



# INFLUENCE OF THERMAL EXCHANGE COEFFICIENT ON THE HEAT RETENTION RATE OF A CONCRETE WALL CONTIGUOUS TO A THERMAL INSULATION TOW-PLASTER

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## ABSTRACT

In this article, we study the influence of the heat transfer coefficient on the heat retention rate per unit length of a wall made of concrete attached to the tow-plaster. The study is done in frequency dynamic regime established conditions. For different values of the exciting pulse, we evaluate the thermal inertia of the wall. The wall has a length of 0.1m including 0.05m of concrete and 0.05m thermal insulating plaster-tow. The thermal conductivity of concrete is about 10 times greater than that of the tow-plaster material. The results show that the thermal behavior of the wall depends partly outdoor climatic constraints. The duration of the outdoor climatic stresses related to the excitation pulse is an important factor on the thermal inertia of the wall. The thermal inertia of the wall is also dependent on the heat exchange coefficient on the surface of the material, its thermophysical properties and initial temperature of the material.

**Keywords:** concrete, tow-plaster, thermal inertia, heat retention, frequency dynamic regime.

## INTRODUCTION

The energy management in buildings and rational use of materials are current major challenges. Several studies have proposed solutions to significantly reduce energy losses [1]. Some studies propose an optimization of the thermal insulating materials used in determining the effective thermal insulation layer [2] or the thickness optimal isolation [3].

The measurement of thermophysical parameters (diffusivity and thermal conductivity) [4, 5] also serves to make a choice on thermal insulation. In this study we use the tow-plaster material [6] as thermal insulation.

The heat retention rate per unit length of material ( $k$ ) is used as a means of evaluation of the thermal inertia of wall.

External climatic stresses translate into the thermal exchange coefficient at the surface of the material and the exciting pulse or the periodicity constraints [7].

## THEORY

### Schema of the study device

The study device is a wall consisting of a concrete part subject to climatic stress at one side. The other side of the concrete is contiguous to a thermal insulation tow-plaster. The other side of the tow-plaster is in contact with the internal environment of a habitat. Temperature variations are modeled in frequency dynamic regime at both inner and outer faces of the wall.

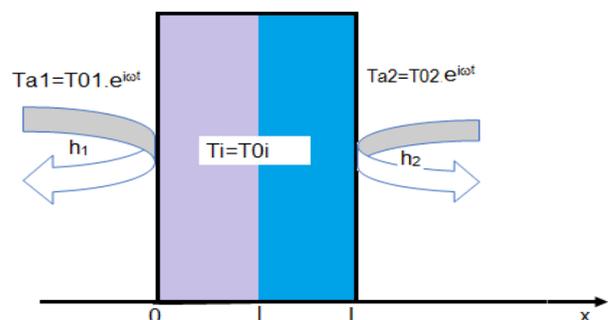
$T_{a1}$  and  $T_{a2}$  are respectively the temperatures outside and inside with respect to the wall in complex modeling.

$T_{01}$  and  $T_{02}$  are respectively the modulus of temperatures to external and internal to the wall.

$T_{0i}$  is the modulus of the initial temperature of the material.

$h_1$  and  $h_2$  are respectively the thermal exchange coefficient at the outer and inner wall surfaces.

$\omega$  is the exciting pulse of external climatic solicitations, it provides access over the duration of climatic solicitations.



**Figure-1.** Concrete wall and tow-plaster thermal insulation. Concrete:  $l = 0.05\text{m}$ ; tow-plaster:  $L = 0.05\text{m}$ ;  $T_{01} = 35^\circ\text{C}$ ;  $T_{02} = 20^\circ\text{C}$  et  $T_{0i} = 25^\circ\text{C}$ .

The initial temperature of the wall ( $25^\circ\text{C}$ ) is intermediate between the modulus of temperatures outside ( $35^\circ\text{C}$ ) and inside ( $20^\circ\text{C}$ ) of the wall.



### Mathematical modeling

Equations (1) and (2) respectively give the heat equations in the concrete slab and the thermal insulation tow-plaster. Equations (3) and (4) give the relationships between diffusivity  $\alpha_i$ , thermal conductivity  $\lambda_i$ , density  $\rho_i$  and specific heat  $c_i$  ( $i=1$  or  $2$  respectively for the concrete material or tow-plaster).

$$\begin{cases} \frac{\partial^2 T_1(x, h_1, h_2, \omega, t)}{\partial x^2} - \frac{1}{\alpha_1} \frac{\partial T_1(x, h_1, h_2, \omega, t)}{\partial t} = 0 & (1) \\ \frac{\partial^2 T_2(x, h_1, h_2, \omega, t)}{\partial x^2} - \frac{1}{\alpha_2} \frac{\partial T_2(x, h_1, h_2, \omega, t)}{\partial t} = 0 & (2) \\ \alpha_1 = \frac{\lambda_1}{\rho_1 c_1} & (3) \\ \alpha_2 = \frac{\lambda_2}{\rho_2 c_2} & (4) \end{cases}$$

Climate constraints allow to obtain the boundary conditions (5), (6) and (7) translating conservation flux to material interfaces; Equation (8) translates assumed perfect contact between the two materials (same temperature on both sides); finally, equation (9) indicates the initial condition.

$$\begin{cases} -\lambda_1 \frac{\partial T_1(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=0} = h_1 (T_{a1} - T(0, h_1, h_2, \omega, t)) & (5) \\ -\lambda_2 \frac{\partial T_2(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=L} = h_2 (T_2(L, h_1, h_2, \omega, t) - T_{a2}) & (6) \\ \lambda_1 \frac{\partial T_1(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=l} = \lambda_2 \frac{\partial T_2(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=l} & (7) \\ T_1(x, h_1, h_2, \omega, t) = T_2(x, h_1, h_2, \omega, t) & (8) \\ T(x, h_1, h_2, \omega, t = 0) = T_i = T_{0i} & (9) \end{cases}$$

The changes of temperature variables is performed (equations (10)) by introducing the initial temperature:

$$\begin{cases} \bar{T}_1 = T_1 - T_{0i} \\ \bar{T}_2 = T_2 - T_{0i} \end{cases} \quad (10)$$

Equations (1) and (2) respectively lead to the equations (11) and (12).

$$\begin{cases} \frac{\partial^2 \bar{T}_1(x, h_1, h_2, \omega, t)}{\partial x^2} - \frac{1}{\alpha_1} \frac{\partial \bar{T}_1(x, h_1, h_2, \omega, t)}{\partial t} = 0 & (11) \\ \frac{\partial^2 \bar{T}_2(x, h_1, h_2, \omega, t)}{\partial x^2} - \frac{1}{\alpha_2} \frac{\partial \bar{T}_2(x, h_1, h_2, \omega, t)}{\partial t} = 0 & (12) \end{cases}$$

Equation solutions (3) and (4) give, respectively, the expressions  $T_1(x, h_1, h_2, \omega, t)$  and  $T_2(x, h_1, h_2, \omega, t)$  of the temperatures inside concrete and tow-plaster in the following forms:

$$T_1(x, h_1, h_2, \omega, t) = (A_1 \sinh(\beta_1 x) + A_2 \cosh(\beta_1 x)) \cdot e^{i\omega t} + T_{0i} \quad (13)$$

$$T_2(x, h_1, h_2, \omega, t) = (A_3 \sinh(\beta_2 x) + A_4 \cosh(\beta_2 x)) \cdot e^{i\omega t} + T_{0i} \quad (14)$$

$$\text{with } \beta_1 = \sqrt{\frac{\omega}{2\alpha_1}} (i + 1) \text{ and } \beta_2 = \sqrt{\frac{\omega}{2\alpha_2}} (i + 1)$$

taking into account the changes variables of temperature, the initial and boundary conditions below allow to obtain coefficients  $A_1, A_2, A_3$  et  $A_4$  [8].

$$-\lambda_1 \frac{\partial \bar{T}_1(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=0} = h_1 (T_{a1} - \bar{T}_1(0, h_1, h_2, \omega, t) - T_{0i}) \quad (15)$$

$$-\lambda_2 \frac{\partial \bar{T}_2(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=L} = h_2 (\bar{T}_2(L, h_1, h_2, \omega, t) + T_{0i} - T_{a2}) \quad (16)$$

$$\lambda_1 \frac{\partial \bar{T}_1(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=l} = \lambda_2 \frac{\partial \bar{T}_2(x, h_1, h_2, \omega, t)}{\partial x} \Big|_{x=l} \quad (17)$$

$$\bar{T}_1(x, h_1, h_2, \omega, t) + T_{0i} = \bar{T}_2(x, h_1, h_2, \omega, t) + T_{0i} \quad (18)$$

$$\bar{T}(x, h_1, h_2, \omega, 0) = 0 \quad (19)$$

The heat flux densities [8], through the various materials are obtained from relations (20) and (21):

$$\begin{cases} \varphi_1(x, h_1, h_2, \omega, t) = -\lambda_1 \frac{\partial T_1(x, h_1, h_2, \omega, t)}{\partial x} & (20) \\ \varphi_2(x, h_1, h_2, \omega, t) = -\lambda_2 \frac{\partial T_2(x, h_1, h_2, \omega, t)}{\partial x} & (21) \end{cases}$$

These results were used to plot the heat transfer curves and evaluate the heat retention rate per unit length of material crossed.

### Heat retention rate per unit length

Rate retention ( $\tau$ ) of heat per unit length of material is given by equation (22) for the concrete and by equation (23) to the tow-plaster [2].

$$\begin{aligned} \tau_1(x, h_1, h_2, \omega, t) &= \frac{T(x_0, h_1, h_2, \omega, t) - T(l_1, h_1, h_2, \omega, t)}{T(x_0, h_1, h_2, \omega, t) * l_1} \quad (22) \end{aligned}$$

$$\begin{aligned} \tau_2(x, h_1, h_2, \omega, t) &= \frac{\varphi(x_0, h_1, h_2, \omega, t) - \varphi(l_2, h_1, h_2, \omega, t)}{\varphi(x_0, h_1, h_2, \omega, t) * l_2} \quad (23) \end{aligned}$$

For a perfect conductor, there is no heat emmagasement phenomenon; the retention rate is virtually zero. The heat flux absorbed on the surface by convection ( $h$ ) is integrally transmitted by conduction ( $\lambda$ ) to the other end of the material. Relaxation phenomena are negligible [9].

As against any material at an initial temperature  $T_{0i}$ , his behavior can not be perfect. We evaluate the heat retention rate with respect to temperature for a length of



material corresponding to external effects practically zero that is to say if the material temperature is in the vicinity of the initial temperature  $T_{0i}$ .

Compared to the flux density, we must remember that it tends to vanish when the behavior in thermal insulation is considerable. The variation of the flux density over a depth or it is not null is used to evaluate the heat retention rate.

**RESULTS**

The various curves are obtained by considering for concrete  $\lambda_1 = 1.3W.m^{-1}.K^{-1}$  and  $\alpha_1 = 5.02.10^{-7}m^2.s^{-1}$  and for tow-plaster:  $\lambda_2 = 0.15W.m^{-1}.K^{-1}$  and  $\alpha_2 = 2.07.10^{-7}m^2.s^{-1}$  [10]

The three series of figures of temperature and heat flux density are given respectively for  $\omega = 10^{-4}rad/s$  (Figures 1 and 2),  $\omega = 10^{-3}rad/s$  (Figures 3 and 4) et  $\omega = 10^{-2}rad/s$  (Figures 5 and 6). The 0.1m wall consists of 0.05 m of concrete (first half depth) and 0.05m thermal insulation tow-plaster.

Analysis of three series of figures shows that the heat transfer is favored for low excitation pulses to the surface of the material which is to say, for a considerable period external climatic solicitations. For large excitation pulses, the relaxation phenomena of the material produces a loss of heat to the material surface. The heat retention rate is low in concrete and substantial in the tow-plaster. This is due in part to the fact that the thermal conductivity and diffusivity of the concrete are higher than those of the thermal insulating tow-plaster.

Figures 2 and 3 shows an important heat transmission in the concrete (small decrease in temperature) and a negative gradient and high of heat flux which also reflects heat retention by the concrete. In the tow-plaster material the temperature drop is greater. The gradient of the flux density is low and tends towards a null. These phenomena correspond to a heat to build up reflecting good quality of the thermal insulating.

Table-1 shows that the retention rate is about five times greater in the tow-plaster material compared to concrete.

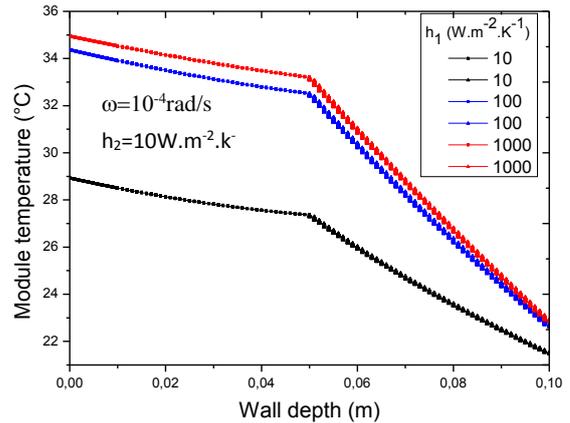


Figure 2: Evolution of temperature with depth

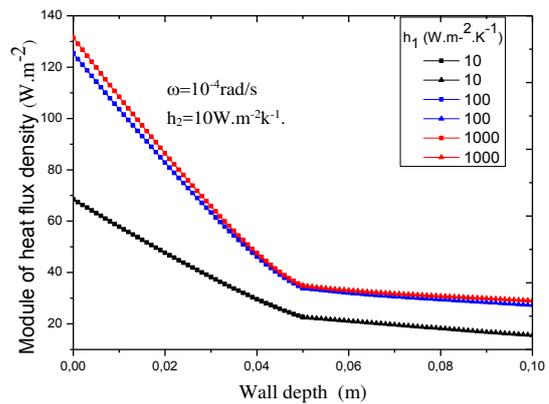


Figure 3: Evolution of flux heat density with depth

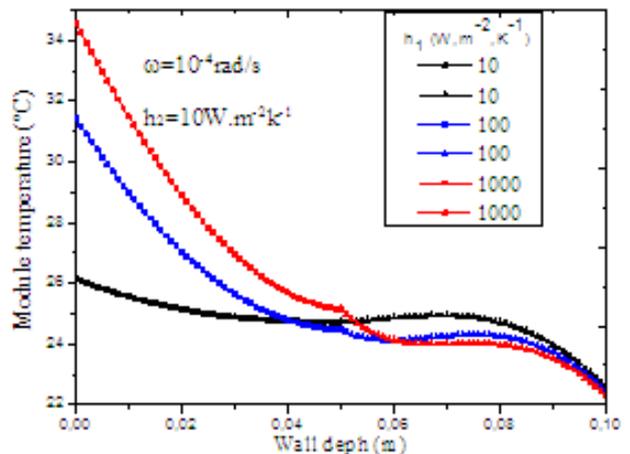


Figure-4. Evolution of temperature with depth.



**Table-1.** Values comparative of the heat retention rate per unit length for  $\omega=10^{-4}$  rad/s and  $h_2=10W.m^{-2}.k^{-1}$ .

	concrete			Tow-plaster		
<b>Depth of estimation</b> ( $10^{-3}$ m)	<b>50</b>	<b>50</b>	<b>50</b>	<b>50</b>	<b>50</b>	<b>50</b>
$h_1(w.m^{-2}.k^{-1})$	10	100	1000	10	100	1000
$\tau_1(x, h_1, h_2, \omega, t)$ ( $cm^{-1}$ )	1.1	1.1	1.0	4.3	6.1	6.3
$\tau_2(x, h_1, h_2, \omega, t)$ ( $cm^{-1}$ )	13.4	14.6	14.7	6.2	3.9	3.4

Figures 4 and 5 show, for a tape medium pulsation, that we have inside the wall a greater heat retention. The essential of the heat from the external face is retained by the concrete ( $T(x, t) \approx T_i$ ). The heat flux density tends towards a substantially null value to the interface (concrete - thermal insulating). It thus appears a cutoff frequency [11] around  $10^{-3}$  rad  $s^{-1}$  from which the heat retention rate becomes considerable for concrete.

The temperature curves showing the tow-plaster insulating material tends to give up heat to the internal environment. The influence of the heat transfer coefficient (h) is relatively small on the thermal inertia of the

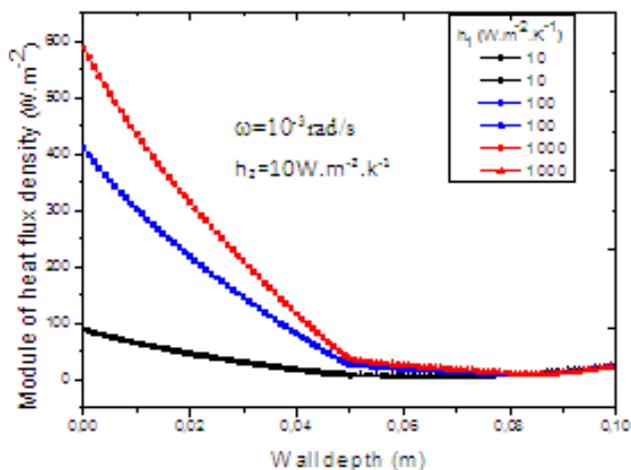
insulator. This phenomenon is manifested in Figure 5 by a positive gradient of the heat flux density in the vicinity of the inner face.

The gradient of the flux density is high in absolute value and negative in the concrete corresponding to a good heat capacity of the concrete to the excitation pulse.

Table-2 show that most of the heat is retained by the concrete at this exciting pulse. The thermal insulating, tow-plaster, acting as a thermal regulator, and with good thermal inertia.

**Table-2.** Values comparative of the heat retention rate per unit length for  $\omega=10^{-3}$  rad/s and  $h_2=10W.m^{-2}.k^{-1}$ .

	Concrete			Tow-plaster		
<b>Depth of estimation</b> ( $10^{-3}$ m)	<b>25</b>	<b>37</b>	<b>50</b>	<b>20</b>	<b>17</b>	<b>14</b>
$h_1(w.m^{-2}.k^{-1})$	10	100	1000	10	100	1000
$\tau_1(x, h_1, h_2, \omega, t)$ ( $m^{-1}$ )	1.8	7.0	7.5	4.4	4.2	4.4
$\tau_2(x, h_1, h_2, \omega, t)$ ( $m^{-1}$ )	11	83	293	100	107	101



**Figure-5.** Evolution flux density heat in depth.

Figures 6 and 7 corresponding to a relatively high excitation pulse that is to say, for relatively short periods of climatic solicitations. The curves of temperature and density heat flux show that the concrete retains a considerable amount of heat. For a relatively high coefficient of heat transfer the temperature of the concrete has a well to a depth of about 2cm; lowering the temperature below the initial temperature of the material. The concrete thus has a high thermal inertia to the excitation pulse.

The heat flux density is substantially null in the tow-plaster material, the essential of the heat being stored by the concrete. The temperature drop in the vicinity of the inner surface and the gradient of the heat flux density is positive. This reflects the thermal inertia of tow-plaster thermal insulating that moderately warm the internal environment.

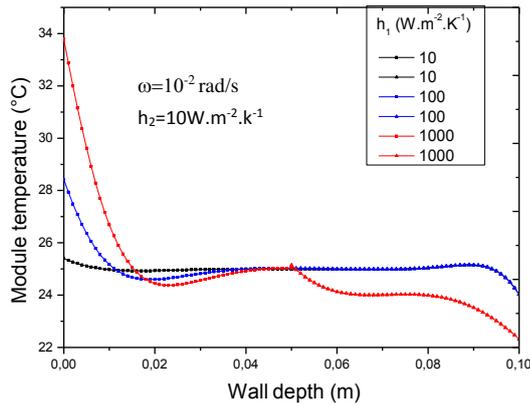


Figure 6: Evolution of temperature with depth

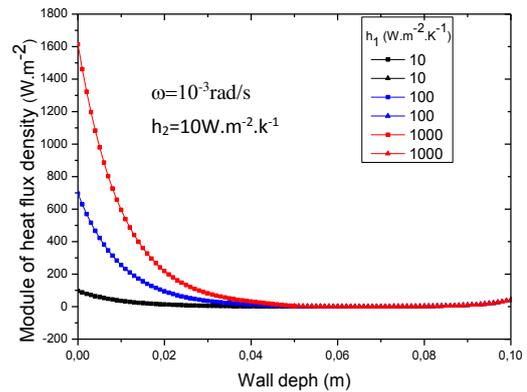


Figure 7: Evolution of flux heat density with depth

**Table-3.** Values comparative of the heat retention rate per unit length for  $\omega=10^{-2}$ rad/s and  $h_2=10W.m^{-2}.k^{-1}$ .

Depth of estimation (10 <sup>-3</sup> m)	Concrete			Tow-plaster		
	9	16	23	7	7	7
$h_1(w.m^{-2}.k^{-1})$	10	100	1000	10	100	1000
$\tau_1(x, h_1, h_2, \omega, t)$ (cm <sup>-1</sup> )	1.8	9.6	16.8	6.0	5.6	5.6
$\tau_2(x, h_1, h_2, \omega, t)$ (cm <sup>-1</sup> )	161	246	389	281	281	281

**CONCLUSIONS**

The study shows that the behavior or the thermal response of the material depends strongly on the external excitation pulse or period of climate external solicitations. There is a cut-off pulse from which the heat retention rate considerable.

For relatively high excitation pulses, the concrete has a good heat storage capacity and has good thermal inertia.

The tow-plaster, glued to concrete, is of great interest in the thermal comfort and can help regulate indoor climate habitat considering its thermal inertia.

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