



## PERFORMANCE OF COMPRESSIVE SENSING ALGORITHMS OVER TIME VARYING FREQUENCY SELECTIVE CHANNEL

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### ABSTRACT

Mobility environment leads to time varying frequency selective channel. Orthogonal Frequency Division Multiplexing (OFDM) be combined with Multiple Input Multiple Output (MIMO) system to increases the system capacity on time varying channel. Time varying frequency selective MIMO channel estimation demands huge number of training signals since the system has huge number of channel coefficients. In practical, most of the channels are composed of a few dominant taps and large part of taps is zero or approximately zero. They are often called sparse multi-path channels. By exploiting the coherent sparsity of the multipath fading channels, Compressive Sensing (CS) based channel estimation method provides better estimation of sparse channel than the conventional estimation methods which are suitable for rich channels and also greatly decrease the pilot overhead burden. This paper evaluates the performance of CS based channel estimation methods for MIMO-OFDM systems over time varying frequency selective channel.

**Keywords:** sparse estimation, time varying frequency selective channel, greedy compressive sensing algorithm, MIMO-OFDM system.

### 1. INTRODUCTION

Today's wireless communication systems require transmission of information over mobility channel with at higher bit rates. The high degree of variation among these channels creates difficulty in estimation of the channel statistics. Channel gain of each path is needed in coherent type signal detection. Time-varying nature of the channel fading and the correlation between the channels due to Doppler frequencies further complicates the system [1] and [2]. Single carrier modulation scheme may not be useful for high rate wireless transmission because it requires high complexity equalizers to handle the Inter Symbol Interference (ISI) in multi path fading channel. Since, the multicarrier scheme is used for a high rate wireless transmission, which does not involve the complexity of channel equalization. Orthogonal Frequency Division Multiplexing (OFDM) is one of such promising multi carrier scheme. OFDM may be combined with Multiple Input Multiple Output (MIMO) system to increase the capacity of time varying channel. In this connection, quality of channel estimation has major impact on the system performance.

Two methods are available to estimate channel state information [3], one is training symbols based channel estimation in that known training signals are transmitted along the data signals and the other is blind estimation, relies only on the statistics of received data symbols to estimate Channel State Information (CSI). Training based methods remain attractive in practice because they decouple symbol detection from channel estimation which reduces complexity. The training based method depends on two steps, sensing and reconstruction [4]. Sensing involves design of training signals used by the transmitter to probe the channel and reconstruction involve the processing the corresponding channel output at the

receiver to recover the CSI. The accuracy of training symbols based method depends on design of training signals and application of effective reconstruction method.

For time invariant channels, a training sequence is usually sent at the beginning of each transmission burst. When the channel is time variant, the preamble based training method may not work well. This motivates periodic insertion of training symbols during the transmission, which is known as pilot symbol aided modulation. It is well suited for all type of channels like time selective, frequency selective and doubly selective channel.

The recent researches shows that physical wireless channels encountered in practice tend to exhibit sparse structures when operating at large bandwidth and symbol duration. The channel impulse response usually have large number of channel taps but very few of them are nonzero. These channels are called sparse channel. These channels can be estimated from the recent advance in sampling theory called Compressive Sensing (CS) which provides a potential solution to reduce the number of pilot subcarriers.

This paper analyzes the performance of sparse channel estimation algorithm, Orthogonal Matching Pursuit (OMP), Stagewise Orthogonal Matching Pursuit (StOMP) and Regularized Orthogonal Matching Pursuit (ROMP) with MIMO OFDM under time varying sparse channel.

The rest of this paper is organized as follows. Section II states the system model and describes the discrete time impulse response of channel. In Section III, CS based channel estimation strategies are described. Section IV presents simulation results of MIMO OFDM with Least Squares (LS), OMP, SOMP and ROMP algorithms. And finally section V concludes the paper.



## 2. SYSTEM MODEL

CS methods concerns the sparse reconstruction problem of estimating the unknown sparse channel vector  $h \in C^n$  from the received vector  $y \in C^m$  can be modeled as

$$y = \phi h + z$$

where CS literatures refer  $\phi \in C^{n \times m}$  as dictionary or sensing matrix,  $z$  is the noise vector and  $h$  is the  $S$  sparse,  $S$  is the number of nonzero channel taps which is much less than total channel taps  $L$ .

CS recovery algorithms are guaranteed for the perfect recovery of the sparse channel vector  $h$  from the above model via Restricted Isometry Property (RIP) [5]-[7] given by

A sensing matrix  $\phi$  satisfies the RIP of order  $k$  if there exists a constant  $\delta$  such that

$$(1 - \delta) \|h\|_2^2 \leq \|\phi h\|_2^2 \leq (1 + \delta) \|h\|_2^2$$

for any  $k$  sparse vector. There is no algorithm to check the RIP since it involves combinatorial computation complexity. Other than RIP the widely used condition is mutual incoherence given by Donoho *et al.* [7]

The mutual incoherence  $\mu$  of a sensing matrix  $\phi$ , is the largest absolute inner product between any two columns  $\phi_i, \phi_j$  of  $\phi$

$$\mu(\phi) = \max_{1 \leq i \leq j \leq n} \frac{|\langle \phi_i, \phi_j \rangle|}{\|\phi_i\|_2 \|\phi_j\|_2}$$

Consider a MIMO-OFDM system with  $N_t$  transmit and  $N_r$  receive antennas. The MIMO channel can be characterized by an array of  $L$  tap finite impulse response filters given by a number of  $N_r \times N_t$  matrices  $H(n)$ , ( $n = 1, \dots, L$ ). In the case of uniform sampling, a wireless channel can often be modeled as a sparse channel that is only a few elements are nonzero in  $L$  taps.

The discrete time channel impulse response between the  $i^{\text{th}}$  transmitter and the  $j^{\text{th}}$  receiver is given [9] by

$$h_{ji}(m) = \sum_{p=1}^S \alpha_p^{ji} \delta((m - \tau_p)T_s)$$

Where  $\alpha_p^{ji}(t) \in C$  and  $\tau_p(t) \in R$  are complex-valued magnitude and real-valued delay spread for path  $p$ , respectively.  $T_s$  is the sampling interval of the system. In high data rate communication systems,  $T_s$  is very small compared to the maximum delay spread, results in a channel with relatively few nonzero taps. From the  $L$  total channel taps,  $S$  of them are nonzero then it is  $S$ -sparse channel and also  $S$  is very much less than  $L$ .

In the sparsity-based channel estimation methods, finds the sparse CSI in each OFDM frame from limited number of noisy measurements of the channel frequency response obtained at pilot locations. The estimated CSI is then, translated into the frequency domain by means of FFT which results in an estimation of the CSI that can be used for data equalization process. It can be represented with the following system equation.

$$y = \phi \cdot h + n$$

Where  $\phi = X_{pilot} \cdot D_{N_p \times L}$ ,  $X_{pilot} = \text{diag}\{X_1, X_2, \dots, X_{N_p}\}$  represents diagonal matrix of  $N_p$  pilots,  $D_{N_p \times L}$  represents submatrix with  $N_p$  rows and  $L$  columns selected from standard  $N \times N$  Discrete Fourier matrix,  $h = [h_1, h_2, \dots, h_L]^T$  represents channel impulse response,  $n = [n_1, n_2, \dots, n_{N_p}]^T$  represent noise vector with each element to be an AWGN variable and  $y = [y_1, y_2, \dots, y_{N_p}]^T$  received pilots.

## 3. CHANNEL ESTIMATION

In a mobility environment, pilot symbols play a significant role in tracking the channel variation over time, if they are appropriately placed over time and frequency. The pilot distribution over the transmission frame in OFDM-based systems has many types, called block, comb and Lattice. All the subcarriers in an OFDM symbol are used as pilots in block pilot allocation. The rest of the symbols in the OFDM frame are used for data. For slowly or time invariant channels, this type of pilot allocation is suitable. Some subcarriers of an OFDM symbol are used as pilots over the entire frame then it is called comb type which is suitable for fast fading channel. In Lattice type, pilot tones are inserted along both time and frequency axis with given periods. The pilots scattered in both time and frequency axis. Comb type pilot insertion is adopted in order to evaluate the channel coefficients.

In OFDM-based systems, different algorithms, which vary in computational complexity and accuracy, can be used for the channel estimation stage. Systems which use coherent detection of the transmitted data, an estimate of the channel response has to be available at the receiver. Thus, the need arises to develop a reliable channel estimator for data detection. The Least Square (LS) and Minimum Mean Square Error (MMSE) criteria are common methods to estimate the channel coefficients. These algorithms are well suited under the assumption that channels are rich but practical channel encountered in practice are sparse in nature. These type of sparse channels are perfectly estimated by the recent advances from the theory of CS.

Sparsity based approaches have two main advantages decreasing mean square error and reducing pilot overhead. Solving linear set of equations by compressive sensing is to achieve the Cramer-Rao lower bound on mean square error [9]. If the structural LS estimator has the knowledge of locations of non zero taps, then it gives best estimate. But there is no information about the location of non zero coefficients of  $h$  at the



receiver. Therefore structural LS estimator cannot be realized. In this case sparsity based approach leads to best result. In pilot aided channel estimation, the pilot subcarriers occupy a fraction of spectrum but they do not carry any information. The spectrum efficiency can be increased by reducing the no of pilot subcarriers. By considering the sparsity of the channel, it is possible to estimate the channel coefficients with fewer numbers of pilots.

Generally, there are two main categories of sparsity based methods in CS. One approach is to minimize the  $l_1$  norm which is based on linear programming techniques.  $l_1$  minimization provides accurate method for sparse signal recovery if it satisfies RIP. But the computational complexity of  $l_1$  minimization is highly impractical. It leads to the need of faster recovery algorithm that works in linear time. Recently several low complexity iterative algorithms are proposed. Fast greedy methods such as OMP iteratively detect and estimate the location and value of the channel taps. These methods are usually faster than  $l_1$  minimization techniques by orders of magnitude while they may fall short of performance [11] and [12]. Simulation is carried out for the greedy algorithms such as OMP, StOMP and ROMP [13]-[18] for the purpose of channel estimation over time varying channel.

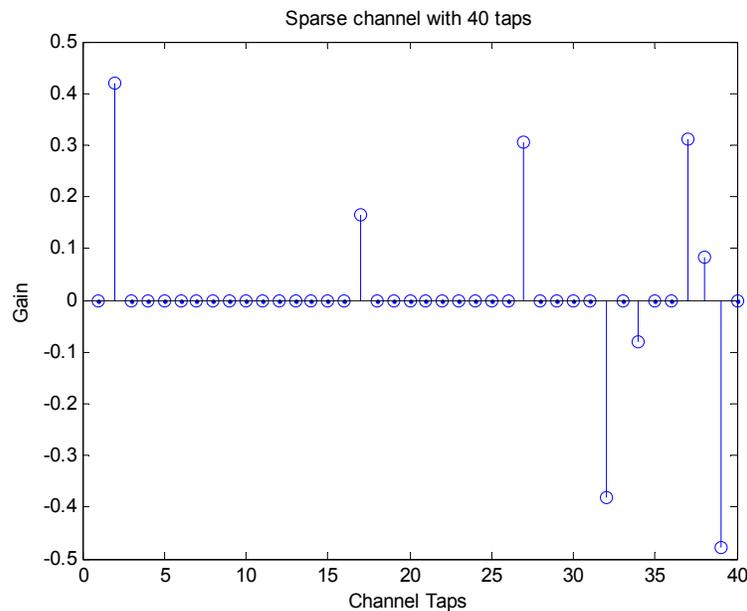
#### 4. SIMULATION RESULTS

The  $1 \times 1$  MIMOOFDM system is considered with 265 subcarriers. Among that 12.5% and 25% are used for pilots with comb type pilot arrangement. The modulation order is set to 4QAM.

**Table-1.** Simulation parameters.

Parameters	Specifications
Number of sub carriers	265
Number of pilots	32 & 64
Carrier frequency	150MHz
$N_r \times N_t$	$1 \times 1$
Modulation	4QAM
channel	Jakes Model

To generate a fading channel gain Jake's model is used in that mobility was considered as 3km/h and 30km/h that leads to Doppler shift of 416.6667 and 4.1667e+003. The following figure shows the sparse channel with 40 taps



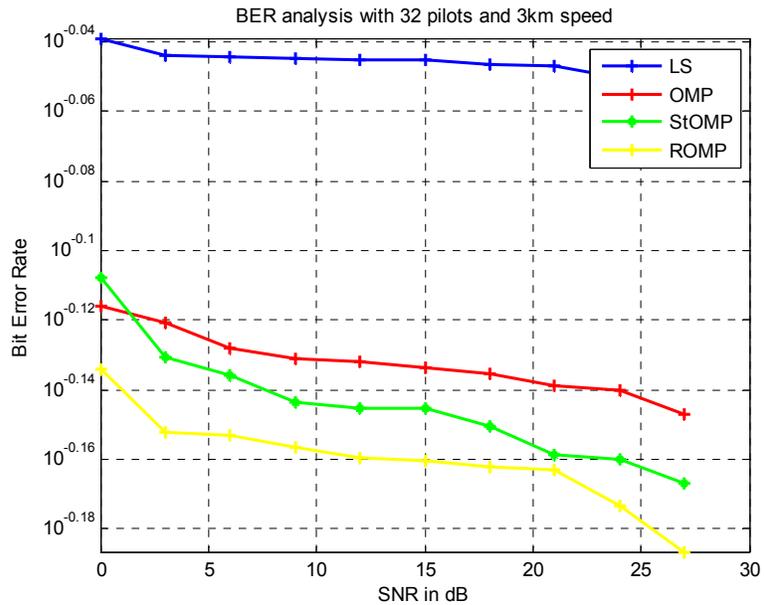
**Figure-1.** Sample sparse channel with sparsity 8 among 40 taps.

The number of non-zero taps is set to 8 out of a total 40 taps for each channel link. The bit error rate is applies as the performance index of the channel estimation algorithms. The performance of OMP, StOMP and ROMP

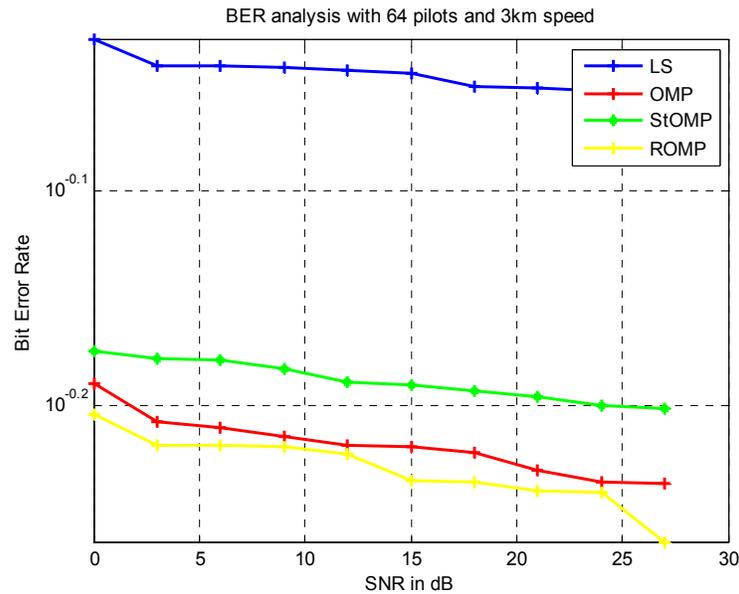
are compared for various SNR ranging from 0 to 30. The following figure shows the performance comparison of various CS algorithms OMP, StOMP and ROMP with traditional LS algorithm.



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**Figure-2.** BER of various channel estimation algorithms with 32 pilots and 3km mobility.



**Figure-3.** BER of various channel estimation algorithms with 64 pilots and 3km mobility.

Figure-2 and Figure-3 shows that CS based methods greatly improve the bit error rate compared to traditional LS method over 3km mobility channel with 32 and 64 pilots. The performance of CS based methods with

32 pilots is better than the performance of LS method with 64 pilots. This shows that CS based algorithms reduces the pilot overhead than traditional method.



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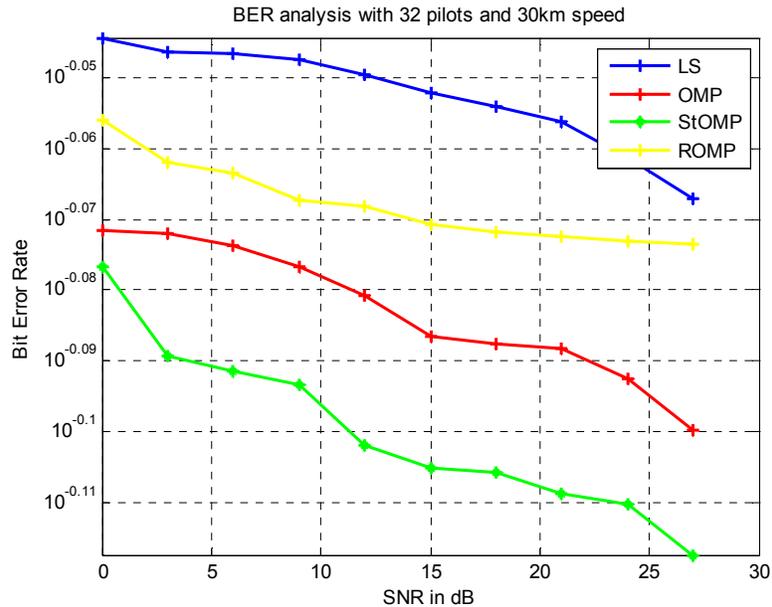


Figure-4. BER of various channel estimation algorithms with 32 pilots and 30km mobility.

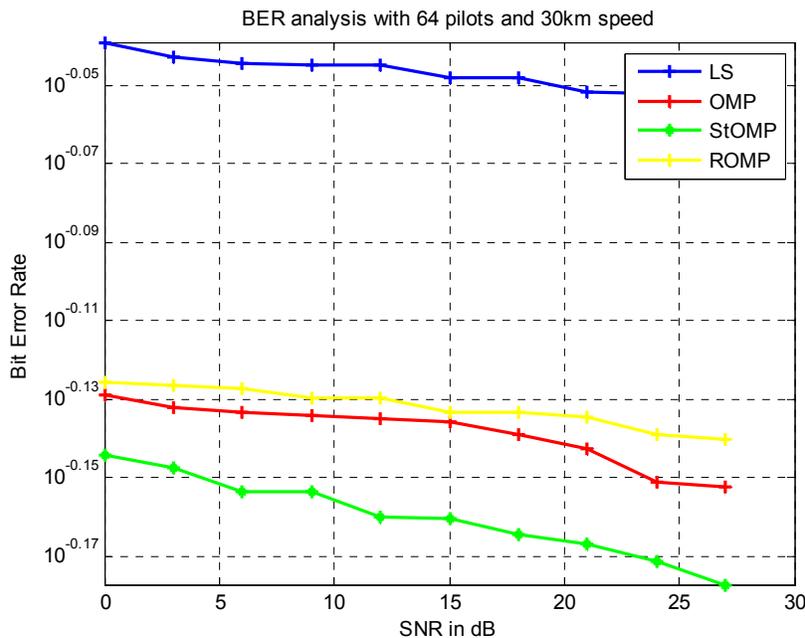


Figure-5. BER of various channel estimation algorithms with 64 pilots and 30km mobility.

Figure-4 and Figure-5 shows the performance of CS based methods over 30km mobility channel with 32 and 64 pilots. It outperforms the traditional LS method but the performance of the CS based methods are reduced because of the Doppler shift introduced by the 30km mobility in the channel than the low mobility channel. Results show that mobility increases, the performance of the CS algorithms are degraded compared to low mobility

even in the high SNR regime with increased number of pilots due to Doppler shift.

## 5. CONCLUSIONS

CS based channel estimation is deployed for MIMO OFDM system over time varying frequency selective channel. In traditional channel estimation, channel frequency response is acquired at the pilot positions and then using those observations rest of the



subcarriers are interpolated. Pilot number increases rapidly, when the channel has large delay spread and contains abundant multipath. With sparse representation the number of pilots can be substantially reduced. It is proved that CS based algorithms out performs the traditional LS algorithm with minimum pilots. To further improve the system performance of time varying frequency selective sparse channel in the mobility and to reduce the pilot overhead, Doppler shift due to mobility has to be mitigated.

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