A FORM GRINDING METHOD FOR MANUFACTURING THE VARIABLE TOOTH THICKNESS HOB ON CNC GEAR GRINDING MACHINE

Van-The Tran
Department of Mechanical Engineering, Hung Yen University of Technology and Education, Hung Yen City, Vietnam
E-Mail: vanct4.hut@gmail.com

ABSTRACT

The variable tooth thickness (VTT) hob is usually applied for longitudinal crowning work gear surface with twist free tooth flank. It is an important cutting tool for manufacturing the high precision helical gear. However, the manufacturing process for a VTT hob has not investigated yet. Therefore, in this study, we proposed a method for generating profile of VTT hob by using form grinding wheel on CNC grinding machine. A mathematical model for the tooth profile of VTT hob is established by setting the center distance between the grinding wheel and hob as a second order function of hob’s longitudinal feed movement. A numeral example is presented to illustrate and verify the merits of the proposed form grinding method.

Keywords: variable tooth thickness hob, grinding wheel, CNC grinding machine.

INTRODUCTION

The grinding process is the most popular finishing process because of its high accuracy, efficiency, and flexibility in tooth flank modification. Actually, two major grinding methods with high productivity for grinding cylindrical workpieces are usually used. One is a single-indexing process called form grinding method applying a form wheel (developed by Höfler, Niles, and Gleason-Pfauter) and other one is a continuous-indexing process called generating grinding method applying a worm wheel (developed by Reishauer and Gleason). The form grinding is the most suitable process for achieving flexible profile and lead modifications on the finished tooth flanks. The newly developed gear grinding machines are all Cartesian-type structures with movable axes controlled by computer numerical control (CNC). Because of state-of-the-art CNC technology, these machines can offer precise simultaneous five-axis movement that enables free-form flank modification.

The details the geometric design, tooth surface envelope theory and tooth contact analysis of various gears is published by Litvin and Fuentes [1]. Practical applications of various gears, including gear processing methods, tool design and the strength analysis based on the finite element method. In recently, Hsu and Fong [2] and Hsu and Su [3] proposed a hobbing method for anti-twist tooth flank of work gear by using variable tooth thickness (VTT). Besides, the topologies, contact ellipses and transmission errors of the double-crowned work gear pairs generation by modified hob in gear-hobbing process are investigated. Some related literatures on form grinding process are published. Yoshino et al. [4] proposed a flank correction method for form grinding through compensation of the wheel profile and the position between the wheel and the work gear. Subsequently, in extensive research, Nishida et al. [5] derived the wheel profile using an analytical method. And Kobayashi et al. [6] proposed an optimum contact-line shape by modulating the setting angle of the wheel for tooth trace modification. Then the accuracy of the gear tooth profile is estimated corresponding to the wheel setting errors [7]. You et al. [8] proposed a model that profiles the CBN shape of the grinding wheels using the geometric properties of the contact points. Radzevich [9] proposed a methodology for calculating the parameters of the plunge shaving cutter and the respective form grinding wheel from measured discrete points of the modified gear. Nishimura et al. [10] offer the ZE series for gear grinding machine that feature higher efficiency, higher precision, lower running cost, and easier operation. In 2009, Chiang et al. [11] proposed a simplified two-dimensional numerical simulation method for form grinding the thread on cylindrical workpieces without solving the simultaneous system equations that produce numerically unstable solutions in the presence of undercutting, interference, or double enveloping. And Chiang and Fong [12] proposed a tilt form grinding method and a methodology to determine the minimum tilt angle for avoiding undercutting and secondary enveloping. Profile and lead modifications are more applied in the past years to fulfill high torque load demands and low running noise [13]. After that, Wei et al. [14] established a mathematical model for calculating the processing error of the rotor tooth profile caused by the form grinding wheel profile. Lately, the free-form flank topographic correction method on a five-axis computer numerical control gear profile grinding machine proposed by Shih and Chen [15, 16].

In this paper, a method for manufacturing the VTT hob is proposed by using form grinding wheel on CNC grinding machine. A mathematical model for the tooth profile of the VTT hob is established. Numerical results show that the proposed VTT hob surface generated by form grinding wheel and theoretical VTT hob surface are close with each other. The proposed VTT hob surface is much smoother than the theoretical VTT hob surface.
MATHEMATICAL MODEL OF THE GRINDING WHEEL

Basically, the theoretical axis profile of a grinding wheel is a projection of the involute curve on traverse plane, as shown in Figure-1. Accordingly, the position vector and unit normal vector of the grinding wheel’s right-hand side profile can be expressed in coordinate system $S_g(x_g,y_g,z_g)$ as follows:

$$\mathbf{r}_g = [x_g(\theta), y_g(\theta), z_g(\theta), 1]^T,$$  \hspace{1cm} (1)

and $\mathbf{n}_g = [n_{x_g}(\theta), n_{y_g}(\theta), n_{z_g}(\theta)]^T,$  \hspace{1cm} (2)

where

$$x_g = r_g (\cos \theta - \theta \cos \alpha_p \sin(\alpha_p - \theta)),$$  \hspace{1cm} (3)

$$y_g = 0,$$  \hspace{1cm} (4)

$$z_g = r_g (\sin \theta - \theta \cos \alpha_p \cos(\alpha_p - \theta)),$$  \hspace{1cm} (5)

$$n_{x_g} = \sin(\alpha_p - \theta),$$  \hspace{1cm} (6)

$$n_{y_g} = 0,$$  \hspace{1cm} (7)

And $n_{z_g} = \cos(\alpha_p - \theta).$  \hspace{1cm} (8)

where $\theta$ is the profile parameters, $r_g$ is the radius of the pitch circle of grinding wheel, and $\alpha_p$ is the profile angle in transverse section, as shown in Figure-1.

Since the grinding wheel surface is a surface of revolution, therefore, the grinding wheel profile is used to generate the grinding wheel surface by rotating the profile defined in Eq. (1) around its axis-symmetric $z_g$. By applying the homogeneous coordinate transformation matrix equation from $S_g$ to $S_w$, the locus and unit normal vector of the grinding wheel surface can be represented in coordinate system $S_w$ as follows:

![Figure-1. Surface parameters of the grinding wheel.](image)
\[ r_w(\theta, \varphi_w) = M_{wg}(\varphi_w) \cdot r_g(\theta), \quad (9) \]

and \[ n_w(\theta, \varphi_w) = L_{wg}(\varphi_w) \cdot n_g(\theta), \quad (10) \]

The matrix \( M_{wg}(\varphi_w) \) describes the coordinate transformation from \( S_g \) to \( S_w \) and is represented as:

\[
M_{wg}(\varphi_w) = \begin{bmatrix}
\cos \varphi_w & \sin \varphi_w & 0 & 0 \\
-\sin \varphi_w & \cos \varphi_w & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad (11)
\]

and the transformation matrix \( L_{wg}(\varphi_w) \) is the sub-matrix of \( M_{wg}(\varphi_w) \) by deleting the last column and row.

After some mathematical operations, the locus of grinding wheel surface can be simplified as follows:

\[
r_w(\theta, \varphi_w) = [x_w(\theta, \varphi_w), y_w(\theta, \varphi_w), z_w(\theta, \varphi_w), 1]^T, \quad (12)
\]

and \( n_w(\theta, \varphi_w) = [n_{x_w}(\theta, \varphi_w), n_{y_w}(\theta, \varphi_w), n_{z_w}(\theta, \varphi_w), 1]^T, \quad (13)\]

where

\[
x_w = r_w(\cos \theta - \cos \alpha_p \sin(\alpha_p - \theta)) \cos \varphi_w, \quad (14)
\]

\[
y_w = r_w(-\cos \theta + \cos \alpha_p \sin(\alpha_p - \theta)) \sin \varphi_w, \quad (15)
\]

\[
z_w = r_w(\sin \theta \cos \alpha_p \cos(\alpha_p - \theta)), \quad (16)
\]

\[
n_{x_w} = \sin(\alpha_p - \theta) \cos \varphi_w, \quad (17)
\]

\[
n_{y_w} = -\sin(\alpha_p - \theta) \sin \varphi_w, \quad (18)
\]

and \( n_{z_w} = \cos(\alpha_p - \theta). \quad (19) \)

By using Eqs. (12) and (13), the 3D model of the grinding wheel surface is shown in Fig. 2.

**MATHEMATICAL MODEL OF THE VTT HOB**

**a. Mathematical model of the theoretical VTT hob**

The profile of the generating surface of the standard rack cutter is the same as that of the standard helical gear. And the theoretical VTT rack cutter profile is generated by modifying the rack cutter tooth thickness of the standard rack cutter as a second order function of the lead parameter, \( s_t = b \lambda^2 \), as shown in Figure-3(a). Accordingly, the position vector and unit normal vector of the right-hand side of the theoretical VTT rack cutter profile can be expressed in coordinate system \( S_t(x_t, y_t, z_t) \) as follows:

\[
r_t = [x_t, y_t, z_t, 1]^T = [n_t \cos \alpha_{ot}, -n_t \sin \alpha_{ot} + b \lambda^2, \lambda^2, 1]^T, \quad (20)
\]

and \( n_t = [n_{x_t}, n_{y_t}, n_{z_t}]^T = [\sin \alpha_{ot}, \cos \alpha_{ot}, -b \lambda^2 \cos \alpha_{ot}]^T, \quad (21) \)

where \( u_t \) and \( v_t \) are the surface parameters, \( \alpha_{ot} \) is the pressure angle of the standard rack cutter and \( b \) is the VTT coefficient of rack cutter, as shown in Figure-3(a).

According to Figure-3(b), by applying the homogenous coordinate transformation matrix equation, the locus and unit normal vectors of VTT rack cutter surface represented in coordinate system \( S_{ht}(x_{ht}, y_{ht}, z_{ht}) \) are attained by

\[
r_{ht}(u_t, v_t, \phi) = M_{ht}(\phi) \cdot r_t(u_t, v_t) = \begin{bmatrix}
\cos \phi & -\cos \beta_{ot} \sin \phi & -\sin \beta_{ot} \sin \phi & r_{ot}(\cos \phi + \phi \sin \phi) \\
\sin \phi & \cos \beta_{ot} \cos \phi & -\sin \beta_{ot} \cos \phi & r_{ot}(\sin \phi + \phi \cos \phi) \\
0 & -\sin \beta_{ot} & \cos \beta_{ot} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \cdot r_t(u_t, v_t), \quad (22)
\]

**Figure-2.** 3D model of the grinding wheel surface.
and \( \mathbf{n}_{ht}(u_t, v_t, \phi_t) = \mathbf{L}_{ht}(\phi_t) \cdot \mathbf{n}_i(u_t, v_t) \), \hspace{1cm} (23)

where \( \mathbf{L}_{ht}(\phi_t) \) is the upper-left \((3 \times 3)\) sub-matrix of the \((4 \times 4)\) homogeneous coordinate transformation matrix \( \mathbf{M}_{ht}(\phi_t) \).

According to the theory of gearing, the equation of meshing between the right-hand side rack cutter surface and left-hand side hob surface can be obtained by

\[
f_{ht}(u_t, v_t, \phi_t) = \frac{\partial [x_{ht}(u_t, v_t, \phi_t), y_{ht}(u_t, v_t, \phi_t), z_{ht}(u_t, v_t, \phi_t)]}{\partial \phi_t} = 0
\]

\hspace{1cm} (24)

**Figure-3.** Surface parameter of the VTT rack cutter and coordinate systems for the schematic generation of the theoretical VTT hob.

**b. Mathematical model of the proposed VTT hob surface generated by form grinding wheel**

For generating the profile of VTT hob, a 3D model of CNC form profile grinding machine is presented in Figure-4. This model has six-axis movements that need for grinding profile of VTT hob: rotational axis \( A_1 \) is applied for setting the crossed angle between the grinding wheel and hob axes, rotational axes \( B_1 \) and \( B_2 \) are applied for controlling rotation angles of the grinding wheel and hob, respectively, horizontal axis \( X_1 \) is applied for controlling the radial feed along the center distance between the grinding wheel and hob, vertical axis \( Y_1 \) is applied for setting the position of the grinding wheel and hob, axis \( Z_1 \) is applied for controlling the longitudinal feed along hob axis. The coordinate systems for grinding process of hob are shown in Figure-5, wherein coordinate systems \( S_w(x_w, y_w, z_w) \) and \( S_h(x_h, y_h, z_h) \) are rigidly connected to the frame of grinding machine, and \( S_A(x_a, y_a, z_a), S_B(x_b, y_b, z_b), S_C(x_1, y_1, z_1) \) and \( S_d(x_d, y_d, z_d) \) are auxiliary coordinate systems for simplification of coordinate transformation. The crossed angle \( \gamma_{hw} \) of the hob and work gear axes is usually a machine-tool setting. To generate the profile of VTT hob, this study proposes a grinding method by changing the center distance between the grinding wheel and hob as a second order function of hob’s longitudinal feed movement in grinding process. The center distance between the grinding wheel and hob is proposed as follows:

\[
E_t = E_o + a_t l_h^2.
\]

\hspace{1cm} (25)

where \( E_o \) is the original center distance between grinding wheel and hob, \( a_t \) is the center distance variation coefficient and \( l_h \) is the hob’s longitudinal feed movement.
The position vector \( \mathbf{r}_w(\theta, \varphi_w) \) (see Eq. (12)) and unit normal \( \mathbf{n}_w(\theta, \varphi_w) \) (see Eq. (13)) present locus of grinding wheel surface, respectively. By applying the homogeneous coordinate transformation matrix equation from \( S_w \) to \( S_h \), the locus and unit normal vector of grinding wheel surface can be represented in coordinate system \( S_h \) as follows:

\[
\mathbf{r}_w(\theta, \varphi_w, \phi_w, l_w) = \mathbf{M}_{hw}(\phi_h, l_h) \cdot \mathbf{r}_w(\theta, \varphi_w),
\]

(26)

and \( \mathbf{n}_w(\theta, \varphi_w, \phi_w, l_w) = \mathbf{L}_{hw}(\phi_h, l_h) \cdot \mathbf{n}_w(\theta, \varphi_w), \)

(27)

where

\[
\mathbf{M}_{hw}(\phi_h, l_h) = \begin{bmatrix}
\cos \phi_h & \cos \gamma_{hw} \sin \phi_h & \sin \gamma_{hw} \sin \phi_h & E \cos \phi_h \\
-\sin \phi_h & \cos \gamma_{hw} \cos \phi_h & \sin \gamma_{hw} \cos \phi_h & E \sin \phi_h \\
0 & -\sin \gamma_{hw} & \cos \gamma_{hw} & -l_h \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(28)

and the transformation matrix \( \mathbf{L}_{hw}(\phi_h, l_h) \) is the sub-matrix of \( \mathbf{M}_{hw}(\phi_h, l_h) \) by deleting the last column and row.

After some mathematical operations, the locus of grinding wheel surface can be simplified as follows:
\[ r_s(\theta, \phi, \gamma, l) = [x_s(\theta, \phi, \gamma, l), y_s(\theta, \phi, \gamma, l), z_s(\theta, \phi, \gamma, l)]^T, \]  

and \[ n_s(\theta, \phi, \gamma, l) = [n_x(\theta, \phi, \gamma, l), n_y(\theta, \phi, \gamma, l), n_z(\theta, \phi, \gamma, l)]^T, \]  

where \[ x_s = (E_i + x_u) \cos \phi_h + z_u \sin \gamma_{hw} + y_u \cos \gamma_{hw} \sin \phi_h, \]  

\[ y_s = (y_u \cos \gamma_{hw} + z_u \sin \gamma_{hw}) \cos \phi_h - (E_i + x_u) \sin \phi_h, \]  

\[ z_s = l_h + z_u \cos \gamma_{hw} - y_u \sin \gamma_{hw}, \]  

\[ n_s = n_x \cos \phi_h + (n_z \sin \gamma_{hw} + n_y \cos \gamma_{hw}) \sin \phi_h, \]  

and \[ n_s = n_y \cos \gamma_{hw} - n_z \sin \gamma_{hw}. \]  

In the gear grinding process, there are two independent kinematic parameters \( l_s \) and \( \phi_h \) that need for finish tooth flank of work gear. Therefore, there are two equations of meshing between the hob and the grinding wheel as follows:

\[ f_1(\theta, \phi, \gamma, l_h) = n_h \frac{\partial}{\partial \phi_h} \begin{bmatrix} x_h(\theta, \phi, \gamma, l_h), y_h(\theta, \phi, \gamma, l_h), z_h(\theta, \phi, \gamma, l_h) \end{bmatrix} = 0, \]  

and \[ f_2(\theta, \phi, \gamma, l_h) = n_h \frac{\partial}{\partial l_h} \begin{bmatrix} x_h(\theta, \phi, \gamma, l_h), y_h(\theta, \phi, \gamma, l_h), z_h(\theta, \phi, \gamma, l_h) \end{bmatrix} = 0, \]

The tooth surface of the hob can be defined by solving Equations (29), (30), (37) and (38), simultaneously.

**NUMERICAL EXAMPLE AND DISCUSSIONS**

The purpose of this example is to verify the merits of proposed form grinding method. The basic data of the hob and the form grinding wheel are given in Table-1. The grinding machine setup data are calculated according to the basic meshing conditions as illustrated in Ref. [1].

**Table-1. Basic data of the hob and the form grinding wheel.**

<table>
<thead>
<tr>
<th>Items</th>
<th>Value (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number teeth of hob ((N_t))</td>
<td>1</td>
</tr>
<tr>
<td>Normal module ((m_pm))</td>
<td>2</td>
</tr>
<tr>
<td>Normal circular-tooth thickness of hob ((s_pm))</td>
<td>3.141 mm</td>
</tr>
<tr>
<td>Normal pressure angle ((\alpha_pm))</td>
<td>20.0°</td>
</tr>
<tr>
<td>Helix angle of hob ((\beta_pm))</td>
<td>87.8° R.H.</td>
</tr>
<tr>
<td>Hob pitch radius ((r_{pm}))</td>
<td>26.05 mm</td>
</tr>
<tr>
<td>Hob form radius ((r_f))</td>
<td>24.21 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Items</th>
<th>Value (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hob outer radius (r_{gl})</td>
<td>28.05 mm</td>
</tr>
<tr>
<td>Hob length ((F_{wl}))</td>
<td>135 mm</td>
</tr>
<tr>
<td>Grinding wheel radius (r_{og})</td>
<td>41.0 mm</td>
</tr>
<tr>
<td>Center distance between grinding wheel and hob ((E_o))</td>
<td>87.05 mm</td>
</tr>
<tr>
<td>Crossed angle between grinding wheel and hob (\gamma_{hw})</td>
<td>87.8°</td>
</tr>
<tr>
<td>Center distance variation coefficient ((a))</td>
<td>1.34×10⁻⁵</td>
</tr>
<tr>
<td>VTT coefficient ((b))</td>
<td>1.0×10⁻⁵</td>
</tr>
</tbody>
</table>

The Topography of the theoretical VTT hob surface and proposed VTT hob surface generated by form grinding wheel are shown in Figures 6-7, respectively. The maximum deviation of the tooth flank of the proposed VTT hob surface, \(M_d = 10.23 \mu m\), is approximately equal to that of the theoretical VTT hob surface, \(M_d = 10.0 \mu m\). And the ratio of deviation of the proposed VTT hob surface, \(R_d = 0.66\), is also approximately equal to that of the theoretical VTT hob surface, \(R_d = 0.67\). It reveals that the proposed VTT hob surface generated by form grinding wheel fits well with the theoretical VTT hob surface. In addition, the proposed VTT hob surface is much smoother than the theoretical VTT hob surface, as shown Figure-7.
CONCLUSIONS

In this study, a new form grinding method for manufacturing the VTT hob is proposed by setting the center distance between the grinding wheel and hob as a second order function of hob’s feed movement (Eq. (25)). Mathematical models for the tooth profile of the theoretical VTT hob and proposed VTT hob are established. The compared results show that the proposed VTT hob surface generated by form grinding wheel fits well with the theoretical VTT hob surface. The proposed VTT hob surface is much smoother than the theoretical VTT hob surface.

REFERENCES


