



MAGNETOHYDRODYNAMIC (MHD) MIXED CONVECTIVE FLOW AND HEAT TRANSFER OVER AN INCLINED PLATE WITH RADIATION EFFECT

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ABSTRACT

A genuine variational principle developed by Gyarmati, in the field of thermodynamics of irreversible processes unifying the theoretical requirements of technical, environmental and biological sciences is employed to study the mixed convection heat transfer about a semi infinite inclined plate in the presence of magnetohydrodynamic (MHD) and thermal radiation effects. The velocity and temperature distributions inside the boundary layer has been considered as a simple polynomial functions and the variational principle is formulated. The Euler-Langrange equations are reduced to coupled polynomial equations in terms of boundary layer thicknesses. The effects of the magnetic parameter (M), the mixed convection parameter (Ri), the angle of inclination (α), the radiation-conduction parameter (Rd), the temperature ratio

 (θ_w) and the Prandtl number (Pr) on the velocity and temperature profiles as well as on the skin friction and heat transfer

parameters are presented and analyzed. For some specific values of the governing parameters, the results agree very well with those available in the literature. The present study establishes a high accuracy of results obtained by this variational technique.

Keywords: magneto hydrodynamics, mixed convection, thermal radiation, heat transfer.

INTRODUCTION

The MHD boundary layer theory has a significant place in the development of the magneto hydrodynamics. In recent years, the study of mixed convection flow and heat transfer for electrically conducting fluids past a surface has attracted much interest of researchers in view of its applications in many engineering problem such as geophysics, astrophysics, boundary layer control in the field of aerodynamics. Alam et al[1, 2] studied the combined effects of viscous dissipation and Joule heating on steady magneto hydrodynamic free convective heat and mass transfer flow of a viscous incompressible fluid past a semi-infinite inclined radiate isothermal permeable moving surface in the presence of thermophoresis. Takhar et al[18] investigated MHD natural convection from a non-isothermal inclined surface with multiple suction/injection slots embedded in a thermally stratified high-porosity medium.

The radiative effects have important applications in physics and engineering. The radiative heat transfer effects on different flows are very important in space technology and high temperature processes, and very little is known about the effects of radiation on the boundary layer of a radiative-MHD fluid past a body. Duwairi and Damseh [7, 8] studied the radiation-conduction interaction in free and mixed convection fluid flow for a vertical flat plate with the presence of a magnetic field effect. The MHD mixed convective heat transfer flow about an inclined plate has been studied by Aydin and Kaya[3]. Mukhopadhyay [14] has unsteady mixed convective flow and heat transfer past a porous stretching surface. Bhattacharyya et al [4] have investigated an MHD boundary layer slip flow and heat transfer over a flat plate. Hassain and Takhar [11] analyzed the effect of radiation using the Rosseland diffusion approximation on the mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Shanmugapriya [17] investigated an MHD mixed convection of a viscous dissipation fluid about a vertical flat plate with uniform suction/injection.

The object of this paper was to investigate the radiation effect on MHD mixed convection flow about an inclined plate. The viscous dissipation effects are negligible and the radiative heat flux in the x-direction is considered negligible in comparison with that in the y-direction. The governing equations describing the problem are transformed into a polynomial equations interms of velocity and thermal boundary layer thicknesses by using the Governing Principle of Dissipative Processes. The velocity, temperature profiles, skin friction and heat transfer are analyzed for various values of governing parameters.

FORMULATION OF THE PROBLEM

Consider a steady, laminar, two-dimensional and MHD mixed convection boundary layer flow of a viscous incompressible fluid along a semi-infinite inclined plate with an acute angle α . The fluid is assumed to be a gray, emitting and absorbing, but non-scattering medium. The *x* co-ordinate is measured from the leading edge of the plate; *y* co-ordinate is measured along the normal to the plate. A magnetic field B_0 is applied in the *y*-direction. The external flow with a uniform velocity U_{∞} takes place in the direction parallel to the inclined plate. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. Under these assumptions and using Boussinesq approximations, the boundary layer equations for this problem are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + gB(T - T_{\infty})\cos\alpha - \left(\frac{\sigma B_0^2}{\rho}\right)(u - U_{\infty}), (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{v}{\Pr}\right) \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial}{\partial y} (q_r).$$
(3)

where subscript indicates partial differentiation, u, v, T, T_x , U_x , B_0 , C_p and q_r represent the velocity component in x-direction, velocity component in y- direction, temperature of the fluid, the free stream temperature, the free stream velocity, the magnetic flux density, specific heat at constant pressure and radiative heat flux in the y direction respectively. The symbols v, g, B, α , ρ , σ mean, kinematic viscosity, the acceleration due to gravity, the coefficient of thermal expansion, the angle of inclination, the fluid density and electrical conductivity of the fluid respectively.

Making use of the Rosseland approximation for radiation for an optically thick layer (Brewster [5]), we have

$$q_r = \frac{-4a}{3k^*} \frac{\partial T^4}{\partial v} \tag{4}$$

where *a* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. If temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series for T^4 about T_{∞} after neglecting higher order terms, is given by

$$T^4 \cong 4T^3_{\infty}T - 3T^4_{\infty} \tag{5}$$

In view of equations (4) and (5), equations (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{\kappa}{\rho C_p}\right)\frac{\partial^2 T}{\partial y^2} - \left(\frac{16\,\sigma T_{\infty}^3}{3\,\rho C_p k^*}\right)\frac{\partial^2 T}{\partial y^2}.$$
 (6)

The appropriate boundary conditions for the velocity and temperature of this problem are

$$\begin{aligned} x &= 0, \quad y > 0, \quad T = T_{\infty}, \quad u = U_{\infty} \\ x > 0, \quad y = 0, \quad T = T_{0}, \quad u = 0, \quad v = 0 \\ y \to \infty, \quad T \to T_{\infty}, \quad u \to U_{\infty} \end{aligned}$$
 (7)

THE FORMULATION OF GOVERNING PRINCIPLE OF DISSIPATIVE PROCESSES (GPDP)

Gyarmati [9, 10] introduced a genuine variational principle called the "Governing Principle of Dissipative Processes" (GPDP) which is given in its universal form

$$\delta \int [\sigma - \Psi - \Phi] dV = 0. \tag{8}$$

The principle (8) is valid for linear, quasi-linear and certain types of non-linear transport processes at any instant of time under constraints that the balance equations

$$\dot{\operatorname{pai}}_{i} + \nabla. \overrightarrow{J}_{i} = \sigma_{i} \quad (i=1,2,3...,f)$$
 (9)

are satisfied. In Equation (8), σ is the entropy production Ψ and Φ are dissipation potentials and V is the total volume of the thermodynamic system. In the Equation (9), \vec{J}_i is the flux and σ_i is the source density of the ith extensive transport quantity a_i . σ can always be written in the bilinear form as

$$\sigma = \sum_{i=1}^{f} \vec{J}_i \cdot \vec{X}_i \ge 0 \tag{10}$$

where J_i and \vec{X}_i are fluxes and forces respectively. According to Onsager's [15, 16] linear theory the fluxes are linear functions, that is

$$\vec{J}_{i} = \sum_{k=1}^{J} L_{ik} \vec{X}_{k}, \quad (i = 1, 2, 3 \dots, f)$$
(11)

or alternatively

$$\vec{X}_{i} = \sum_{k=1}^{f} R_{ik} \vec{J}_{k}, \quad (i = 1, 2, 3, \dots, f)$$
 (12)

The constants L_{ik} and R_{ik} are conductivities and resistances respectively and they satisfy the reciprocal relations [15]

$$L_{ik} = L_{ki} \text{ and } R_{ik} = R_{ki} (i, k = 1, 2, 3, ..., f)$$
 (13)

The matrices of L_{ik} and R_{ik} are mutual reciprocals. That is

$$\sum_{m=1}^{J} L_{im} R_{mk} = \sum_{m=1}^{J} L_{mk} R_{im} = \delta_{ik} (i, k = 1, 2, 3, ..., f)$$
(14)

where δ_{ik} is the Kronecker delta. The local dissipation potentials Ψ and Φ are defined as

$$\Psi\left(\vec{X},\vec{X}\right) = (1/2)\sum_{i,k=1}^{f} L_{ik} \vec{X}_{i} \cdot \vec{X}_{k} \ge 0,$$
(15)

$$\Phi\left(\vec{J},\vec{J}\right) = (1/2)\sum_{i,k=1}^{f} R_{ik} \vec{J}_{i} \cdot \vec{J}_{k} \ge 0, \qquad (16)$$

Since in the case of transport processes X_i can be generated as gradients of certain "T" variable, it is written as

$$\vec{X}_i = \nabla \Gamma_i. \tag{17}$$

The principle (8) with the help of equations (10), (13), (15), (16) and (17), takes the form

$$\delta \int_{V} \left[\sum_{i=1}^{f} \vec{J}_{i} \cdot \nabla \Gamma_{i} - (1/2) \sum_{i,k=1}^{f} L_{ik} \nabla \Gamma_{i} \cdot \nabla \Gamma_{k} - (1/2) \sum_{i,k=1}^{f} R_{ik} \vec{J}_{i} \cdot \vec{J}_{k} \right] dV = 0.$$
(18)

The principle (8) is also in energy picture as

$$\int_{V} [T\sigma - \Psi^* - \Phi^*] dV = 0.$$
⁽¹⁹⁾

Here $T\sigma$ is the energy dissipation and the dissipation potentials Ψ^* and Φ^* are given by

$$\Psi^* = T \Psi \text{ and } \Phi^* = T \Phi.$$
(20)

It is found that GPDP in energy picture given by Eq.(19) is always advantageous for dealing with thermohydrodynamic systems. This variational principle has been already applied for various dissipative systems and was established as the most general and exact principle of macroscopic continuum physics. For the description of viscous flow systems Vincze [19] used the GPDP to derive the equation of thermohydrodynamics. Many other variational principles have already been showed as partial forms of Gyarmati's principle.

The balance equations of the system play a central role in the formulation of Gyarmati's variational principle and hence the governing equations (1), (2) and (6) are written in the balance forms as

$$\nabla \vec{V} = 0, \ (\vec{V} = iu + jv). \tag{21}$$

$$\rho(\vec{V}.\nabla)\vec{V} + \nabla.\vec{P} = g\beta\rho(T - T_{\infty}) - \sigma B_0^2[(\vec{i}.\vec{V}) - U_{\infty}], \qquad (22)$$

$$\rho c_{p}(\vec{V}.\nabla)T + \nabla.\vec{J}_{q} = 0.$$
⁽²³⁾

These equations represent the mass, momentum and energy balances respectively. In Equation (22) $\overline{\overline{P}}$ denotes the pressure tensor which can be decomposed as

$$= \stackrel{=}{p} \stackrel{=}{\delta} \stackrel{=}{P} \stackrel{v_s}{\delta} + \stackrel{e}{P}$$
 (24)

where p is the hydrostatic pressure, δ is the unit tensor, and \tilde{P}° is the symmetrical part of the viscous pressure tensor, whose trace is zero. In the energy picture, the energy dissipation for the present system is given by

$$T\sigma = -J_q(\partial \ln T/\partial y) - P_{12}(\partial u/\partial y)$$
(25)

the heat flux
$$J_q$$
 and P_{12} the only component of

momentum flux \overline{P} satisfy the constitutive relations connecting the independent fluxes and forces as

$$J_q = -L_\lambda(\partial \ln T/\partial y)$$
, and $P_{12} = -L_s(\partial u/\partial y)$. (26)

Here $L_{\lambda} = \lambda T$ and $L_s = \mu$ where λ and μ are the thermal conductivity and viscosity respectively. It is well known that *lnT* is the proper state variable instead of *T* when the principle assumes energy picture. With the help of Equation (26) the dissipation potentials in the energy picture are found as follows.

$$\Psi^* = (1/2)[L_{\lambda}(\partial \ln T/\partial y)^2 + L_S(\partial u/\partial y)^2], \qquad (27)$$

$$\Phi^* = (1/2)[R_{\lambda}J_q^2 + R_s P_{12}^2].$$
(28)

Where $L_{\lambda} = R_{\lambda}^{-1}$ and $L_{s} = R_{s}^{-1}$. Using the equations (25), (27) and (28) Gyarmati's variational principle in the energy picture (19) is formulated in the following form

$$\delta \int_{0}^{T} \int_{0}^{\infty} \left[-J_{q} (\partial \ln T / \partial y) - P_{12} (\partial u / \partial y) - (L_{\lambda} / 2) (\partial \ln T / \partial y)^{2} \right]$$

$$-(L_s/2)(\partial u/\partial y)^2 - (R_\lambda/2)J_q^2 - (R_s/2)P_{12}^2]dydx = 0, \quad (29)$$

in which 'l' is the representative length of the surface.

SOLUTION PROCEDURE

As a starting point of the variational treatment for the present problem the velocity distribution inside the boundary layer is approximated as a fourth degree polynomial function.

$$u/U_{\infty} = 2y/d_{1} - 2y^{3}/d_{1}^{3} + y^{4}/d_{1}^{4}, (y < d_{1})$$

$$u = U_{\infty}, (y \ge d_{1})$$

$$\frac{(T - T_{\infty})}{(T_{0} - T_{\infty})} = 1 - 2y/d_{2} + 2y^{3}/d_{2}^{3} - y^{4}/d_{2}^{4}, (y < d_{2})$$

$$T = T_{\infty}. (y \ge d_{2})$$
(30)

Where d_1 and d_2 are hydrodynamical and thermal boundary layer thicknesses respectively. The velocity and thermal profiles (30) satisfy the following compatibility conditions:

$$y = 0; \quad u = 0, \quad v = 0, \quad T = T_0,$$

$$y = d_1; \quad u = U_{\infty}, \quad v = 0, \quad u_y = 0, \quad u_{yy} = 0,$$

$$y = d_2; \quad T = T_{\infty}, \quad T_y = 0, \quad T_{yy} = 0.$$
(31)

The transverse velocity component v is obtained from the mass balance Equation (1) as

$$v = U_{\infty} \left[y^2 / d_1^2 - 3y^4 / 2d_1^4 + 4y^5 / 5d_1^5 \right] d_1' \qquad (32)$$

To formulate Gyarmati's variational principle the velocity and temperature functions (30) are substituted in the momentum and energy balance Equations (2) and (6), and on direct integration with respect to y with the help of smooth fit boundary conditions $(u_y = 0 \text{ and } T_y = 0)$ the fluxes P_{12} and J_q are obtained respectively as given below.

$$-P_{12}/L_{s} = (U_{\infty}^{2}d_{1}^{'}/v)[0.117460317 - 2y^{3}/3d_{1}^{3} + 7y^{5}/5d_{1}^{5} - 11y^{6}/15d_{1}^{6} -3y^{7}/7d_{1}^{7} + 2y^{8}/5d_{1}^{8} - 4y^{9}/45d_{1}^{9}] + (gB(T_{0} - T_{\infty})/v)\cos\alpha[-y + y^{2}/d_{2} - y^{4}/2d_{2}^{3} + y^{5}/5d_{2}^{4} + d_{1} - d_{1}^{2}/d_{2} + d_{1}^{4}/2d_{2}^{3} - d_{1}^{5}/5d_{2}^{4}] - (\sigma B_{0}^{2}U_{\infty}/v\rho)[y - y^{2}/d_{1} + y^{4}/2d_{1}^{3} - y^{5}/5d_{1}^{4} + 3d_{1}/10]$$
(33)

$$\begin{split} &-J_{q}/L_{\lambda} = \left(\frac{U_{\infty}T_{\infty}}{\nu}\right) \frac{\Pr(\theta_{w}-1)}{\left(1+\frac{4}{3Rd}\right)} d_{2}^{'} \left[4y^{3}/3d_{1}d_{2}^{2}-12y^{5}/5d_{1}d_{2}^{4}+4y^{6}/3d_{1}d_{2}^{5}\right] \\ &-4y^{5}/5d_{1}^{3}d_{2}^{2}+12y^{7}/7d_{1}^{3}d_{2}^{4}-y^{8}/d_{1}^{3}d_{2}^{5}+y^{6}/3d_{1}^{4}d_{2}^{2}-3y^{8}/4d_{1}^{4}d_{2}^{4} \\ &+4y^{9}/9d_{1}^{4}d_{2}^{5}-4d_{2}/15d_{1}+3d_{2}^{3}/35d_{1}^{3}-d_{2}^{4}/36d_{1}^{4} + \left(\frac{U_{\infty}T_{\infty}}{\nu}\right) \frac{\Pr(\theta_{w}-1)}{\left(1+\frac{4}{3Rd}\right)} d_{1}^{'} \end{split}$$

$$\begin{aligned} & \left[-2y^3/3d_1^2d_2+6y^5/5d_1^2d_2^3-2y^6/3d_1^2d_2^4+3y^5/5d_1^4d_2^4-9y^7/7d_1^4d_2^3\right.\\ & \left.+3y^8/4d_1^4d_2^4-4y^6/15d_1^5d_2+3y^8/5d_1^5d_2^3-16y^9/45d_1^5d_2^4+2d_2^2/15d_1^2\right.\\ & \left.-9d_2^4/140d_1^4+d_2^5/45d_1^5\right] \end{aligned} \tag{34}$$

The prime indicates the differentiation with respect to x.

With the help of equations (30), (32), (33) and (34) the GPDP given by (29) is formulated and the integration of the Lagrangian with respect to y is carried out. The variational principle, after simplification, is written in a simple form

$$\delta \int_{0}^{l} L[d_{1}, d_{2}, d_{1}', d_{2}'] dx = 0, \qquad (35)$$

where $L[d_1, d_2, d'_1, d'_2]$ is the Lagrangian density of the principle.

The boundary layer thicknesses d_1 and d_2 are the independent parameters to be calculated and the Euler-Lagrange equations corresponding to these variational principles are

$$(\partial L/\partial d_1) - (d/dx)(\partial L/\partial d_1) = 0, \tag{36}$$

and
$$(\partial L/\partial d_2) - (d/dx)(\partial L/\partial d_2) = 0.$$
 (37)

The equations (36) and (37) are second order ordinary differential equations in terms of d_1 and d_2 respectively. Instead of solving these equations directly by using numerical method, the following transformations are used in the variational principle to obtain analytical solution for the present problem,

$$d_1 = d_1^* \sqrt{vx/U_{\infty}} \text{ and } d_2 = d_2^* \sqrt{vx/U_{\infty}}.$$
 (38)

The Euler-Lagrange equations of the transformed principle assume the simple forms

$$(\partial L/\partial d_1^*) = 0, \tag{39}$$

$$(\partial L/\partial d_2^*) = 0, (\Pr \ge 1)$$
(40)

The coefficients of the equations (39) and (40) depend on the independent parameters Pr, Gr, Re, Ri, M, Rd and θ_w where

Pr = v / α , (Pr andtl number) $Gr = g\beta(T_0 - T_\infty)x^3 / v^2$, (Grashof number) Re = $U_\infty x / v$, (Re ynolds number) $Ri = Gr / Re^2$, (Richardson number) $M = (\sigma B_0^2 x) / (\rho U_\infty)$, (Magnetic parameter) $Rd = \kappa k^* / (4aT_\infty^3)$, (Radiation parameter) $\theta_w = T_0 / T_\infty$, (Surface temperature ratio)

The equations (39) and (40) are obtained as coupled algebraic equations in non-dimensional boundary layer thicknesses d_1^* and d_2^* and the coefficients of these equations depend on the independent parameters of the problem M, Ri, α , Rd, $\Pr and \theta_w$. These equations can be solved for the following range: $M = 1, 2, 3, Ri = 1, 2, 3, \alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, Rd = 1, 2, 3, Pr = 1$ and $\theta_w = 1.7$.

ANALYSIS OF RESULTS

After getting d_1^* and d_2^* the local skin friction values and local heat transfer values are calculated with the help of the following relations respectively.

$$\eta = y \sqrt{U_{\infty} / vx},\tag{41}$$

$$\tau = \sqrt{\nu x / U_{\infty}^{3}} (-P_{12} / L_{s})_{y=0}, \tag{42}$$

$$N_{ul} = \sqrt{\nu x / U_{\infty} (T_0 - T_{\infty})^2 (-J_q / L_{\lambda})_{y=0}}.$$
 (43)

The results have been compared by work done by Chamkha *et al* [6], Lin and Lin [13], Yih [20] and Kunzetsor and Nield [12]. A good agreement is seen between the results.

Figure-1 shows the effect of the mixed convection parameter Ri on the dimensionless velocity and temperature profiles. The velocity and temperature gradients increase with Ri and the momentum and thermal boundary layer decrease. The effect of the mixed convection parameter Ri on the local skin friction and the local heat transfer parameters are shown in Figure-2. Both the skin friction and the local heat transfer parameters increase with an increase in the mixed convection parameters Ri.



Figure-3 depicts the velocity and the temperature profiles for various angles of inclination $\alpha = (0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ})$. From this figures it is clearly observed that increasing the angle of inclination decreases the velocity inside the hydrodynamic boundary layer. The thermal boundary layer thickness increases by increasing the angle of inclination, with an accompanying decrease in the wall temperature gradient. In Figure-4, the effect of the angle of inclination α on the local skin friction and local heat transfer parameters are displayed. This figure shows that the local skin friction and local heat transfer parameters decrease with an increase in the angle of inclination α .



In Figure-5, velocity and temperature profiles are exhibited for the different values of the magnetic parameter M. The increasing of the magnetic parameter Mincreases the velocity and temperature profiles. Both the local skin friction and heat transfer parameters increase with the magnetic parameters M in Figure-6.



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 d_1

 d_2



CONCLUSIONS

The present study gives the numerical solution for steady mixed convection heat transfer about a semi plate presence infinite inclined in the of magnetohydrodynamic (MHD) and thermal radiation effects. Using simple transformation technique the governing partial differential equations are simplified into polynomial equations, the coefficients of which are functions of independent parameters $M, Ri, \alpha, Rd, Pr and \theta_{w}$.

From the present numerical investigation, the following conclusions can be drawn:

- An increase in the mixed convection parameter increases the local skin friction and local heat transfer parameters.
- An increase in the radiation parameter decreases the local skin friction and increases the local heat transfer parameters.
- An increase in the radiation parameter decreases the local skin friction and increases the local heat transfer parameters.
- An increase in the surface temperature parameter increases the local skin friction and decreases the local heat transfer parameters.
- An increase in the magnetic parameter increases the local skin friction and the local heat transfer parameters.
- An increase in the angle of inclination decreases the local skin friction and local heat transfer parameters.

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Nomenclature

- *x, y* -coordinates in horizontal and vertical directions, respectively
- *u*, *v* -velocity component in the x and y directions, respectively
- *T* temperature of fluid

- T_0 temperature of plate
- T_{∞} temperature of ambient fluid
 - hydrodynamical boundary layer thickness
 - thermal boundary layer thickness
- P_{12} momentum flux
- J_q thermal flux
- L_s, L_λ conductivities

 d_1^* , d_2^* - non dimensional boundary layer thicknesses

- *g* acceleration of gravity
- c_p specific heat the convective fluid
- *Re* Reynolds number
- *M* Magnetic parameter
- *Pr* Prandtl number
- *Gr* Grashof number
- *Rd* Radiation parameter
- *Ri* Mixed convection parameter

Greek symbols

- α angle of inclination
- η similarity variable
- ξ non-similarity variable
- θ_{w} temperature ratio
- δ symbol for variation
- κ coefficient of thermal conductivity
- *v* kinematic viscosity
- μ dynamic viscosity
- Ψ, Φ dissipation potentials in entropy picture
- Ψ^{*}, Φ^{*} local dissipation potentials in energy picture
- ρ density of the fluid
- σ entropy production

Subscripts

- ∞ free stream condition
- *0* temperature at the wall

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