-69). IEEE.

[19] Chen K., Xi J., Song L.and Yu Y. 2013, July. An object image edge detection based quality-guided phase unwrapping approach for fast three-dimensional measurement. In Advanced Intelligent Mechatronics (AIM), 2013 IEEE/ASME International Conference on (pp. 571-576). IEEE.

[20] i X. and Xia X. G. 2008. A fast robust Chinese remainder theorem based phase unwrapping algorithm. IEEE Signal Processing Letters. (15): 665-668.

[21] Sharoze A., Habibulla K., Idris S. and Firoz A. 2015. I-PUMA: Fast Phase Unwrapping Via IBFS Graph Cuts. International Journal of Engineering and Technology. 7(1): 254-265.

[22] Sharoze A., Habibulla K., IdrisS. and Firoz A. 2015. Very Fast Phase Unwrapping Via Grid-Cuts: G-PUMA.International Journal of Applied Engineering Research. 10, No. 14, 34120-34131.

[23] Ferraioli G., Shabou A., TupinF.andPascazio V. 2009, May. Fast InSAR multichannel phase unwrapping for DEM generation. In Urban Remote Sensing Event, 2009 Joint (pp. 1-6). IEEE.

[24] An L., Xiang Q. S. and Chavez S. 2000. A fast implementation of the minimum spanning tree method for phase unwrapping. Medical Imaging, IEEE Transactions on. 19(8): 805-808.

[25] Eineder M., HubigM.andMilcke B. 1998, July. Unwrapping large interferograms using the minimum cost flow algorithm. In Geoscience and Remote Sensing Symposium Proceedings, 1998. IGARSS'98. 1998 IEEE International (Vol. 1, pp. 83-87). IEEE.

[26] Costantini M., Farina A.andZirilli F. 1999. A fast phase unwrapping algorithm for SAR interferometry. Geoscience and Remote Sensing, IEEE Transactions on. 371): 452-460.

[27] Schofield M. A. and Zhu Y. 2003. Fast phase unwrapping algorithm for interferometric applications. Optics letters. 28(14): 1194-1196.

[28] Jeught S. V. D., Sijbers J. andDirckx J. J. 2015. Fast Fourier-Based Phase Unwrapping on the Graphics Processing Unit in Real-Time Imaging Applications. Journal of Imaging. 1(1): 31-44.

[29] Jamriška O., Sýkora D. andHornung A. 2012, June. Cache-efficient graph cuts on structured grids. In Computer Vision and Pattern Recognition (CVPR), 2012 IEEE Conference on (pp. 3673-3680). IEEE.

[30] D. Ghiglia and M. Pritt. 1998. Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software. Hoboken, NJ, USA: Wiley.