



MODELING OF CONSTITUTIVE MODEL TO PREDICT THE DEFORMATION BEHAVIOUR OF COMMERCIAL ALUMINUM ALLOY AA7010 SUBJECTED TO HIGH VELOCITY IMPACTS

M. K. Mohd Nor

Crashworthiness and Collisions Research Group, Faculty of Mechanical Engineering and Manufacturing, Tun Hussein Onn University,
Beg Berkunci Parit Raja, Batu Pahat, Johor, Malaysia
E-Mail: khir@uthm.edu.my

ABSTRACT

A finite strain constitutive model for orthotropic metals was developed within a consistent thermodynamic framework of irreversible process in this paper to capture the deformation behaviour of commercial aluminum alloy. The important features of this formulation are the multiplicative decomposition of the deformation gradient and a new decomposition of deviatoric and spherical parts of Mandel stress tensor. The elastic free energy function and the yield function are defined within an invariant theory. The Hill's yield criterion was adopted and the thermally micromechanical-based model, Mechanical Threshold Model (MTS) was used as a referential curve. The model complexity was further extended by coupling the formulation with the shock Equation of State (EOS). The proposed formulation was implemented into the Lawrence Livermore National Laboratory-DYNA3D code and validated against the available experimental data of Taylor Cylinder Impact test of commercial aluminum alloy AA7010. The proposed formulation showed a good agreement with respect to the reference experimental data.

Keywords: finite strain, constitutive model, commercial aluminum alloy, elastoplasticity, equation of state.

INTRODUCTION

During the last two centuries, scientists and engineers have explored and studied the theory of plasticity field. The development of this theory has shown momentous progress more recently. The technological demands on this field come from manufacturing processes such as stamping, rolling, car crashworthiness and defence. The ability to appropriately capture the behaviour of deformation processes in these applications is becoming more and more important.

In practice, in the real world, most of the engineering materials such as aluminum alloy and composites are orthotropic. Sheet forms of aluminium alloy are examples of orthotropic materials. The constitutive models intended to represent plastic behaviour are of great importance in the current design and analysis of forming processes due to their broad engineering application (Cazacu and Barlat, 2003). The characterization of plastic deformation for such materials is still an open and exciting area of study.

Much research has been carried out, leading to results in various technologies involving analytical, experimental and computational methods. Even though there has been significant progress in computational methods, there are still many issues relating to mechanical characteristics which have to be answered. This is important as simulation accuracy has to be improved. The prime motivation in this work, therefore, is to propose an approach capable for modelling of deformation behaviour in commercial aluminum alloy subjected to high velocity impacts.

NEW CONSTITUTIVE MODEL FORMULATION

The construction of the new material model for orthotropic materials in the finite strain regime is formulated based on the multiplicative decomposition of the deformation gradient \mathbf{F} :

$$\mathbf{F} = \mathbf{F}_e \cdot \mathbf{F}_p \quad (1)$$

where \mathbf{F}_e and \mathbf{F}_p represent elastic and plastic deformation gradient respectively. Equally, an incompressibility constraint, $\det(\mathbf{F}_p) = 1$ is assumed to hold for plastic deformation. The new material model is developed and integrated in the isoclinic intermediate configuration $\bar{\Omega}_i$.

New Mandel Stress Tensor

Mandel stress tensor was combined with the new stress tensor decomposition (Vignjevic *et al.*, 2007). A rigorous derivation of the new stress tensor decomposition is not covered in this section however can be found in (Mohd Nor, 2008)]. This new stress tensor decomposition has been used to presents a new yield criterion for orthotropic sheet metals under plane-stress conditions (Mohd Nor *et al.* 2013). In this work, the predictions of the new effective stress expression were compared with the experimental data of 6000 series aluminium alloy sheet (A6XXX-T4) and Al-killed cold-rolled steel sheet SPCE where a good agreement was obtained.

The formulation was then further explored by (Mohd Nor *et al.* 2013) to test its ability to describe shockwave propagation. The proposed formulation was implemented into the LLNL-DYNA3D code and validated against the experimental Plate Impact test data. A good agreement between experimental and simulation



was obtained for two principal directions of material orthotropy.

The combination between the new stress tensor decomposition and the Mandel stress tensor gives

$$\hat{\Sigma} = \det(\mathbf{F}) \cdot \hat{\mathbf{R}}^T \cdot \left(\mathbf{S} + \frac{\sigma\psi}{\psi\psi} \cdot \psi \right) \cdot \hat{\mathbf{R}}^{-T}. \quad (2)$$

where ψ , $\hat{\mathbf{R}}$ and \mathbf{S} define the direction of the new volumetric axis in stress space, the orthogonal rotation tensor and the deviatoric stress tensor respectively. ψ is formulated as

$$\psi_{(ii)} = \frac{C_{i1} + C_{i2} + C_{i3}}{\sqrt{\frac{(C_{11} + C_{12} + C_{13})^2 + (C_{12} + C_{22} + C_{23})^2 + (C_{13} + C_{23} + C_{33})^2}{3}}} \quad (3)$$

where C_{ij} are members of the material stiffness matrix. Repeated indices in brackets in the above equation indicate no summation. It should be noted that ψ_{ij} becomes δ_{ij} when dealing with isotropic materials. Hence the new decomposition reduces to the conventional decomposition developed for isotropic materials. The representation of this decomposition ψ_{ij} and δ_{ij} in 3-dimensional stress spaces is shown in Figure-1 by green and red lines respectively.

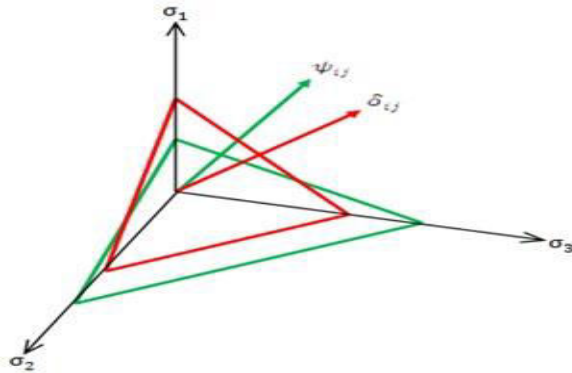


Figure-1. ψ and δ as a vectors in a principal stress space.

From this figure, it can be observed that this decomposition leads to a shift of the pressure vector away from the common alignment (equal angle with the principal stress directions). Equally, it can be observed that the direction of the volumetric axis ψ_{ij} is not making the same angle with the principal stress directions. Based on the definition of the new stress tensor decomposition, any considered orthotropic materials will uniquely define their own deviatoric plane within the stress space.

Orthotropy of Elastic Free Energy Function

The symmetric group of elastic orthotropy $\hat{\mathcal{P}}_e$ was defined in the isoclinic configuration using the Helmholtz free energy as a function of Elastic Green-Lagrange strain tensor $\hat{\mathbf{E}}_e$ and structural tensors $\hat{\mathbf{M}}_{ii} | i =$

1,2,3. This symmetric group $\hat{\mathcal{P}}_e$ can be further defined in terms of the integrity basis consisting of a set of invariants of the Green-Lagrange strain tensor and the structural tensors. The constant fourth-order tensor $\hat{\mathbf{M}}$ that consists of the structural tensors was then introduced by the second derivative of $\hat{\mathcal{P}}_e$. This tensor $\hat{\mathbf{M}}$ was finally set equal to a fourth-order elasticity or material stiffness tensor $\hat{\mathbf{C}}$ to derive new elastic orthotropic constants, $\lambda, \mu, \sigma_i | i=1, \dots, 7$.

Flow Rule

The isotropic plastic hardening which describes the expansion of the yield locus is assumed to be controlled by an equivalent plastic strain $\bar{\epsilon}_p$ and a referential flow stress-equivalent plastic strain curve of the thermally micromechanical-based model Mechanical Threshold Stress (MTS), (Follansbee and Kocks, 1988).

Orthotropic Yield Function

An appropriate yield function is extremely important from a numerical point of view by the introduction of yield locus where the initial shape and subsequent evolution during plastic deformation can be precisely captured by an 'effective' constitutive relation. Based on this motivation, an orthotropic yield function was constructed using the classical Hill's yield criterion (Hill, 1948). The classical Hill's yield criterion has been widely used in industrial simulations and provides reasonably good results as well as being numerically efficient (Duchêne *et al.*, 2008). The dependence on material anisotropy during plastic deformation is modeled by the introduction of the structural tensors $\hat{\mathbf{M}}_{ii} | i = 1, 2, 3$ defined with respect to the isoclinic configuration $\hat{\mathbf{\Omega}}_i$ as follows:

$$\hat{f} = \hat{f}(\hat{\Sigma}', \hat{\mathbf{M}}_{ii}, \alpha). \quad (4)$$

where $\hat{\Sigma}'$ and α refer to the deviatoric part of the new Mandel stress tensor and an isotropic hardening variable respectively. Accordingly, the plastic anisotropy of the new material model can be expressed as

$$\hat{f} = \sqrt{\hat{\Sigma}': \hat{\mathbb{H}}: \hat{\Sigma}'} - \hat{f}(\alpha) = 0. \quad (5)$$

where $\hat{\mathbb{H}}$ is a fourth-order tensor defined in the isoclinic configuration $\hat{\mathbf{\Omega}}_i$. This anisotropy tensor $\hat{\mathbb{H}}$ describes the distortion of the new material model yield surface by representing the plastic anisotropy state using the Hill's orthotropic parameters and the structural tensors. The identification of the fourth-order plastic anisotropy tensor $\hat{\mathbb{H}}$ follows the identical method used to define the material parameters of the orthotropic elasticity tensor \mathbb{C} , as discussed in the previous section. Therefore, the constant fourth-order tensor $\hat{\mathbf{M}}$ is set equal to $\hat{\mathbb{H}}$, to derive constants for plastic orthotropy $\lambda, \mu, \sigma_i | i=1, \dots, 7$.

The hardening is modeled by an isotropic hardening. Therefore, the yield surface is expected to



maintain its initial shape (change in size, not shape). This means the Hill's material parameters are kept constant during plastic deformation. As mentioned in the preceding section, the yield surface expansion is controlled by the physically and a thermally micromechanical-based model known as MTS model. This model has been proved appropriate in representing strain, strain rate and temperature dependence of orthotropic materials (Panov, 2006).

Coupling with Equation of State (EOS)

In modeling the propagation of strong shock waves and the non-linear elastoplasticity behavior of metals at very high pressures, it is necessary to use an EOS in addition to a strength model. The EOS is a closure equation which allows for the solution of the balance equations across the shockwave which usually defines the pressure as a function of internal energy, pressure and deviatoric stress resulting in a nonlinear problem. The Mie-Grüneisen EOS (Malvern, 1969) that available in LLNL-DYNA3D was used in this work. This analytic EOS is frequently used when modeling solid materials. It was coupled with the proposed formulation, and incorporated into the stress update algorithm which is performed in the local material coordinate system for each element.

NUMERICAL SIMULATION

The proposed constitutive model was implemented into the LLNL-DYNA3D code in the next stage of this work and named Material Type 93. All parameters defined for the new constitutive model were integrated in detail into this code. This process involved alteration of several subroutines in the code. The capability of the newly formulation to represent the deformation behaviour of commercial aluminum alloy AA7010 within a three-dimensional stress state was then investigated. The Finite Element (FE) model used to validate the proposed formulation was modelled to generate a 3D deformation field (3D mode) is shown in Figure-2.

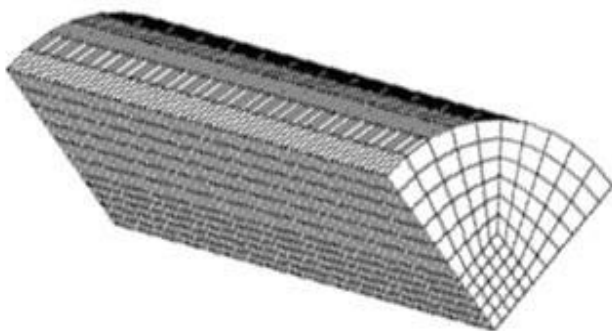


Figure-2. Finite element (FE) model for Taylor cylinder impact test simulation.

The material parameters of aluminum alloy AA7010 are given in (Mohd Nor, 2012). In order to

reduce the simulation time of this experimental test, the number of elements to model the cylinder was reduced by modelling a quarter of the cylinder with 9.30mm diameter and 46.50mm length. Furthermore, the cylinder was modelled by using a butterfly mesh method with 6375 solid elements, as depicted in Figure-2.

The anvil was modelled as a rigid wall, therefore the impact-interface friction between the solid cylinder rod and the anvil was negligible. 200ms^{-1} impact velocity was used in this analysis. To validate the proposed formulation, the final radius and length of the deformed cylinder profile obtained experimentally were compared with the results generated by new material model. The analysis was simulated until $120\mu\text{s}$ to ensure the final deformed shape of the cylinder profile was really obtained. The material axes definition was set as global orthotropic (AOPT 2).

RESULTS AND DISCUSSIONS

The deformation behaviour captured by the proposed constitutive model at $5\mu\text{s}$, $20\mu\text{s}$ and $50\mu\text{s}$ is shown in Figure-3. It can be clearly seen that a mushroom-shape was developed in the deformed cylinders near to the impact area, due to the plastic compressive wave that exceeded the yield strength of the specimen being generated in the cylinder. Greater mushrooming is therefore expected for the case the cylinder is impacted with a higher velocity.

In this Figure, it can be observed that the plastic wave has caused a severe deformation in a radial motion moving away from the cylinder's axial axis. Therefore, the highest effective strain is always concentrated in the middle of the cylinder footprints. A higher impact velocity again would produce a higher effective plastic strain around the localized deformation area compared to a lower impact velocity.

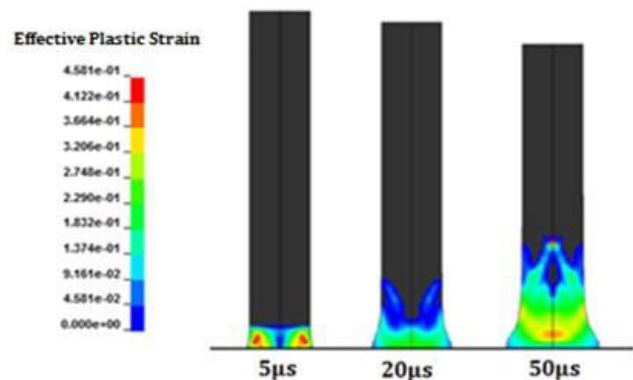


Figure-3. Deformed profiles (mushroom-shape) of the impact cylinder at various instants.

This radial deformation mode at the bottom of the cylinder then caused an axial shortening of the cylinders. The propagation of the elastic compression wave was observable in the simulation test. Consequently, this deformation behaviour ensured there was no plastic



deformation developed at the top of the cylinders as the effective plastic strain continue to raised from the bottom to the top of the cylinders and have seen ceased somewhere in the middle of the cylinders.

By using the simulation test data, a radial strain vs. distance from impact end curves of the new material model against the experimental result were plotted in Figure-4. In this figure, the major and minor side profiles of the deformed cylinder were compared in the case of 200ms^{-1} impact velocity. Generally, it can be seen that the increment in the footprint radius has an inverse relation with the shortening of the cylinder length. These criteria are directly influenced by the value of impact velocity.

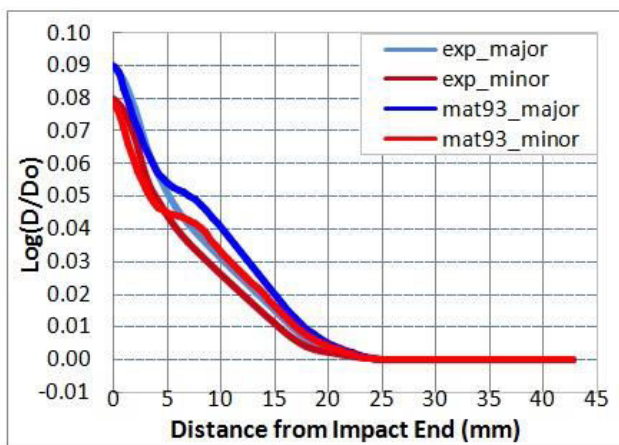


Figure-4. Major and minor side profile of taylor cylinder experimental test results against simulation results (plotted as radial strain vs. distance).

In addition, Figure-4 shows a good agreement between Material Type 93 and the experimental results in terms of both final footprint radius and length of the deformed cylinder in the case of 200ms^{-1} impact velocity. The only deviation between the simulation and the experimental results is related to the developed mushroom-shape. It can be observed that the new material model has captured a slightly different mushroom-shape with respect to the experimental results. However the deviation seems insignificant.

The deformation is developed at the bottom, near to the impact area and raised to the top of the cylinder. This is due to the propagation of the plastic compressive wave which is accompanied by plastic deformation. In the new material model, the plastic deformation or effective plastic strain is controlled by a flow stress formulation which refers to the MTS physically-based model. Since the MTS model has been widely accepted as one of the most appropriate physically-based models, the mushroom-shape disagreement between simulation and experiment of this analysis is potentially affected by the parameters used to characterize the proposed material model. The MTS parameters used in this analysis were previously derived with new procedures in Villi's work [9]. Further work is required to justify this hypothesis. However, it is reasonable to suggest that potentially only a

few parameters might not be correctly derived since the footprint radius and the final length of the deformed cylinder were satisfactorily captured. Instead of the MTS parameters, the deviation was actually expected in the first place since there was no damage and failure models were incorporated in the new material model formulation. The improvement when these models are adopted can be observed in Villi's work [9].

From the analysis performed in this section, it is noticed that the proposed formulation of the new constitutive model, Material Type 93 is capable of producing a good agreement with respect to the three-dimensional stress-state of commercial aluminum alloy AA7010 using Taylor Impact test data.

CONCLUSIONS

A new constitutive model appropriate to predict the deformation behaviour of commercial aluminum alloy AA7010 subjected to high velocity impact is proposed in this paper. The formulation takes into consideration the influence of strain rate and temperature by adopting the new generalized pressure formulated for orthotropic materials.

The proposed constitutive model used a new Mandel stress tensor that was combined with the new stress tensor decomposition. In addition, the Mechanical Threshold Model (MTS) was adopted as a referential curve to control the yield surface expansion that accounts for isotropic plastic hardening. Furthermore, the formulation was developed in the isoclinic configuration by using a multiplicative decomposition of the deformation gradient framework. The complexity was increased by combining the proposed formulation with equation of states (EOSs).

The numerical algorithm for this new constitutive model was structured in the second phase to implement the proposed formulation into the LLNL-DYNA3D. All parameters defined for the new constitutive model were integrated in detail into this code. This process involved alteration of several subroutines in the code. The newly implemented material model was finally validated against the Taylor Cylinder Impact test results. The results were satisfactory with respect to the experimental data.

The proposed formulation was the key novelty of this work – and in fact a new finding in this field. This achievement is a good indication for more appropriate orthotropic material models in future in order to help towards a better understanding of the complexity of material orthotropy undergoing finite deformation.

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