



A FAST AND STABLE ORIENTATION SOLUTION OF THREE CAMERAS-BASED UAV IMAGERIES

Martinus Edwin Tjahjadi

Department of Geodesy, ITN-Malang, Malang, Indonesia

E-Mail: edwint.tjahjadi@gmail.com

ABSTRACT

In this research, a fixed-wing UAV based spatial information acquisition platform which can carry three cameras on board is developed and evaluated. A position and attitude determination of all collected photos is a prerequisite for unmanned aerial vehicles (UAVs) - based photogrammetry and computer vision applications to derive spatial data. In photogrammetric practices, an approximate position and attitude of each collected digital photos must be determined in advance to enable to start bundle adjustment procedures. There are numerous methods for solving this problem arises from both photogrammetric and computer vision communities. However, due to a high measurement cost of geodetic type of GPS observations, a minimal number of control points are used to solve a closed form of space resection problems. Two variant methods of a closed form solution using three control points only are evaluated. A method so called "a perspective similar triangle" arises from computer vision community and "a photo scaled variation" method are evaluated and compared. Both methods have a stable direct solution in presence of image noise; nonetheless this study reveals their robustness and stability of the solution with vertical, oblique, high oblique and convergent photos.

Keywords: UAV, space resection, exterior orientation, closed-form.

1. INTRODUCTION

Space resection in photogrammetry as defined by [1] or a 2D-3D image orientation [2] is a the process to determine an image's spatial position and orientation parameters based on the photogrammetric measurements of ground control points captured on the image. These parameters are called exterior orientation parameters and they include three ground space coordinates of the perspective (camera) centre and three rotation parameters which describe the orientation of the ground space coordinate system with respect to the camera coordinate system.

A closed-form solution means that there is no need for a priori value or initial guess and iteration [2]. A priori fixed of arithmetic operations characterises indeterminacy, instability, or multiplicity of the solution using sufficiently minimum three non-collinear ground control points, possibly a planar plane with equal heights. The indeterminacy of the solution is caused by some unstable geometric structure, i.e. a small perturbation in the position of the camera centre could lead a big change in the result particularly in "dangerous cylinder" cases [3], [4]. The instability of the solution mostly influenced by the permutation of the chosen triangle vertexes. The accuracy of the result could vary about 10^3 times [5]. The multiplicity of the solution is due to the fact that different methods of the solution lead to the different quartic equations [6], [4]. Therefore, a geometrically isomorphic negative solution always exists for every real positive solution. In general four real solutions are possible. The correct solution can be found by using equation solver [7], [8], [9], [10] or by adding a fourth given point [11], [12]. However, if the inappropriate equation solver is used, the

performance of the polynomial solver degenerates significantly.

Haralick *et al* [13] have investigated the six variants of algorithms of Grunert 1841 and compared them with respect to numerical accuracy and stability. Despite the clear superiority of the algorithm of Finsterwalder ascertained by Haralick *et al*, Li and Xu [5] found that all existing three point closed-form resection algorithm rooted from Grunert's method suffer from the permutation, geometric singularity and equation solver problems since these method utilise the three side lengths of the target triangle (i.e. three non-collinear observed ground points) as geometrical constraint, for example, the distances among the three estimated ground points are constrained to equal the side lengths of target triangle.

This constraint is avoided by Li and Xu by using a so called "perspective similar triangle" (PST) to reposition ground control points in a way that it can reduce quartic polynomial to become a quadratic one, hence only two real solution exists. Meanwhile, in photogrammetric community, a variant of this method were invented by Munji and Hussain [14]. To avoid the constraint they assumed that a camera focal length can be varied across the image. So the three imaged ground points will have different focal lengths as well as slightly different positions with the original ones in terms of camera coordinate system. The solution is based on the analysis of the scale variations between side lengths of the triangle vertexes of the imaged ground control points. Their method also ends up with the two real solutions. In this paper, Munji and Hussain method is termed as "Photo Scale Invariant" (PSI) method. This paper aims to evaluate feasibility these unique methods to provide fast and stable



the exterior orientation parameter of an image capture by camera mounted on a UAV for mapping related purposes.

2. STATEMENT OF PROBLEM

Most of the closed-form solution of the space resection problem amounts to the two steps of procedures. The first step is to determinethe distances from the perspective centre of a camera to three known ground control points coordinates. However, this paper presents an alternative approach by setting a scalefactor a constant term. Munjy and Hussain method [14] achieves a constant scale factor by repositioning image coordinates of other 2 control points on the image, whilst Li and Xu [5] method achieves this by repositioning the object space coordinate of one control point only. Next, the second step is a computation of the exterior orientation parameters. This paper uses a closed form three dimensional coordinate transformation or an absolute orientation problem. Finally, the suitability of the two methods for UAV's aerial mapping, particularly in the collinearity based least squares adjustment, is discussed.

3. PERSPECTIVE SIMILAR TRIANGLE METHOD

The perspective similar triangle (PST) is coined by Li and Xu which assumed that three ground points coordinates (X, Y, Z) of P_1, P_2, P_3 are observed yielding a known image coordinate (x, y) of points p_1, p_2 and p_3 with calibrated focal length f (Figure-1).

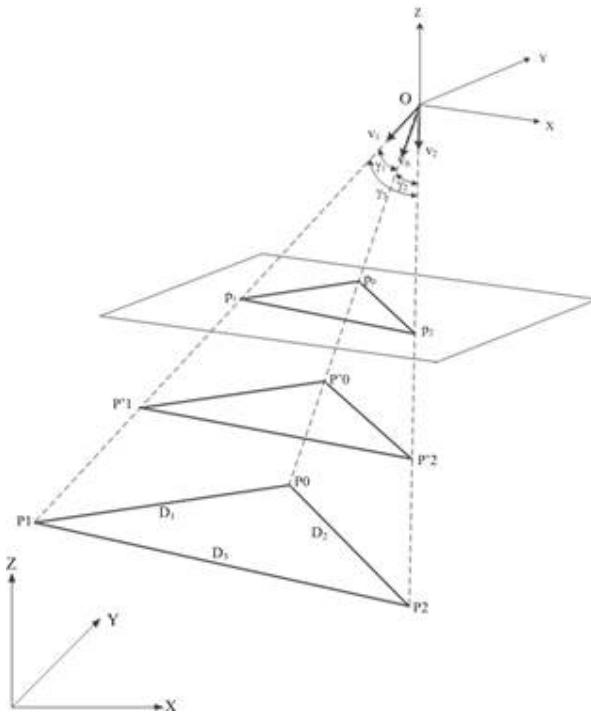


Figure-1. Spatial resection with three given ground points.

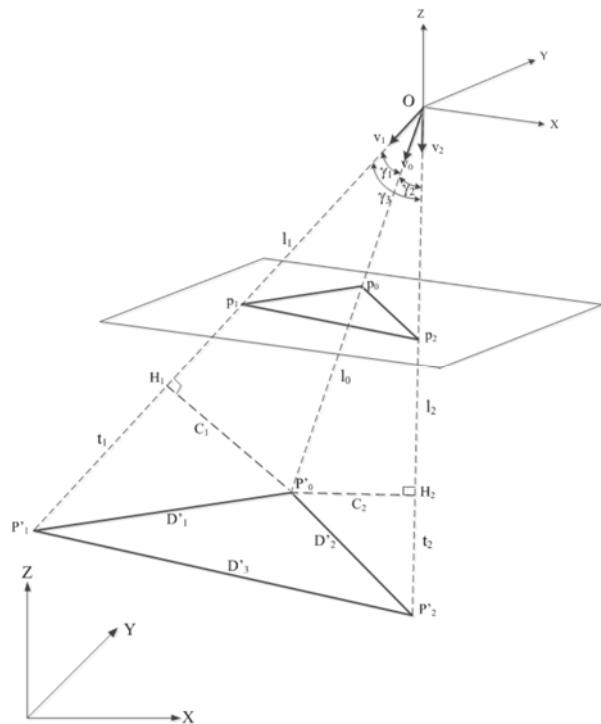


Figure-2. The geometry of perspective similar triangle.

In order to solve the exterior orientation parameter of the perspective centre O, distances $s_i = |P_i - O| (i = 0, 1, 2)$ between the point O to each ground point must be computed first, and it always yields multiple roots (ref). To reduce the number of the computed roots, they convert the problem by using artificial ground points which positioned arbitrarily at a vector line of distance s_i , as illustrated at Figure-2, to create a triangle $P'_0P'_1P'_2$ which are “similar” to the triangle $P_0P_1P_2$. Li and Xu show that there exists a unique similar triangle $P'_0P'_1P'_2$ for any triangular ground point $P_0P_1P_2$ projected onto image plane as $p_0p_1p_2$ and be with the same image projection geometry. As a result, the number of possible solution has not changed as that of the original space resection problem.

To convert to the PST, an arbitrary base point (i.e. P'_0) on the line Op_0 is selected. By constructing a plan passing P'_0 and being parallel to $P_0P_1P_2$, the vertex of P'_1 and P'_2 can be determined along the line Op_1 and Op_2 respectively which intersect this plan. Once the distance s'_i of $P'_0P'_1P'_2$ are solved, the distance of the O to the s_i can be determined by multiplying a scale factor k on s'_i due to this similarity of these two triangles, [5] applied two geometrical constraints, namely, a constraint of the ratio of the corresponding side lengths of similar triangles and a constraint of the corresponding angles of similar triangles equal each other. These constraints can be formulated as follows:

$$A_1 t_1^2 - t_2^2 + A_2 = 0 \quad (1)$$



$$A_3t_1 + A_4t_2 + A_5t_1t_2 + A_6t_1^2 + A_7 = 0 \quad (2)$$

where

$$\begin{aligned} l_0 &= 1; \quad C_1 = l_0 \sin \gamma_1; \quad C_2 = l_0 \sin \gamma_2; \\ l_1 &= l_0 \cos \gamma_1; \quad l_2 = l_0 \cos \gamma_2; \\ k &= D_2 / D_1; \quad A_1 = k^2; \quad A_2 = k^2 C_1^2 - C_2^2; \\ A_3 &= l_2 \cos \gamma_3 - l_1; \quad A_4 = l_1 \cos \gamma_3 - l_2; \quad A_5 = \cos \gamma_3; \\ A_6 &= (D_3^2 - D_1^2 - D_2^2) / (2D_1^2); \\ A_7 &= l_0^2 - l_1^2 - l_2^2 + l_1l_2 \cos \gamma_3 + A_6C_1^2 \end{aligned}$$

A variable t_2 can be computed by combining Equation (1) and Equation (2) such that:

$$t_2 = -\frac{A_3t_1 + A_6t_1^2 + A_7}{A_4 + A_5t_1} \quad (3)$$

So, the variable t_2 can be eliminated by substituting Equation (3) into Equation (1), and a fourth order polynomial of t_1 can be derived,

$$B_4t_1^4 + B_3t_1^3 + B_2t_1^2 + B_1t_1 + B_0 = 0 \quad (4)$$

where

$$\begin{aligned} B_0 &= A_7^2 - A_2A_4^2; \quad B_1 = 2(A_3A_7 - A_2A_4A_5); \\ B_2 &= A_3^2 + 2A_6A_7 - A_1A_4^2 - A_2A_5^2; \\ B_3 &= 2(A_3A_6 - A_1A_4A_5); \quad B_4 = A_6^2 - A_1A_5^2 \end{aligned}$$

The next step is to determine t_2 by t_1 . It is clear that only if $A_4 + A_5t_1 \neq 0$, t_2 can be solved by Equation (3) or it uses $t_2 = \pm \sqrt{A_1t_1^2 + A_2}$. The scale factor of similar triangles can be calculated from

$$\lambda = D_1 / \sqrt{t_1^2 + C_1^2} \quad (5)$$

then the distances between O to ground triangle vertexes $P_1P_2P_3$ can be computed from

$$s_0 = \lambda l_0; \quad s_1 = \lambda(l_1 + t_1); \quad s_2 = \lambda(l_2 + t_2) \quad (6)$$

[5] claimed that PST solution has less complexity than ordinary closed form solution since it has only two unknown variables of t_1 , t_2 and constant terms. They

argued that the PST is more convenient to compute and analyse and it would lead a better numerical performance.

4. PHOTO SCALE INVARIANT METHOD

The photo scale invariant (PSI) is invented by [14]. This method is based on the widely known photogrammetric postulate that the photo scale in a perspective image is unequal across the image plane. In other words, if the camera focal length is allowed to vary across the image or at each imaged ground point, a constant photo scale in the image space can be imposed.

If the photo scale between a three set of ground points is forced to remain constant, it will cause a three dimensional perturbation in the camera coordinate system. As a result, a new set of image coordinate with a different focal length at each triangle vertex. This new set of coordinates can be perceived as an expression of a scaled and rotated three dimensional model of the ground points. Using a closed-form absolute orientation method, three dimensional transformations the three ground coordinate points and the newly formed three dimensional model the computation of the exterior orientation parameter can be performed.

They argued that the fourth order of polynomial equation can be derived by varying the image coordinates and the focal length at each imaged ground point. By fixing the focal length point p_0 (f_0), the changed image coordinates p_1 and p_2 caused by the focal length variation can be expressed as:

$$\hat{x}_i = \frac{x_i}{f_0} f_i; \quad \hat{y}_i = \frac{y_i}{f_0} f_i \quad i = 1, 2 \quad (7)$$

The horizontal distances and photo scales on the image between the three points after the changes due to [14] is as follows:

$$d_{ij}^2 = \frac{1}{f_0^2} \left((f_i x_i - f_j x_j)^2 + (f_i y_i - f_j y_j)^2 \right) \quad (8)$$

$$\mu = \left(d_{ij}^2 + (f_i - f_j)^2 / D_{ij}^2 \right)^{1/2} \quad (9)$$

where $i = 0, 1 \quad j = i + 1, 2$.

Substituting Equation (8) and Equation (9) to remove photo scales will result in equations,

$$f_1^2 + a_1 f_1 + b_1 f_2 + c_1 f_2^2 = \beta_1 \quad (10)$$

$$d_1 f_1^2 + e_1 f_1 + g_1 f_1 f_2 + h_1 f_2^2 = \beta_2 \quad (11)$$



with

$$\alpha_1 = D_{01}^2/D_{02}^2; \quad \alpha_2 = D_{01}^2/D_{12}^2$$

$$r_i^2 = x_i^2 + y_i^2 + f_0^2; \quad i = 0,1,2$$

$$a_0 = r_1^2/f_0^2$$

$$a_1 = -2(x_0 x_1 + y_0 y_1 + f_0^2)/f_0/a_0$$

$$b_1 = 2\alpha_1(x_0 x_2 + y_0 y_2 + f_0^2)/f_0/a_0$$

$$c_1 = -\alpha_1 r_2^2/r_1^2$$

$$\beta_1 = (\alpha_1 - 1)r_0^2/a_0$$

$$d_1 = (1 - \alpha_1)r_1^2/f_0^2$$

$$e_1 = -2(x_0 x_1 + y_0 y_1 + f_0^2)/f_0$$

$$g_1 = 2\alpha_2(x_1 x_2 + y_1 y_2 + f_0^2)/f_0^2$$

$$h_1 = -\alpha_2 r_2^2/f_0^2$$

$$\beta_2 = -r_0^2$$

From Equation (10), f_2 can be substituted as

$$f_2^2 = \frac{1}{c_1}(\beta_1 - f_1^2 - a_1 f_1 - b_1 f_2) \quad (12)$$

Substituting Equation (12) into Equation (11) yields:

$$f_2 = \frac{\chi_1 - \chi_2 f_1^2 - \chi_3 f_1}{g_1 f_1 - \chi_4} \quad (13)$$

where

$$\chi_1 = \beta_2 - \frac{h_1}{c_1} \beta_1; \quad \chi_2 = d_1 - \frac{h_1}{c_1}$$

$$\chi_3 = e_1 - \frac{a_1 h_1}{c_1}; \quad \chi_4 = \frac{b_1 h_1}{c_1}$$

Then substituting Equation (13) into Equation (10) will end up with the fourth order polynomial as follows:

$$\eta_4 f_1^4 + \eta_3 f_1^3 + \eta_2 f_1^2 + \eta_1 f_1 + \eta_0 = 0 \quad (14)$$

with

$$\eta_4 = g_1^2 + c_1 \chi_2^2$$

$$\eta_3 = -2g_1 \chi_4 + g_1^2 a_1 - b_1 g_1 \chi_2 + 2c_1 \chi_2 \chi_3$$

$$\eta_2 = \chi_4^2 - 2a_1 \chi_4 g_1 - b_1 g_1 \chi_3 + b_1 \chi_4 \chi_2 - 2c_1 \chi_1 \chi_2 + c_1 \chi_3^2 - \beta_1 g_1^2$$

$$\eta_1 = a_1 \chi_4^2 + b_1 \chi_1 g_1 + b_1 \chi_4 \chi_3 - 2c_1 \chi_1 \chi_3 + 2\beta_1 \chi_4 g_1$$

$$\eta_0 = -b_1 \chi_1 \chi_4 + c_1 \chi_1^2 - \beta_1 \chi_4^2$$

At this point, it worth noting both methods arrive at a fourth order polynomial function of a variable, and the geometric meaning of this variable of each method is easy to express, but Equation (14) clearly shows the polynomial variable is the focal length at point p_1 . Equation (4) and Equation (14) will yield one to four real roots. However, [14] stated that if four roots exist, two roots which are closest in value to f_0 are chosen.

5. DETERMINATION OF EXTERIOR ORIENTATION PARAMETERS

After having determined the three distances from the perspective centre to the ground points whilst using the PST method, it is necessary to find the 6 parameter of the position and attitude of the camera. The 3D coordinates of the three points ($p_0 p_1 p_2$) in the camera system:

$${}^c \mathbf{x}_i = s_i \mathbf{v}_i; \quad i = 0,1,2 \quad (15)$$

Where ${}^c \mathbf{x}_i$ are 3D coordinates of p_0, p_1, p_2 ; \mathbf{v}_i are the unit vector from the perspective centre to the ground points. When using the PSI method, the perturbed image coordinates of Equation (7) is similar to the ${}^c \mathbf{x}_i$. The relationship between the ground coordinate system and the camera system is expressed as [15]:

$${}^c \mathbf{x}_i = {}^c \mathbf{R}_o \left({}^o \mathbf{x}_i - {}^o \mathbf{x}_O \right) \quad (16)$$

Where ${}^c \mathbf{R}_o$ is a rotation matrix between ground and camera coordinate system, ${}^o \mathbf{x}_i$ are 3D coordinate of $P_0 P_1 P_2$, and ${}^o \mathbf{x}_O$ is the coordinates of the perspective centre in terms of ground coordinate system. The ${}^c \mathbf{R}_o$ and ${}^o \mathbf{x}_O$ are six unknowns of the exterior orientation parameters.

The rotation parameter can be determined by creating a tripod both in ground and camera coordinate systems [15] (Figure-3). The tripod can be defined using the directions of vector $P_0 P_2$ and $P_0 P_1$, where $P_0 P_2$ is defined as the X axis (${}^o X$). The Y and Z axis can be derived by using a cross product of the vectors (i.e. b and c).

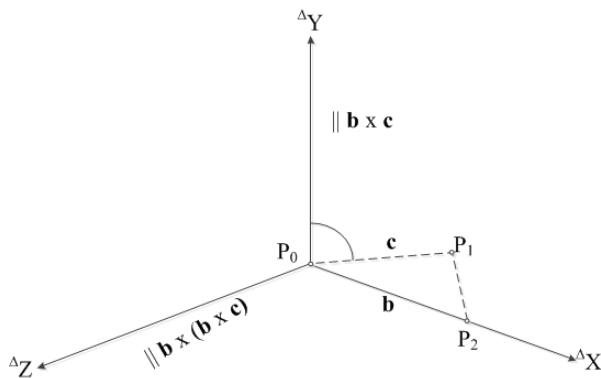


Figure 3. P_0 defines the origin of a tripod, and P_0P_2 defines X axis of a tripod.

Due to [15] the rotation matrices can be computed as

$${}^o\mathbf{R}_\Delta = \begin{bmatrix} {}^o\mathbf{b} & N({}^o\mathbf{b} \times {}^o\mathbf{c}) & N({}^o\mathbf{b} \times ({}^o\mathbf{b} \times {}^o\mathbf{c})) \end{bmatrix} \quad (17)$$

and

$${}^c\mathbf{R}_\Delta = \begin{bmatrix} {}^c\mathbf{b} & N({}^c\mathbf{b} \times {}^c\mathbf{c}) & N({}^c\mathbf{b} \times ({}^c\mathbf{b} \times {}^c\mathbf{c})) \end{bmatrix} \quad (18)$$

with

$${}^o\mathbf{b}={}^o\mathbf{x}_2-{}^o\mathbf{x}_0; \quad {}^o\mathbf{c}={}^o\mathbf{x}_1-{}^o\mathbf{x}_0$$

$${}^c\mathbf{b}={}^c\mathbf{x}_2-{}^c\mathbf{x}_0; \quad {}^c\mathbf{c}={}^c\mathbf{x}_1-{}^c\mathbf{x}_0$$

Equation (17) and (18) represent the columns of the rotation matrix of the tripod axes, thus the rotation matrix becomes:

$${}^c\mathbf{R}_o={}^c\mathbf{R}_\Delta \quad \Delta\mathbf{R}_o={}^c\mathbf{R}_\Delta \quad {}^o\mathbf{R}_\Delta^t \quad (19)$$

Once the rotation matrix ${}^c\mathbf{R}_o$ computed, the spatial position of the camera can be obtained with:

$${}^o\mathbf{x}_O={}^o\mathbf{x}_0-{}^c\mathbf{R}_o^t \quad {}^c\mathbf{x}_0 \quad (20)$$

6. RESULT AND DISCUSSIONS

To test both algorithms a small C++ program was written and implemented on our ongoing development photogrammetric software. Image data set is obtained from [16]. Image data sets were capture by an aerial camera with has 151.876mm focal length, presented in

Table-1. Ground point coordinate data are presented in Table-2.

Table-1. Image coordinate.

Point	Photo coordinate (mm)	
	x	y
1	-53,845	65,230
2	104,500	68,324
3	4,701	-12,153
4	-61,372	-79,559
5	93,825	-62,060

Table-2. Ground point coordinate.

Point	Ground point coordinates (metre)		
	X	Y	Z
1	6934,954	23961,105	160,136
2	7860,202	23941,563	152,653
3	7261,078	23491,497	142,208
4	6836,650	23087,475	137,719
5	7791,556	23166,680	138,827

Table-1 and Table 2 show 5 data point set. However, only the first three points (i.e. point 1- 3) are randomly selected as input data for computing closed-form space resection solutions. The exterior orientation parameter is not uniquely determined from 3-point set due to multiple solution phenomena. The number of the solutions directly corresponding to the number of real roots of the 4-th order polynomial.

Both the PST and PSI methods use the 4-th order polynomial, hence there should be four possible solutions exist. In contrast with the PST method, the PSI solution reduces this possibility by selecting only two roots which are the closest with the initial focal length. Table-3 reveals all two possible solutions of the PSI method. These solutions is utilised for initial values to start iterative least square computation of space resection based on collinearity equation. Only convergent solution presented in Table-3.

The PST method presented in Table-4 provides 3 real solutions. All possible solutions are used as initial values for least square - collinearity based space resection. All five ground points are utilised to produce a final and accurate result and all the iterative computation can converge precisely. The result is shown in collinearity solution in Table-3.

**Table-3.** The PSI method.

PSI Method			Collinearity Solution
EO par	Solution1	Solution 2	
Omega	1.987790	-1.256302	-1.7332 degrees
Phi	-11.76349	0.483372	0.8049 degrees
Kappa	8.140598	-3.391176	-2.1777 degrees
X	6559.293091	7255.7194	7248.4668m
Y	28062.83449	23506.438	23593.7277m
Z	-49.6996792	1029.3776	1058.1499m

Table-4. The PST method.

PST Method			
EO par	Solution1	Solution 2	Solution 3
Omega	-10.23130	5.553352636	5.553352599s
Phi	0.035203028	0.051866261	0.051866186
Kappa	-1,95047858	-2.24100500	-2.241006175
X	7259.869764	7269.50146	7084.977074
Y	23752.90105	23506.91717	23757.4309
Z	-787.972796	-689.024247	-220.6579054

7. CONCLUSIONS

On consideration of the problem of closed form solution using three point set, we evaluated two methods which are not relying on Grunet based solutions. Both methods use a scaled model to solve for quartic polynomial roots. The PSI method uses a scale variation of the image distances between ground points. Meanwhile, the PST method relies on the similar triangle concept. The distances between the perspective centre and the ground points are computed indirectly through this triangle. In terms of speed and stability of the computation result, the PST is more reliable.

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