



DESIGN AND IMPLEMENTATION OF FRACTIONAL ORDER CONTROLLER FOR HEAT EXCHANGER

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ABSTRACT

A shell and tube heat exchanger is a most common type of heat exchanger in oil refineries and other large chemical processes and suited for high pressure applications. In heat exchanger one fluid runs through the tubes, and another fluid flows over the tubes to transfer heat between the two fluids. A model for a heat exchanger is designed and a Fractional Order (FO) Proportional- Integral- Derivative (PID) controller has been proposed in this paper which works on the closed loop error and its fractional derivative and fractional integrator. FOPID is a PID controller whose derivative and integral orders are of fractional rather than integer. The extension of derivative and integral order from integer to fractional order provides more flexibility in design of the controller, thereby controlling wide range of dynamics of a system. Particle Swarm Optimization (PSO) is used to find the FOPID Controller parameters, Proportional (K_p), Integral (K_i), Derivative (K_d) gains, integral order (λ), and the derivative order (μ).

Keywords: heat exchanger, PID, FOPID, ZN method, particle swarm optimization.

1. INTRODUCTION

A heat exchanger is equipment for efficient transfer of heat from one medium to another. This media is separated by a solid wall to prevent mixing [1]. Shell and tube heat exchangers are probably the most common type of heat exchangers applicable for a wide range of operating temperatures and pressures. The outlet temperature of the shell and tube heat exchanger system has to be kept at a desired set point according to the process requirement. The most popular controllers are the PID controllers [2] which has parameters to be tuned to get a well specification for the system both in time domain and frequency domain.

The most important advantages of the $PI^\lambda D^\mu$ controller [3] is that it provides very good control on dynamical systems and it is affected much lesser for variations in control system parameters. PID controller generalization has been proposed by Podlubny as $PI^\lambda D^\mu$ controller which is known as fractional order PID controller, where λ is the non-integer order of integrator and μ is the non-integer order of the differentiator term. It has been demonstrated that the response of the $PI^\lambda D^\mu$ controller is good on comparing with classical PID controller. Many tuning techniques for obtaining the parameters of the controllers were introduced during last few decades [4]. Some tuning rules for robustness to plant

uncertainty for $PI^\lambda D^\mu$ controller are given in literature [2-13]. However in order to achieve better results, there are still needs for new methods to obtain the parameters of $PI^\lambda D^\mu$ controllers. In this paper, PSO based FOPID controller is designed and implemented on Heat Exchanger system.

2. FRACTIONAL ORDER ($PI^\lambda D^\mu$) CONTROLLER

The most common form of a fractional order PID controller is the $PI^\lambda D^\mu$ controller involving an integrator of order λ and a differentiator of order μ where λ and μ can be any real numbers. The transfer function of such a controller has the form

$$G_c(s) = \frac{U(s)}{E(s)} = k_p + k_i \frac{1}{s^\lambda} + k_d s^\mu, (\lambda, \mu > 0) \quad (1)$$

Where $G_c(s)$ is the transfer function of the controller, $E(s)$ is an error, and $U(s)$ is controller's output. The integrator term is $1/s^\lambda$, that is to say, on a semi-logarithmic plane, there is a line having slope -20dB/decade. The control signal $u(t)$ can then be expressed in the time domain as

$$u(t) = k_p e(t) + k_i D^{-\lambda} e(t) + k_d D^\mu e(t) \quad (2)$$

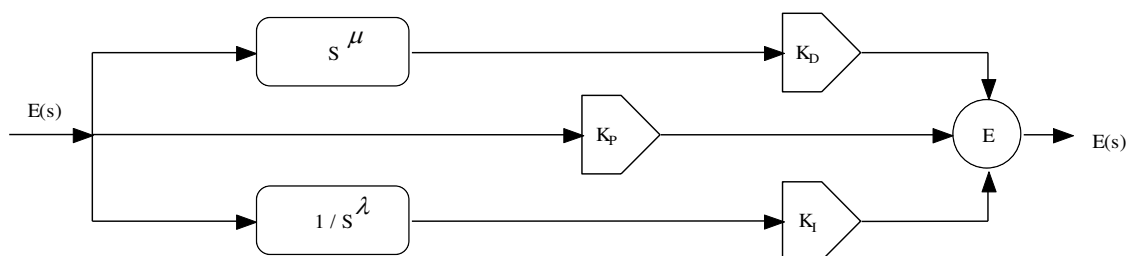


Figure-1. Block diagram of FOPID Controller.



Figure-1 shows the block-diagram configuration of FOPID. Clearly, selecting $\lambda = 1$ and $\mu = 1$, a classical PID controller can be recovered.

It can be expected that the $PI^\lambda D^\mu$ controller may enhance the systems control performance. One of the most important advantages of the $PI^\lambda D^\mu$ controller is that it provides very good control on dynamic system and it is affected much lesser for variations in control system parameter. [5, 6]

3. SYSTEM DESCRIPTION

Two different sections of heat exchangers are shown in Figure-2.

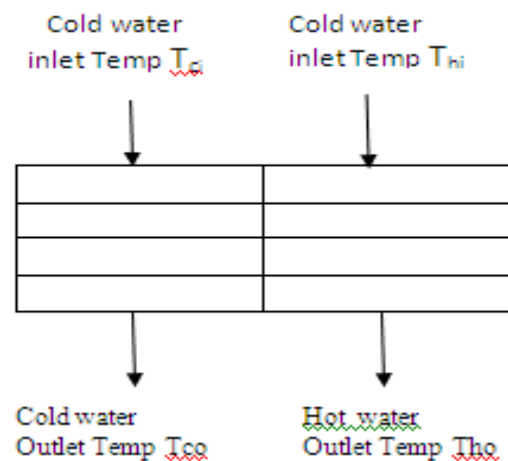


Figure-2. Shell and Tube sections of a Heat Exchanger.

- The following assumptions were made while designing the mathematical model of shell and tube heat exchanger.
- The control volumes are small and have a constant temperature.
- The heat exchanger is insulated and there is no heat loss from the heat exchanger to the surrounding.

Table-1. Input values for the heat exchanger mathematical model.

Inputs	Value	Units
Density of Water (ρ_s, ρ_t)	1000	Kg/m ³
Specific Heat Capacity of Water (C_s, C_t)	4230	J/Kg/ ° C
Shell Heat Transfer Area (A_s)	0.281	m ²
Tube Heat Transfer Area (A_t)	0.253	m ²
Shell Side Volume (V_s)	2.62×10^{-4}	m ³
Tube Side Volume (V_t)	1.43×10^{-4}	m ³
Heat Transfer Coefficient of Shell (h_s)	2162	W/m ² ° C
Heat Transfer Coefficient of Tube (h_t)	2162	W/m ² ° C
Mass Flow Rate of Cold Water (m_s)	0-0.11	Kg/S
Mass Flow Rate of Hot Water (m_t)	0.282	Kg/S
Cold Water Inlet Temp (T_{ci})	33	° C
Hot Water Inlet Temp (T_{hi})	55	° C
Number of Control Volume (N)	10	N/A

Rate of energy stored in the control volume is equal to the rate of gain of energy from the neighboring control volume. The energy balance equation on the shell control volume is given by

$$\frac{\rho_s c_s v_s}{N} * \frac{dT_{co}}{dt} = m_s c_s (T_{ci} - T_{co}) + \frac{h_s A_s}{N} (T_{ho} - T_{co}) \quad (3)$$

The energy balance equation on the shell control volume is given by

$$\frac{\rho_t c_t v_t}{N} * \frac{dT_{ho}}{dt} = m_t c_t (T_{hi} - T_{ho}) + \frac{h_t A_t}{N} (T_{co} - T_{ho}) \quad (4)$$

Input values for the heat exchanger mathematical model. As listed in Table-1 the hot water outlet temperature is considered as controlled variable where as the cold water flow rate to the shell side is treated as manipulated variable. The flow rate of the hot water is treated as disturbance [14].



4. MATHEMATICAL MODELLING AND DESIGN OF PID CONTROLLER

When a mathematical model of a system is available, the parameters of the controller can be explicitly determined. [7, 8] However, when a mathematical model is unavailable, the parameters must be determined experimentally. Controller tuning is the process of determining its parameters which produce the desired output. Controller tuning allows for optimization of a process and minimizes the error between the variable of the process and its set point [15]. Many types of controller tuning methods are available in literature such as the trial and error method, and process reaction curve methods. One of the most common classical controllers tuning method is the Ziegler-Nichols method. This method is often used when the mathematical model of the system is not available.

Process reaction curve method is to approximate a higher order process a first order with dead time. The open loop response of the process is obtained as 'S' shaped curve or sigmoid curve as shown in Figure-3.

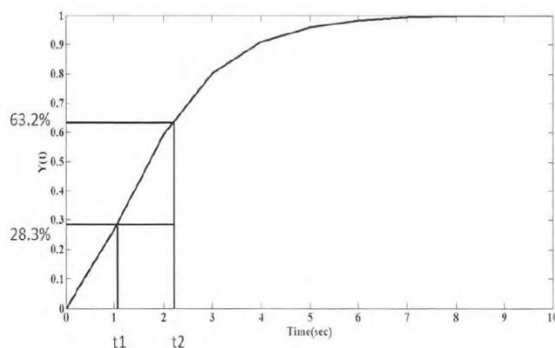


Figure-3. Process reaction curve.

Using Two point method, the given higher order system is approximated into first order system with delay (FOPTD). The time taken to reach 63.2% of the final steady state value is assumed as t_2 similarly the time taken to reach 28.3% of the final steady state value, is assumed as t_1 .

From this t_1 , t_2 the process parameter are obtained from

Time constant (τ) = $1.5 (t_2 - t_1)$

Time delay (T_d) = $t_2 - \tau$

Gain (K_p) = $\Delta \text{Output} / \Delta \text{Input}$

The PID controller can be determined by the process reaction curve method as given in Figure-3. The PID controller was first placed on the market in 1939 and has remained the most widely used controller in process control until today. An investigation performed in 1989 in Japan indicated that more than 90% of the controllers used in process industries are PID controllers and advanced versions of the PID controller. The purpose of controller is to make the process variable y follow the set-point value r . To achieve this purpose, the manipulated variable u is changed at the command of the controller. The error is

defined by $e = r - y$. The compensator $C(s)$ is the computational rule that determines the manipulated variable u based on its input data, which is the error e . The last thing to notice is that the process variable y is assumed to be measured by the detector, which is not shown explicitly here, with sufficient accuracy instantaneously that the input to the controller can be regarded as being exactly equal to y .

P element: Proportional to the error at the instant t , and it is called "present" error.

I element: proportional to the integral of the error up to the instant t , which can be interpreted as the accumulation of the "past" error.

D element: proportional to the derivative of the error at the instant t , which can be interpreted as the prediction of the "future" error.

PID Controller algorithm is given by,

$$U(t) = K [e(t) + \frac{1}{T_i} \int_0^t e(\tau) \tau + T_d \frac{de(t)}{dt}] \quad (5)$$

This can be represented as,

$$U(t) = K e(t) + K_i \int_0^t e(\tau) \tau + K_d \frac{de(t)}{dt} \quad (6)$$

Where, T_d is the derivative time constant. With the presence of derivative term $\frac{de}{dt}$, the controller anticipates what the error will be in the immediate future and applies a control action which is proportional to the current rate of change of error.

The transfer function of PID Controller is

$$G_c(s) = K_c [1 + \frac{1}{T_i s} + T_d s] \quad (7)$$

To obtain the K_p (proportional gain), a constant of integral term (K_i), the constant of derivative term K_d , the fractional order of differentiator μ and the fractional order of integrator λ . The method uses classical Zeigler – Nichols tuning rule to obtain K_p , K_i and K_d [16] which is shown in the Table-2.

Table-2. Controller parameters by Z – N Tuning method.

Controllers	K_c	T_i	T_d
P	$\frac{\tau}{K_p t_d}$	---	---
PI	$\frac{0.9 \tau}{K_p t_d}$	$3.3 t_d$	---
PID	$\frac{1.2 \tau}{K_p t_d}$	$2 t_d$	$0.5 t_d$

5. DESIGN OF FOPID CONTROLLER USING PSO



ALGORITHM

Particle Swarm Optimization proposed by Eberhart and James Kennedy [17]. This method is dependent on simulation of social behavior and universal optimization technique. Exchange of data in individuals called particles in swarm is the base for PSO. According to the neighboring particle, which attains the best, an individual particle modifies its trajectory to reach its own best position. Later the best position among the population is identified and particle moves towards the global best and in such a case entire swarm is considered as neighbourhood. [18]

N particles form a swarm and working in D-dimensional search space. The random velocity may be

$$V_{id} = V_{id} + c_1 * rand() * (P_{id} - X_{id}) + c_2 * rand() * (P_{gd} - X_{id}) \quad (8)$$

$$X_{id} = X_{id} + V_{id} \quad (9)$$

Here c_1 cognitive learning rate and c_2 is called social learning rate which are positive constants,

$$V_{id} = w * V_{id} + c_1 * rand() * (P_{id} - X_{id}) + c_2 * rand() * (P_{gd} - X_{id}) \quad (10)$$

$$X_{id} = X_{id} + V_{id} \quad (11)$$

Here w is inertia factor that balances the global wide range exploitation and the local nearby exploration abilities of the swarm.

The improvement of the velocity update equation is investigated and imposed the limit on the velocity of particles. If the particles are not imposed to positions then it may results in fly out of search space, which sometimes guides to produce invalid solutions.

In all iterations general methods are implemented to ensure particles in predefined search space. To check the position of particles some measurements were taken and to rectify invalid solutions. One rectification measure

assigned to each particle. Each particle will modifies its flying speed based on its own and companion's experience at every iteration.

The i th particle is denoted as $X_i = (x_{i1}, x_{i2}, \dots, x_{iD})$, whose best previous solution ($pbest$) is represented as $P_i = (p_{i1}, p_{i2}, \dots, p_{iD})$. Current velocity (position change rate) is described by $V_i = (v_{i1}, v_{i2}, \dots, v_{iD})$. Finally the best solution achieved so far by the whole swarm ($gbest$) is represented as $P_g = (P_{g1}, P_{g2}, \dots, P_{gD})$. [19]

At each time step, each particle moves toward $pbest$ and $gbest$ locations. The fitness function verifies the particles to determine whether the best fitting solution is arrived. The particles are manipulated according to the following equations [20].

and $rand()$ is random function in the range of $[0, 1]$. The particles are having the velocity range within $[Vmin, Vmax]$. A time increasing inertia factor is designed by Eberhart and shi to overcome time lag.

is to impose limit on positions, as it does on the velocity. If an element of the position is smaller than X_{min} , it is set equal to X_{min} , if greater than X_{max} .

Another is to reject the invalid particle, and then repeatedly evaluate the velocity update equation, formula and [10], until the position updating equation formula [11] obtaining a valid solution. Though those measures can restrict particles in defined search space, at the same time, they bring some excess computation. An improved PSO with momentum factor is introduced to overcome this disadvantage. The new technique can limit the particles in defined search space without checking the boundary at every iteration. Then improved particles are manipulated by the following equations:

$$V_{id} = w * V_{id} + c_1 * rand() * (P_{id} - X_{id}) + c_2 * rand() * (P_{gd} - X_{id}) \quad (12)$$

$$X_{id} = (1 - mc) * X_{id} + mc * V_{id} \quad (13)$$

Where mc is momentum factor ($0 < mc < 1$), and $v_{min} = X_{min}, v_{max} = X_{max}$.

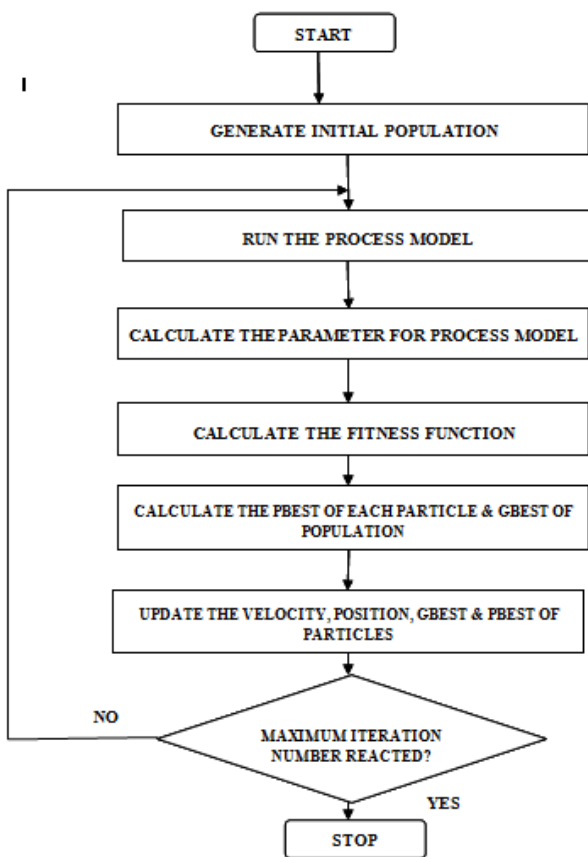


Figure-4. Flowchart of PSO algorithm.

6. SIMULATION RESULTS

A method for tuning of PID using conventional method and fractional order PID controller using PSO based FOPID has been designed. In this paper Z- N based PID controller is designed and implemented which is compared with fractional order PID Controller tuned by PSO algorithm.

The hot water outlet temperature is considered as the process variable. First capacity of designed controllers to follow the set point changes is tested.

Initially set point of 45 °C given at time $t = 0$. And set point change of + 2°C is given at time $t = 50$ and time $t = 100$. The response of the process is shown in Figure-5 for designed controllers.

The designed controllers also tested for load disturbances. Load disturbance of +10% in hot water flow rate is given at time $t = 50$ and load disturbance of -10 % is given at $t = 100$. The response of the process is shown in Figure-7 for the controllers. The simulation results show that the PSO based FOPID controller has the best performance because it has zero steady-state error at lowest time. It takes short time to reach the steady state. Furthermore, it has less (Integral Square Error) and IAE (Integral Absolute Error). Also it gives very less settling time and very less steady state error. Hence, it can be concluded that FOPID controller is the best controller for the Shell and Tube Heat Exchanger system.

Figures 6 and 8 shows the variation in manipulated variable that is cold water flow rate of the

system for servo and regulatory responses. From the response it is clear that cold water flow rate is within the specified range for servo and regulatory responses.

Table-3 shows the controller parameters for Heat Exchanger system for conventional PID and PSO based FOPID. From the Table-4, the proposed PSO algorithm based FOPID method gives much better performance with respect to conventional PID especially for Steady state error, settling time, ISE and IAE.

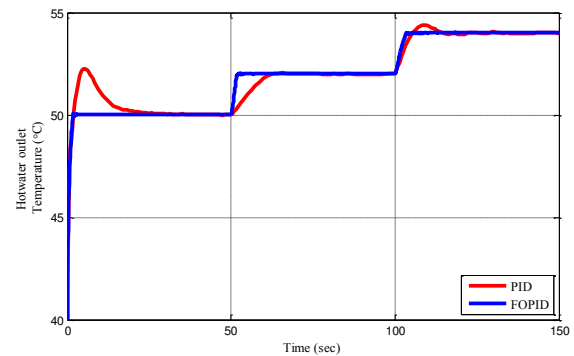


Figure-5. Servo response of PID and FOPID (Hot water temperature).

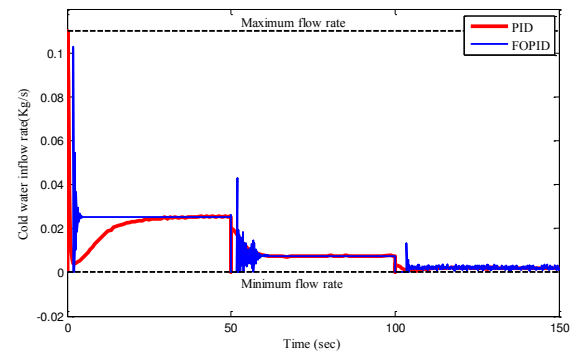


Figure-6. Servo response of PID and FOPID (Cold water flow).

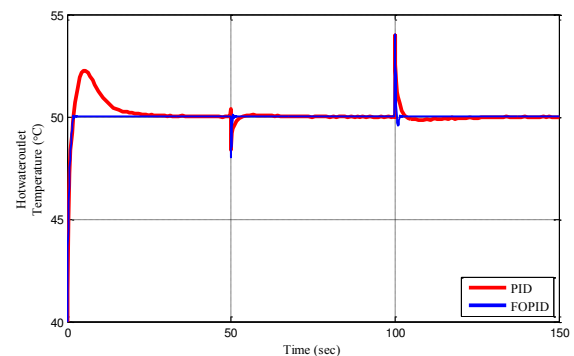


Figure-7. Regulatory response of PID and FOPID (Hot water temperature).

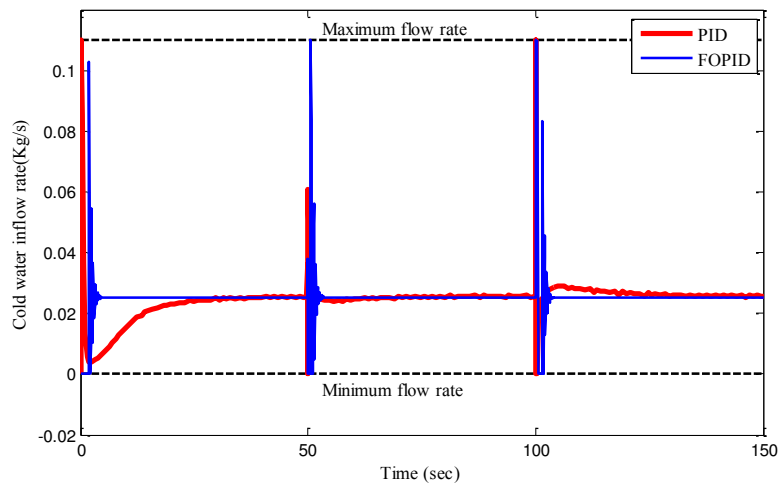


Figure-8. Regulatory response of PID and FOPID (Cold water flow).

Table-3. Controller parameters.

Controller type	Proportional gain (Kp)	Integral gain (Ki)	Deferential gain (Kd)	Lamda (λ)	Mue (μ)
PID	-0.0028	-0.0018	-0.0081	-	-
FOPID	-0.4989	-0.5019	-0.1209	0.1298	0.2025

Table-4. Comparison of PID and FOPID controller.

Controller type	Peak value	Peak time (s)	Over shoot percentage (Mp)	Settling time (s)	Servo		Regulatory	
					IAE	ISE	IAE	ISE
PID	50.08	1.929	0.16%	3.254	2.0553e+05	2.9379e+06	8.7365e+04	8.9695e+05
FOPID	52.2	5.682	4.5%	24.89	1.0537e+05	7.9450e+05	5.6034e+04	2.4233e+05

From the Table-4, the proposed PSO based FOPID method gives much better performance with respect to conventional PID controller.

7. CONCLUSIONS

In this paper conventional PID controller and PSO based fractional order controller have been designed for a Heat Exchanger Process. In this paper some criterion like small settling time, less steady-state error, overshoot, Less ISE and less IAE are considered to select the best controller. The best controllers never achieve all these criterions at same time, so it is necessary to decide which criterion one prefers the most. For a heat exchanger system, the most required criterion is that the system has no steady-state error or minimum error. Between these controllers, a comparison has been done to see which controller can meet the criterion. It is observed from the results the FOPID controller gives better results than conventional PID Controller.

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