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# MINIMIZING IDLE TIME OF CRITICAL MACHINE IN PERMUTATION FLOW ENVIRONMENT WITH WEIGHTED SCHEDULING

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**ABSTRACT**

The weighted scheduling problem is practically obvious and need more investigation. In this research work is focused on the weighted scheduling problem with critical machine consideration of permutation flow type manufacturing environment. The objectives are makespan minimization as well as maximize the utilization of critical machine. The problem initially simplified and solved by heuristic method and validated. A special type of heuristic is suggested to solve such critical machine based weighted scheduling problems. The proposed heuristics were validated with multilevel problems. The proposed heuristic outperforms in reduction of ideal time, of critical machine as well as makespan.

**Keywords:** flow shop, idle time of machines, scheduling, sequencing, critical machine.

## INTRODUCTION

In present global spirited manufacturing scenario, the manufacture meets a variety a products with a shorter span of time because of continuous up gradation of the design of the components. To meet the Competitive Business Environment, Sequencing and Scheduling plays Very important role in manufacturing industries. In this virtue, the manufacturing industries must meet certain due date commitments to their customers since failure to meet such deadlines will cause them to lose revenue or heavy penalties; in addition, the Manufacturers need to schedule their resources or critical resources efficiently to remain viable relative to their competitors. Usually in flow shop scheduling the same sequence or order of the job has to be implemented in machines.

The priority is provided for the jobs based on the due date or critical distinctiveness of the order delivery. Gupta and Sharma [1] studied N × 2 flow shop problems to minimize rental cost under the predefined rental policy in which probabilities have been allied with processing time which includes job - block criteria. This paper is an attempt to develop the study conducted by Gupta and Sharma [2] by introducing the concept of probability associated with no jobs in a specified sequence with the view of minimizing makespan time.

In general, such problems are not easier to solve by enumeration. Regrettably generalized sequencing models are also not available to cope with all cases. The solution method in such cases was first developed by S.M. Johnson and the procedures are often referred as Johnson's role. Johnson [3] has proposed the two stage scheduling problem with the make span objective.

The NP-completeness of the flow shop-scheduling problem has been discussed extensively by Quan-Ke Pan and Ling Wang [4]. In the 1965's Palmer [5] was the one that initiated heuristic procedures. The first significant work in progress of a proficient heuristic is due to Campell, Dudek and Smith [6]. Their algorithm fundamentally consists of splitting the given m-machine problem into a series of an equivalent two-machine flow shop problem and then solving by means of Johnson's rule. In 1977, Dannenbring [7] was the one who developed a procedure called 'rapid access', which attempts to combine the advantages of Palmer's slope index and CDS procedures. His method is found to yield a better quality solution than those of Palmer's and CDS methods, but it requires much more computational effort. King and Spachis during the year 1980 [8] treats the make span problem as a counterpart to that of minimizing the total delay and run-out delay. They have proposed heuristics that aim at harmonizing the two consecutive job time-block profiles by considering these delays.

Pugazhenthi and Anthony propose a heuristic to minimizing the idle time, of critical machine when the material flows in manufacturing in various causes [9-13]. The critical machine based weighted scheduling problem is unique and highly practical relevance. In this paper they said the reduction of idle time, of critical machine as well as minimizing the makespan.

### Assumptions

This research assumes the following factors,

a. Jobs are independent of each other.
b. Breakdown time is considered based on the inconvenience to do work.
c. The critical Machine breakdown is considered.
d. Pre-emission is not allowed, i.e. once a job started on a machine, the process on that machine can't be stopped unless the job is completed.

## PERMUTATION FLOW SHOP WEIGHTED SCHEDULING PROBLEM

The weighted scheduling problem is a type of problem in which jobs are given weightage based on delivery requirements. Here below six heuristics are developed and tested and explain them stage by stage. The general structure of n-jobs, m-machines problem is shown in Table-1.

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**Table-1.** General structure of the processing times for N X M flowshop problem with weightage.

| Job | Processing time on machines | | | | | Weightage |
|---|---|---|---|---|---|---|
| | $m_1$ | $m_2$ | $m_3$ | . . | $m_M$ | $w_j$ |
| $j_1$ | $t_{11}$ | $t_{21}$ | $t_{31}$ | . . | $t_{M1}$ | $w_1$ |
| $j_2$ | $t_{12}$ | $t_{22}$ | $t_{32}$ | . . | $t_{M2}$ | $w_2$ |
| $j_3$ | $t_{13}$ | $t_{23}$ | $t_{33}$ | . . | $t_{M3}$ | $w_3$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| $J_N$ | $t_{1N}$ | $t_{1N}$ | $t_{1N}$ | $t_{1N}$ | $t_{MN}$ | $w_n$ |

For the heuristics requirements in the Table-2 the processing times of jobs multiplied by the weighted percentage to simplify it intoa regular flow shop problem. The last column of the table shows that the total resultant time for each job.

**Table-2.** General structure of the resultant processing times of N X M permutation flowshop problem.

| Job | Resultant processing time on machines | | | | | | |
|---|---|---|---|---|---|---|---|
| | $m_1$ | $m_2$ | $m_3$ | | | $m_M$ | $T_j'$ |
| $j_1$ | $t'_{11}$ | $t'_{21}$ | $t'_{31}$ | .. | .. | $t'_{M1}$ | $T_1'$ |
| $j_2$ | $t'_{12}$ | $t'_{22}$ | $t'_{32}$ | .. | .. | $t'_{M2}$ | $T_2'$ |
| $j_3$ | $t'_{13}$ | $t'_{23}$ | $t'_{33}$ | .. | .. | $t'_{M3}$ | $T_3'$ |
| . | . | . | . | . | . | . | |
| . | . | . | . | . | . | . | |
| $j_N$ | $t'_{1N}$ | $t'_{2N}$ | $t'_{3N}$ | | | $t'_{MN}$ | $T_N'$ |

**Heuristic-1**

**Stage-I:** To find ($t'_{ij}$) the product of individual process time with a weightage value of the respective job. Hence the problem is reduced as a normal Flowshop problem.

$t'_{ij} = t_{ij} * w_i$

Thus the problem is reduced as shown in Table-2.

**Stage-II:** To find total of resultant processing times for each job ($T_j'$)

$T_1' = t'_{11} + t'_{21} + t'_{31} + t'_{41} + t'_{51} + .... + t'_{M1}$
$T_2' = t'_{12} + t'_{22} + t'_{32} + t'_{42} + t'_{52} + .... + t'_{M2}$
$T_3' = t'_{13} + t'_{23} + t'_{33} + t'_{43} + t'_{53} + .... + t'_{M3}$
.
.
$T_N' = t'_{1N} + t'_{2N} + t'_{3N} + t'_{4N} + t'_{5N} + .... + t'_{MN}$

**Stage-III:**

a) Arrange the resultant total processing times in non-increasing order.
b) Now split the entire sequence in to two half (if odd number of job consider one additional job in the second half.
c) Then rearrange the first half of total resultant times in increasing order and keep the second half as it is.
d) Now join first and second half. The resultant order is the required schedule.

**Heuristic-2**

**Stage-I:** To find ($t'_{ij}$) the product of individual process time with a weightage value of the respective job. Hence the problem is reduced as a normal fellowship problem.

$t'_{ij} = t_{ij} * w_i$

Thus the problem is reduced as shown in Table-2.

**Stage-II:** To find total of resultant processing times for each job ($T_j'$)

$T_1' = t'_{11} + t'_{21} + t'_{31} + t'_{41} + t'_{51} + .... + t'_{M1}$
$T_2' = t'_{12} + t'_{22} + t'_{32} + t'_{42} + t'_{52} + .... + t'_{M2}$
$T_3' = t'_{13} + t'_{23} + t'_{33} + t'_{43} + t'_{53} + .... + t'_{M3}$
.
.
$T_N' = t'_{1N} + t'_{2N} + t'_{3N} + t'_{4N} + t'_{5N} + .... + t'_{MN}$

**Stage-III:**

a) Arrange the resultant total processing times in non-increasing order.
b) Now split the entire sequence into two halves (if odd number of jobs, consider one additional job in the second half.
c) Then rearrange the Second half of total resultant times in increasing order and keep the first half as it is.
d) Now join first and second half. The resultant order is the required schedule.

**Heuristic-3**

**Stage-I:** To find ($t'_{ij}$) the product of individual process time with a weightage value of the respective job. Hence the problem is reduced as a normal flowshop problem.

$t'_{ij} = t_{ij} * w_i$

Thus the problem is reduced as shown in Table-2.

**Stage-II:** To find a total of resultant processing times for each job ($T_j'$)

$T_1' = t'_{11} + t'_{21} + t'_{31} + t'_{41} + t'_{51} + .... + t'_{M1}$
$T_2' = t'_{12} + t'_{22} + t'_{32} + t'_{42} + t'_{52} + .... + t'_{M2}$
$T_3' = t'_{13} + t'_{23} + t'_{33} + t'_{43} + t'_{53} + .... + t'_{M3}$
.
.
$T_N' = t'_{1N} + t'_{2N} + t'_{3N} + t'_{4N} + t'_{5N} + .... + t'_{MN}$

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**Stage-III:**

Arrange the resultant total processing times in non increasing order. The resultant order is the required schedule.

## NUMERICAL EXAMPLE-I

Consider an industrial case study problem. There are four machines in the flowshop and four jobs to be processed. The weightage for jobs provided based on the due date. The weightage was given in percentage for each job and hence the sum of weightage for four jobs is 100%. The stage wise (machine wise) processing times for each job furnished in Table-3. There are three heuristics proposed and their numerical examples are discussed below.

**Table-3.** Processing Times (minutes) of the problem and weightage for Jobs.

| Job | Processing time on machines | | | | Weightage |
|---|---|---|---|---|---|
| | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $w_j$ |
| $J_1$ | 6 | 6 | 9 | 5 | 30% |
| $J_2$ | 7 | 3 | 4 | 5 | 40% |
| $J_3$ | 8 | 2 | 3 | 2 | 20% |
| $J_4$ | 6 | 5 | 5 | 3 | 10% |

The resultant processing times after considering the weightage for jobs were furnished in Table-4. Total of resultant processing times for each job ($T_j$') also computed and furnished in the last column of the Table-4.

**Table-4.** The resultant processing times with respect to weightage for Jobs in minutes.

| Job | Resultant processing time on machines | | | | |
|---|---|---|---|---|---|
| | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $T_j$' |
| $J_1$ | 1.8 | 1.8 | 2.7 | 1.5 | 7.8 |
| $J_2$ | 2.8 | 1.2 | 1.6 | 2.0 | 7.6 |
| $J_3$ | 1.6 | 0.4 | 0.6 | 0.4 | 3.0 |
| $J_4$ | 0.6 | 0.5 | 0.5 | 0.3 | 1.9 |

**Heuristic 1-Example**

The stage I and Stage II were commonly completed and furnished in Table-4.

**The Stage III:**

a) Arrange the job wise total of resultant processing time ($T_j$') values in descending order.
{$T_1$',$T_2$',$T_3$',$T_4$'} *that is* $T_1$'>$T_2$'>$T_3$'>$T_4$'= 7.8 > 7.6 > 3.0 >1.9

b) Now split the entire sequence into two halves (if odd number of jobs, consider one additional job in the second half.

c) {$T_1$',$T_2$'} ,{$T_3$', $T_4$'}Then rearrange the first half in ascending order and keep the second half as it is.
{$T_2$',$T_1$'},{$T_3$',$T_4$'} that is $T_2$'< $T_1$'

d) Now join first and second half. {$T_2$',$T_1$',$T_3$',$T_4$'}. The resultant order is the required schedule. Hence the schedule is {$j_2$,$j_1$,$j_3$,$j_4$}

The manual simulation of the proposed schedule {$j_2$, $j_1$, $j_3$, $j_4$} is furnished in Table-6. In which the $I_1$, $I_2$ are the initial and intermediate ideal time of machine at $i^{th}$ stage respectively. The initial ideal time means that the machine waits up to get a first job for processing from the time of beginning of processing of first job in the first stage. The intermediate ideal time is the machine waits for job after processing the job (s). The in and out times are the time at which the job processing begins at the machine and its end time (i.e., Process End time = process start time + processing time at the stage of the job). The makespan is the completion time of last job at the last stage. Here the makespan is 40 minutes. The total ideal time ($I_1$+$I_2$ =I for all machines) of machines when execution of processing of jobs, according to the proposed schedule {$j_2$, $j_1$, $j_3$, $j_4$} is57 minutes.

**Table-5.** Manual simulation of processing of jobs with schedule {$j_2$,$j_1$, $j_3$, $j_4$}.

| Job | $m_1$ | | $m_2$ | | $m_3$ | | $m_4$ | |
|---|---|---|---|---|---|---|---|---|
| | In | Out | In | Out | In | Out | In | Out |
| $J_2$ | 0 | 7 | 7 | 10 | 10 | 14 | 14 | 19 |
| $J_1$ | 7 | 13 | 13 | 19 | 19 | 28 | 28 | 33 |
| $J_3$ | 13 | 21 | 21 | 23 | 28 | 31 | 33 | 35 |
| $J_4$ | 21 | 27 | 27 | 32 | 32 | 37 | 37 | 40 |
| *Ideal time* | 0 | 0 | 7 | 9 | 10 | 6 | 14 | 11 |

The detailed numerical illustration discussed for heuristic1. Similarly, by following the stages suggested in the heuristic, the schedules can be identified from the heuristic 2 and heuristic3 are {$j_1$, $j_2$, $j_4$, $j_3$} and {$j_1$, $j_2$, $j_3$, $j_4$} respectively. The manual simulation outputs are consolidated and presented in the Table-6. It is observed from the Table 6 that the heuristic 2 yielded minimum makespan.

## VALIDATION OF HEURISTIC PERFORMANCE

The Palmer suggested heuristic for due date based scheduling [14] and its extension was dealt by Hundal and Rajgopal later [15]. The validation of heuristics was carried out by two stages. The first stage is the competition of schedule generated by famous heuristic say palmer heuristic. Herepalmer heuristic is used for

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primary verification of generated schedule. The palmer slopping sloping index was calculated using the standard equation. The equation for job 1 is furnished below (equation -1) and job wise sloping indexes ($S_j$) were tabulated in Table-7.

$$S_1 = (m-1)*t_{im} + (m-3)*t_{im-1} + \ldots - (m-3)\, t_{i2}-(m-1)t_{i1} \quad \text{-- (1)}$$

**Table-6.** The Job wise palmer sloping index ($S_j$).

| Job | $S_j$ |
|---|---|
| $J_1$ | 0 |
| $J_2$ | -5 |
| $J_3$ | -17 |
| $J_4$ | -9 |

The job wise palmersloping indexes are: for job1$S_1$ is 0, for job 2 $S_2 =$ -5, for job 3, $S_3 =$ -17 and for job 4, $S_4$= -9. By arranging the values of job wise palmer sloping index in descending order we get a sequence as $S_1$-$S_2$-$S_4$-$S_3$. The obtained schedule is $\{j_1, j_2, j_4, j_3\}$.The manual simulation for the schedule explained in Table-7. And it is noted that the palmer heuristic produced the same schedule which produced by heuristic 2. From Table-6, the heuristic 2 out performs than other heuristics in yielding minimum makespan.

**Table-7.** Manual Simulation of Processing of jobs with Schedule $\{j_1, j_2, j_4, j_3\}$.

| Job | $m_1$ | | $m_2$ | | $m_3$ | | $m_4$ | |
|---|---|---|---|---|---|---|---|---|
| | In | Out | In | Out | In | Out | In | Out |
| $j_1$ | 0 | 6 | 6 | 12 | 12 | 21 | 21 | 26 |
| $j_2$ | 6 | 13 | 13 | 16 | 21 | 25 | 26 | 31 |
| $j_4$ | 13 | 19 | 19 | 24 | 25 | 30 | 31 | 34 |
| $j_3$ | 19 | 27 | 27 | 29 | 30 | 33 | 34 | 36 |
| *Ideal time* | *0* | *0* | *6* | *7* | *12* | *0* | *21* | *0* |

## PERMUTATION FLOW SHOP WEIGHTED SCHEDULING PROBLEM WITH CRITICAL MACHINE CONSTRAINT

The weighted scheduling problem in a permutation flowshop environment with critical machine has an additional constraint of effectively utilizing the Critical machine by best schedule. In specific, in addition to makspan minimization the schedule must have the aim to minimize the intermediate ideal time of critical machine. An efficient schedule is one which maximizes the utilization of all resources, thereby reducing the makespan. In a recent manufacturing environment, taking machine hire or having sophisticated equipment to do special type of work etc. are considered as critical machine to maximize the profit.Pugal and Antony xavior (2014) reviewed about the critical machine in flowshop environment in which they suggested how to identify the critical machine unless otherwise specified. For this case study the second stage is critical. Here three more new heuristics were developed in which the trigonometric index calculated based on the critical machine consideration.

### Heuristic-4

**Stage-I:** The product of individual process time with a weightage value of the respective job. Hence the problem is reduced as a normal flowshop problem.

$t'_{ij} = t_{ij} * w_i$

Thus the problem is reduced as shown in Table-2.

**Stage-II:** Arrange the trigonometric function that is$t'_{MN}$ in descending order and find the trigonometric equation for n-jobs.

$$y_j = \sum_{i=1}^{i=z-1} (\cos t'_{ij})^2 + (\tan t'_{ij})^2 + \sum_{i=z+1}^{i=m} (\cos t'_{ij})^{-2}$$

Where,

$y_j$= the trigonometric slope for $j^{th}$ job

**Stage-III:**

a) Arrange the trigonometric slope ($y_j$) in descending order.
b) Now split the entire sequence into two halves (if odd number of jobs, consider one additional job in the second half.
c) Then rewrite the first half in ascending order and keep the second half as it is.
d) Now join first and second half. The resultant order is the required schedule.

### Heuristic-5

**Stage-I:** The product of individual process time with a weighted value of the respective job. Hence the problem is reduced as a normal flowshop problem.

$t'_{ij} = t_{ij} * w_i$

Thus the problem is reduced as shown in Table-2.

**Stage-II:** Arrange the trigonometric function that is $t'_{MN}$ in descending order and find the trigonometric equation for n-jobs.

$$y_j = \sum_{i=1}^{i=z-1} (\cos t'_{ij})^2 + (\tan t'_{ij})^2 + \sum_{i=z+1}^{i=m} (\cos t'_{ij})^{-2}$$

Where,

$y_j$= the trigonometric slope for $j^{th}$ job

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**Stage-III:**

a) Arrange the trigonometric slope ($y_j$) in descending order.
b) Now split the entire sequence into two half (if odd number of job, consider one additional job in the second half.
c) Then rearrange the second half trigonometric slopes ($y_j$) in ascending order and keep the first half trigonometric slopes ($y_j$) as it is.
d) Now join first and second half. The resultant order is the required schedule.

**Heuristic-6**

**Stage-I:** The product of individual process time with a weightage value of the respective job. Hence the problem is reduced as a normal flowshop problem.

$t'_{ij} = t_{ij} * w_i$

Thus the problem is reduced as shown in Table-2.

**Stage-II:** Arrange the trigonometric function that is $t'_{MN}$in descending order and find the trigonometric equation for n-jobs.

$$y_j = \sum_{i=1}^{i=z-1} (\cos t'_{ij})^2 + (\tan t'_{ij})^2 + \sum_{i=z+1}^{i=m} (\cos t'_{ij})^{-2}$$

Where,

$y_j$= the trigonometric slope for $j^{th}$ job

**Stage-III:**

a) Arrange the trigonometric slope ($y_j$) in descending order.
b) Now split the entire sequence into two half (if odd number of job, consider one additional job in the second half.
c) Then rearrange the first and second half trigonometric slopes ($y_j$) in ascending order separately.
d) Now join first and second half. The resultant order is the required schedule.

**NUMERICAL EXAMPLE-II**

The numerical example-II illustrates the execution procedures for heuristic 4 to heuristic 6. Here we consider the same case study problem where z=m/2.

**Heuristics-4**

**Stage-I:** In this stage the individual process time of the job at stage is multiplied with its weightage percentage ($t'_{ij} = t_{ij} * w_i$). The resultant value is resultant processing time. It is shown in Table-4. Hence the problem is reduced as a normal flowshop problem. Here z is the critical machine.

**Stage-II:** Computation of trigonometric index ($Y_j$), where z is the critical machine number. As per case study problemthe critical machine is machine 2. The $Y_j$values were computed and tabulated in Table-9.

**Table-8.** The Job wise trigonometric sloping index ($Y_j$).

| Job | ***Cri. M/c - m*₂*( Yⱼ)*** |
|---|---|
| $J_1$ | 219.49 |
| $J_2$ | 1186.14 |
| $J_3$ | 2.8264 |
| $J_4$ | 0.6 |

**Stage-III:**

a) Arranging the trigonometric slope ($Y_j$) in descending order as {($\boldsymbol{Y_2}$ =1186.14), ($\boldsymbol{Y_1}$= 219.49), ($\boldsymbol{Y_3}$= 2.8264) , ($Y_4$= 0.6)}
b) Now split the entire sequence into two equal half ($Y_2$, $Y_1$), ($Y_3$, $Y_4$)
c) Then rewrite the first half in ascending order {($\boldsymbol{Y_1}$= 219.49), ($\boldsymbol{Y_2}$ =1186. 14)} and keep the second half as it is.
d) Now join first and second half {$Y_1$, $Y_2$, $Y_3$, $Y_4$}. The resultant order is the required schedule {$J_1$, $J_2$, $J_3$, $J_4$}.

The manual simulation of the schedule {$J_1$, $J_2$, $J_3$, $J_4$} is illustrated in the Table 10.

**Table-9.** Manual simulation of processing of jobs with schedule {$J_1$, $J_2$, $J_3$, $J_4$}.

| Job | $m_1$ | | $m_2$ | | $m_3$ | | $m_4$ | |
|---|---|---|---|---|---|---|---|---|
| | **In** | **Out** | **In** | **Out** | **In** | **Out** | **In** | **Out** |
| $j_1$ | 0 | 6 | 6 | 12 | 12 | 21 | 21 | 25 |
| $j_2$ | 6 | 13 | 13 | 15 | 21 | 25 | 25 | 30 |
| $j_4$ | 13 | 21 | 21 | 23 | 25 | 28 | 30 | 32 |
| $j_3$ | 21 | 27 | 27 | 32 | 32 | 37 | 37 | 40 |
| *Ideal time* | 0 | 0 | 6 | 11 | 12 | 4 | 21 | 5 |

Similarly the schedules generated by the schedules can be identified by followings its simple stages. The schedules were generated by using heuristic 8 to heuristic 14 and the same were manually simulated. The results were tabulated in Table-11.

From the Table-11 it is observed that the heuristic 9 yields minimum akespan and also it should be noted that total minimum intermediate machine ideal time. In particular near the minimum ideal time for machine 2 and remaining two machines no ideal time.

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**Table-10.** Performance of heuristics.

| Heuristic | Schedule | Machine ideal time | | | | | | | | | | | Makespan |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Initial | | | | | Intermediate | | | | | Total | |
| | | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $I_1$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $I_2$ | I | $C_j$ |
| 1 | $\{j_2,j_1,j_3,j_4\}$ | 0 | 7 | 10 | 14 | 31 | 0 | 9 | 6 | 11 | 26 | 57 | 40 |
| 2 | $\{j_1,j_2,j_4,j_3\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 7 | 0 | 0 | 7 | 46 | 36 |
| 3 | $\{j_1,j_2,j_3,j_4\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 11 | 4 | 5 | 20 | 59 | 40 |

**Table-11.** The performance of critical machine based heuristics.

| Heuristic | Schedule | Machine ideal time | | | | | | | | | | | Makespan |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Initial | | | | | Intermediate | | | | | Total | |
| | | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $I_1$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $I_2$ | I | $C_j$ |
| 1 | $\{j_1,j_2,j_3,j_4\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 11 | 4 | 5 | 20 | 59 | 40 |
| 2 | $\{j_2,j_1,j_4,j_3\}$ | 0 | 7 | 10 | 14 | 31 | 0 | 6 | 5 | 9 | 20 | 51 | 38 |
| 3 | $\{j_1,j_2,j_4,j_3\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 7 | 0 | 0 | 7 | 46 | 36 |

**Table-12.** The performance of remaining possible schedules.

| Schedule | Machine ideal time | | | | | | | | | | | Makespan $C_j$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Initial | | | | | Intermediate | | | | | Total | |
| | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $I_1$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $I_2$ | $I$ | |
| $\{j_2,j_1,j_3,j_4\}$ | 0 | 7 | 10 | 14 | 31 | 0 | 9 | 6 | 11 | 26 | 57 | 40 |
| $\{j_4,j_3,j_1,j_2\}$ | 0 | 6 | 11 | 16 | 33 | 0 | 8 | 7 | 14 | 29 | 62 | 45 |
| $\{j_1,j_2,j_3,j_4\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 11 | 4 | 5 | 20 | 59 | 40 |
| $\{j_3,j_4,j_2,j_1\}$ | 0 | 8 | 10 | 13 | 31 | 0 | 9 | 11 | 19 | 39 | 70 | 47 |
| $\{j_4,j_2,j_3,j_1\}$ | 0 | 6 | 11 | 16 | 33 | 0 | 11 | 10 | 16 | 37 | 70 | 47 |
| $\{j_4,j_2,j_1,j_3\}$ | 0 | 6 | 11 | 16 | 33 | 0 | 7 | 5 | 10 | 22 | 55 | 41 |
| $\{j_4,j_1,j_3,j_2\}$ | 0 | 6 | 11 | 16 | 33 | 0 | 8 | 2 | 8 | 18 | 51 | 39 |
| $\{j_4,j_1,j_2,j_3\}$ | 0 | 6 | 11 | 16 | 33 | 0 | 7 | 2 | 11 | 20 | 53 | 42 |
| $\{j_3,j_2,j_4,j_1\}$ | 0 | 8 | 10 | 13 | 31 | 0 | 9 | 11 | 19 | 39 | 70 | 47 |
| $\{j_3,j_2,j_1,j_4\}$ | 0 | 8 | 10 | 13 | 31 | 0 | 8 | 10 | 16 | 34 | 65 | 44 |
| $\{j_3,j_1,j_4,j_2\}$ | 0 | 8 | 10 | 13 | 31 | 0 | 8 | 7 | 15 | 30 | 61 | 43 |
| $\{j_3,j_1,j_2,j_4\}$ | 0 | 8 | 10 | 13 | 31 | 0 | 8 | 7 | 14 | 29 | 60 | 42 |
| $\{j_2,j_4,j_3,j_1\}$ | 0 | 7 | 10 | 14 | 31 | 0 | 10 | 12 | 15 | 37 | 68 | 44 |
| $\{j_2,j_4,j_1,j_3\}$ | 0 | 7 | 10 | 14 | 31 | 0 | 6 | 6 | 12 | 24 | 55 | 41 |
| $\{j_2,j_3,j_4,j_1\}$ | 0 | 7 | 10 | 14 | 31 | 0 | 10 | 11 | 18 | 39 | 70 | 47 |
| $\{j_2,j_3,j_1,j_4\}$ | 0 | 7 | 10 | 14 | 31 | 0 | 9 | 10 | 15 | 34 | 65 | 44 |
| $\{j_3,j_4,j_1,j_2\}$ | 0 | 8 | 10 | 13 | 31 | 0 | 6 | 8 | 17 | 31 | 62 | 45 |
| $\{j_4,j_3,j_2,j_1\}$ | 0 | 6 | 11 | 16 | 33 | 0 | 11 | 10 | 16 | 37 | 70 | 47 |
| $\{j_3,j_4,j_2,j_1\}$ | 0 | 8 | 10 | 13 | 31 | 0 | 9 | 11 | 19 | 39 | 70 | 47 |
| $\{j_4,j_3,j_1,j_2\}$ | 0 | 6 | 11 | 16 | 33 | 0 | 8 | 7 | 14 | 29 | 62 | 45 |
| $\{j_1,j_4,j_3,j_2\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 8 | 1 | 3 | 12 | 51 | 39 |
| $\{j_1,j_4,j_2,j_3\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 7 | 0 | 1 | 8 | 47 | 37 |
| $\{j_1,j_3,j_2,j_4\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 10 | 4 | 4 | 18 | 57 | 40 |
| $\{j_1,j_3,j_4,j_2\}$ | 0 | 6 | 12 | 21 | 39 | 0 | 8 | 1 | 3 | 12 | 51 | 39 |

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**Table-13.** Validation of heuristics with various sizes of problems with various critical machines.

| Problem no. | No. of machine | No. of jobs | Critical machine (CM) | Proposed heuristic 2 | | Palmer heuristic | | Proposed heuristic 6 | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Total completion time | Idle time | Total completion time | Idle time | Total completion time | Idle time |
| **1** | 5 | 10 | 3 | 2111 | 12 | 2104 | 0 | 1953 | 15 |
| | | | 4 | | 56 | | 38 | 1810 | 19 |
| | | | 5 | | 41 | | 28 | 2117 | 0 |
| | | | 2 | | 46 | | 18 | 1700 | 27 |
| **2** | 10 | 10 | 7 | 2210 | 42 | 2203 | 35 | 2025 | 28 |
| | | | 9 | | 125 | | 89 | 2172 | 65 |
| | | | 6 | | 42 | | 28 | 1930 | 17 |
| | | | 4 | | 25 | | 32 | 2221 | 0 |
| | | | 2 | | 23 | | 14 | 2355 | 15 |
| | | | 3 | | 13 | | 0 | 2343 | 3 |
| **3** | 15 | 10 | 2 | 3640 | 19 | 3627 | 15 | 3575 | 0 |
| | | | 4 | | 88 | | 27 | 3264 | 23 |
| | | | 3 | | 42 | | 37 | 3264 | 24 |
| | | | 7 | | 95 | | 58 | 3689 | 49 |
| | | | 6 | | 107 | | 25 | 3734 | 75 |
| **4** | 5 | 20 | 5 | 3914 | 65 | 3901 | 85 | 3420 | 57 |
| | | | 3 | | 132 | | 52 | 3726 | 35 |
| | | | 2 | | 168 | | 85 | 3454 | 0 |
| | | | 4 | | 88 | | 37 | 4064 | 29 |
| **5** | 10 | 20 | 7 | 4429 | 222 | 4414 | 142 | 4337 | 135 |
| | | | 5 | | 182 | | 102 | 4216 | 73 |
| | | | 4 | | 162 | | 142 | 4314 | 119 |
| | | | 3 | | 67 | | 64 | 4410 | 52 |
| | | | 9 | | 82 | | 79 | 4367 | 67 |
| | | | 8 | | 122 | | 98 | 4390 | 78 |

## SEARCH FOR OPTIMUM SCHEDULE

The search for optimum schedule is a kind of validation of heuristic performance (Saravanan and Raju [16]. They suggested approach of validation with all possible schedules. It is an approach of locating optimal solution and validating the obtained solution with optimal. This can help to ensure about the present solution is either optimal or near optimal. Hence the remaining possible schedules were derived and they were manually simulated to search for further optimum. The results were tabulated in Table-12. From the Table-6, Table-11 and Table-12 it is observed that only schedule $\{j_1, j_2, j_4, j_3\}$ gives minimum makespan about 36 minutes. Such schedule is generated by the heuristic 2 for the permutation flow shop with weighted scheduling problems and heuristic 6 for critical machine based weighted scheduling problem in the permutation flowshop environment.

## VALIDATION OF HEURISTICS

The best performing heuristics such as heuristic 2, Palmer and heuristic 6 were considered. The illustrated case extended to various N x M problems. The heuristics were validated with and tabulated the results in Table-13. In heuristic 2 and Palmer there is no facility to consider critical machine. So the value of machine waiting time when execution of proposed schedules in the case of heuristic 2 and Palmer Heuristic. The heuristic 6 yields minimum makespan consistently and optimize the critical machine waiting time at higher levels (refer 9$^{th}$ and 10$^{th}$ column in Table-13). Table-13 illustrates the validation

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results. The number of machines varied as 5, 10 and 15 and the number of jobs varied as 10 and 20. Altogether 25 cases were examined by varying the critical machine (refer 3rd column in Table 13 is optimized.

## RESULT AND DISCUSSIONS

The validation of heuristics at various levels of problems were carried out and their results were tabulated in Table-13. The makespan performances of heuritics were depicted in Figure-1 to Figure-5. The critical machine based heuristic reduced makespan significantly (Averagely 5.30% than Heuritic 2 and 4.98% Palmer method). The ideal time reduction performances were shown in Figure-6 to Figure-10.

The results show that the critical machine based heuristic (heuristic -6) outperforms in critical machine ideal time drastically in 51.4% than Heuritic 2 and 23.29% than Palmer method. Hence it is understood that the heuristic 6 outperforms in reduction of throughput related measures like makespan, ideal time of critical machines, resource utilization and in process inventory times.

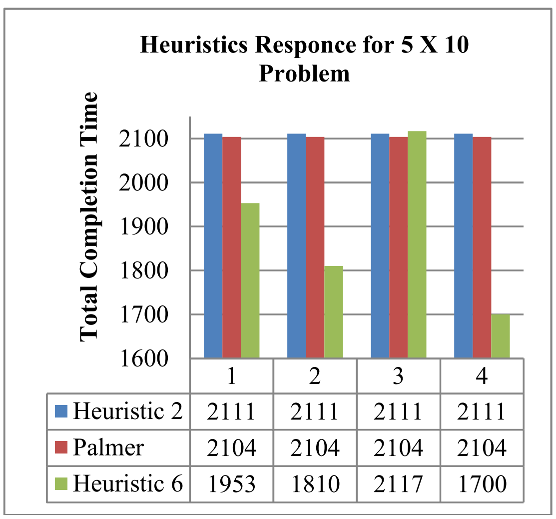


**Figure-1.** Comparison of the heuristic effect of 5 x 10 machine problem.

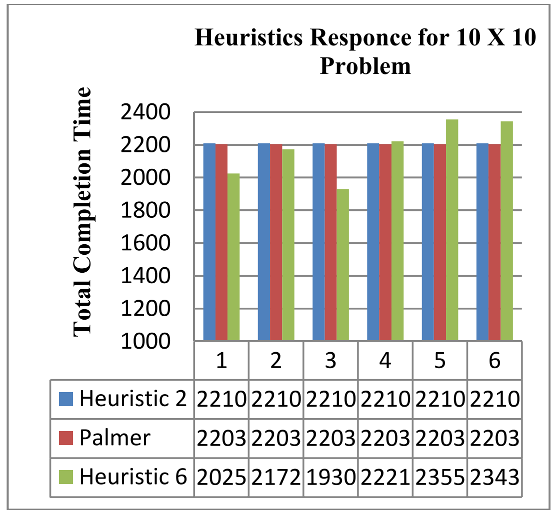


**Figure-2.** Comparison of the heuristic effect of 10 x 10 machine problem.

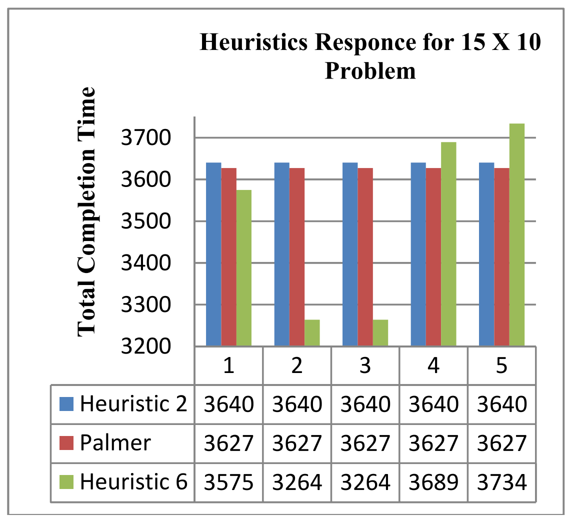


**Figure-3.** Comparison of the heuristic effect of 15 x 10 machine problem.

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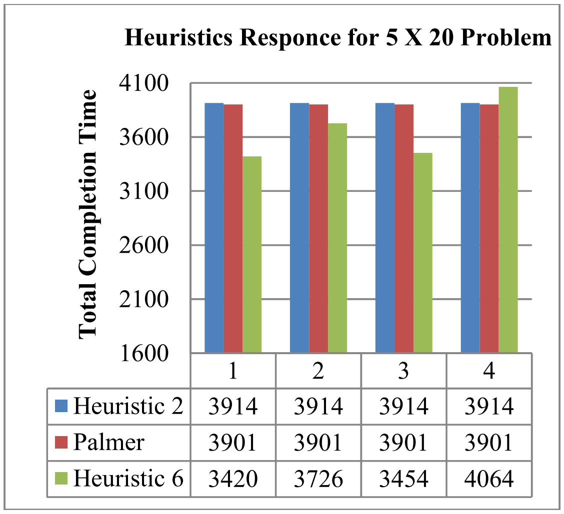


**Figure-4.** Comparison of the heuristic effect of 5 x 20 machine problem.

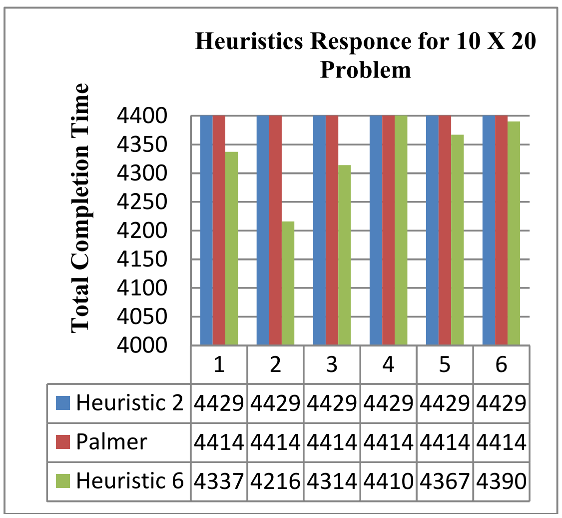


**Figure-5.** Comparison of the heuristic effect of 10 x 20 machine problem.

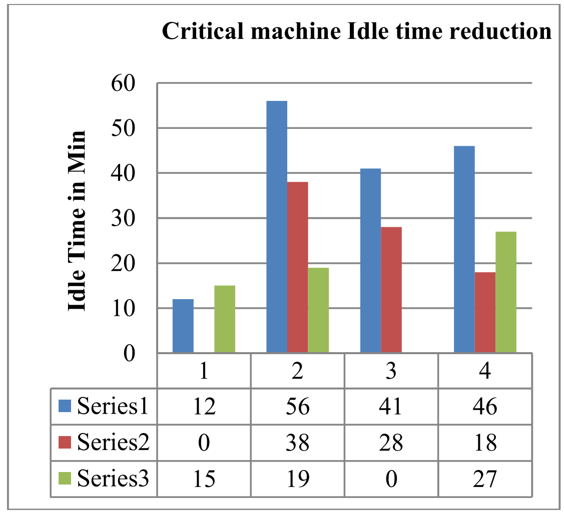


**Figure-6.** Comparison of the heuristic response to Critical machine Idle time reduction of 5 x 10 machine problem.

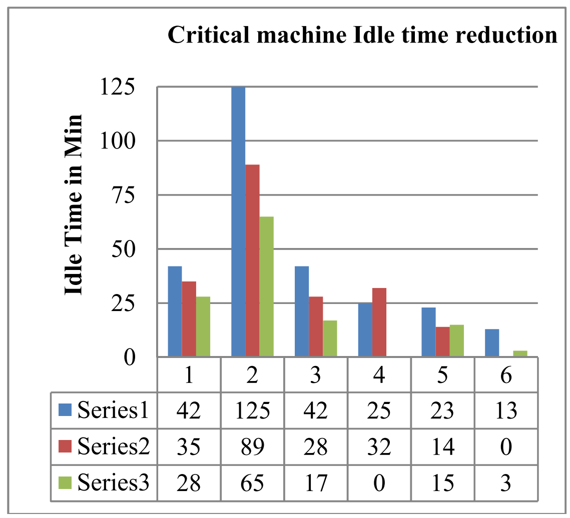


**Figure-7.** Comparison of the heuristic response to Critical machine Idle time reduction of 10 x 10 machine problem.

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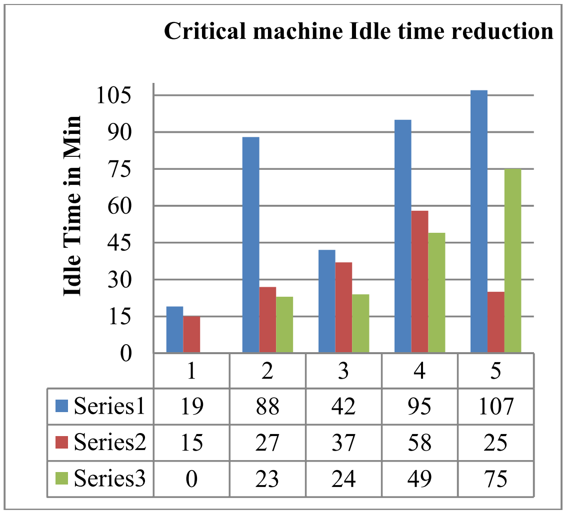
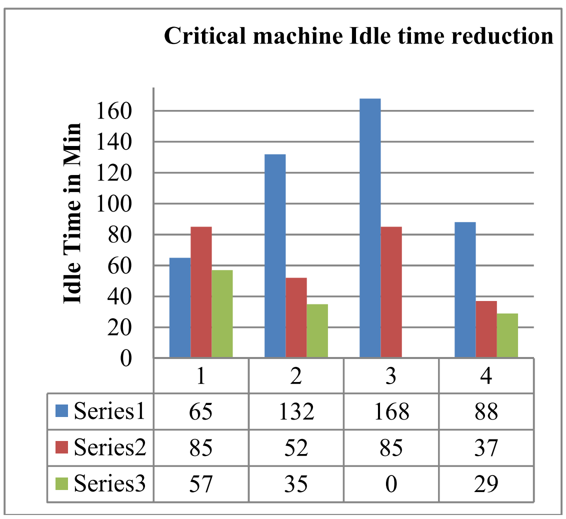


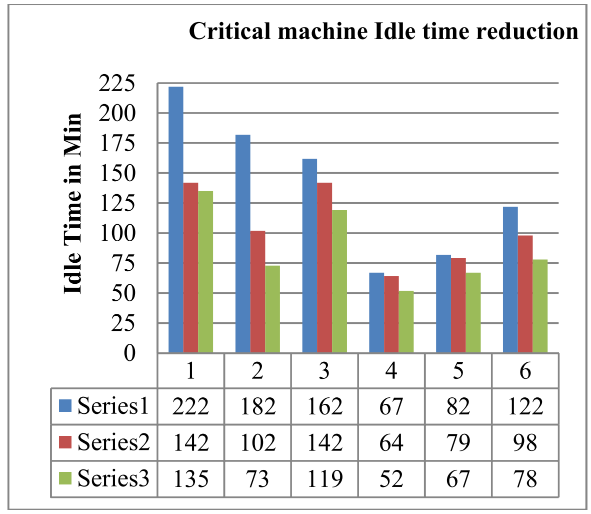

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Series1 | 19 | 88 | 42 | 95 | 107 |
| Series2 | 15 | 27 | 37 | 58 | 25 |
| Series3 | 0 | 23 | 24 | 49 | 75 |

**Figure-8.** Comparison of the heuristic response to Critical machine Idle time reduction of 15 x 10 machine problem.




| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Series1 | 65 | 132 | 168 | 88 |
| Series2 | 85 | 52 | 85 | 37 |
| Series3 | 57 | 35 | 0 | 29 |

**Figure-9.** Comparison of the heuristic response to Critical machine Idle time reduction of 5 x 20 machine problem.




| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Series1 | 222 | 182 | 162 | 67 | 82 | 122 |
| Series2 | 142 | 102 | 142 | 64 | 79 | 98 |
| Series3 | 135 | 73 | 119 | 52 | 67 | 78 |

**Figure-10.** Comparison of the heuristic response to Critical machine Idle time reduction of 10 x 20 machine problem.

## CONCLUSIONS

The weighted scheduling problem in the permutation flow type manufacturing environment was discussed with the additional criteria of critical machine consideration. Initially the problem is dealt without consideration of critical machine. Three heuristics were developed and executed. The developed schedules were manually simulated for illustration and consolidated. The heuristic 2 out performs. The heuristic 2 is validated with palmer method.

Secondly the critical machine consideration was taken into account and developed three more heuristics. The developed heuristics were experimenting with the case study problem and the same was illustrated in numerical example II. The heuristic 6 found out performed than other two heuristics, but generated same schedule as it generated by heuristic 2 and palmer heuristic. So the search for the optimum solution procedure was carried out in which all remaining possible schedules were examined and found that the heuristic generated schedule is optimum.

The heuristic 2, heuristic 6 and polymer heuristic was further validated with extended problems other than 4X4 problem. There were 5 different problem sizes with 25 types of critical machine consideration were included in this investigation. The results show that heuristic 6 out performs in case of minimizing makespan as well as minimizing critical machine ideal time. There are no considerations of critical machines in heuristic 2 and Palmer method. The ideal time reduction in critical machine helps to achieve the objective of makespan minimization. Hence it is evident that the heuristic 6 improves the resource utilization as well as in process inventory times by minimizing the makespan and critical machine ideal time.

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