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INTEGRAL BACKSTEPPING CONTROLLER FOR AN UNDERACTUATED X4-AUV

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ABSTRACT

The autonomous underwater vehicle (AUV) mostly has fewer control inputs than the degree of freedoms (DOFs) in motion and be classified into underactuated system. It is difficult tasks to stabilize that system because of the highly nonlinear dynamic and model uncertainties. Hence, it usually required nonlinear control method and this paper presents the stabilization of an underactuated X4-AUV using integral backstepping control method. The X4-AUV system is executed by separating system into two parts subsystem which is translational and rotational subsystems. Integral backstepping control is applied for translational and rotational subsystem. The effectiveness of the proposed control technique for an underactuated X4-AUV demonstrates through simulation.

Keywords: integral backstepping, underactuated system, X4-AUV.

INTRODUCTION

Underwater robotics is an important research area due to its great applications: i.e., from a scientific research of ocean, surveillance, inspection of commercial undersea facilities, military operations and many more. Nevertheless, controlling such system is a challenging task because the dynamic model has nonlinearity and uncertain external disturbances besides difficulties in hydrodynamic modelling. Thus, it attracted further research and attention correlate with underactuated AUV, defined as the system with a fewer number of control inputs than a number of DOF and generally falls in nonholonomic systems.

Control of nonholonomic systems is theoretically challenging and practically interesting. Brockett's Theorem (Brocket, 1983) defined those systems cannot be stabilized to a point with pure smooth (or even continuous) state feedback control, usual smooth and time invariant. A stabilization problem consists of designing control law which guarantees equilibrium of a closed loop system is asymptotically stable or at least locally asymptotically stable. Therefore, control problems for underactuated systems usually required nonlinear control techniques. There are numerous nonlinear control techniques can be applied for controller and backstepping approach has gained the attention recently.

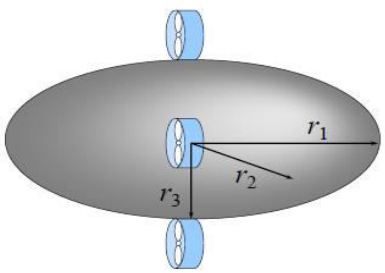


Figure-1. X4-AUV with an ellipsoidal hull shape.

A model of X4-AUV with six DOFs and four control input (thrusters) is presented. It categorized in underactuated AUV and has equations of the motion appear as second-order nonholonomic constraints. Zain (Zain *et al.*, 2010) proposed an X4-AUV with an ellipsoidal hull shape as shown in Figure-1. The slender body of ellipsoidal hull shape make it works efficiently than conventional X4-AUV (Okamura, 2009) in term of drag pressure.

This study proposed an integral backstepping control strategy to stabilize position and angles of an understated X4-AUV. The main idea in integral backstepping controller design is adding the integral of tracking error between the original system input and the input to design as a first step of control strategies. The proposed technique widely applied and effective in stabilizing the quadrotor helicopters which also generally falls into underactuated system (Tahar *et al.*, 2011), (Bouabdallah and Siegwart, 2007).

In this paper, an integral backstepping is applied to stabilize an underactuated X4-AUV with four thrusters and six DOFs in motion. The X4-AUV is executed by nonlinear control strategy with separate into two parts subsystem: i.e., translational and rotational subsystems. The controller for translational subsystem stabilizes the position and for rotational subsystem achieves the desired roll, pitch, and yaw angles.

COORDINATE SYSTEM

A special reference frame must establish in order to describe the motion of the underwater vehicle. There are two coordinate systems: i.e., an inertial coordinate system (or fixed coordinate system) and motions coordinate system (or body-fixed coordinate system). The coordinate frame $\{E\}$ is composed of the orthogonal axes $\{E_xE_yE_z\}$ and is called as an inertial frame. This frame is

commonly placed at a fixed place on Earth. The axes E_x and E_y form a horizontal plane, and E_z is the direction of the field of gravity. The body-fixed frame $\{B\}$ is composed of the orthogonal axes $\{X, Y, Z\}$ and is attached to the vehicle. The body axes, two of which coincide with the principle axes of inertia of the vehicles are defined by Fossen (Fossen and Sagatun, 1991) as follows: X is the longitudinal axis (directed from aft to fore); Y is the transverse axis (directed to starboard); Z is the normal axis (directed from top to bottom). Figure-2 shows the coordinate systems of an AUV, which consist of a right-hand inertial frame $\{E\}$ in which the downward vertical direction is to be positive, and a right-hand body frame $\{B\}$.

Letting $\boldsymbol{\xi} = [x \quad y \quad z]^T$ denote the centre of mass of the body in the inertial frame, and defining the rotational angles of the X, Y, and Z axes as $\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]$, the rotational matrix R from the body frame $\{B\}$ to the inertial frame $\{E\}$ is reduced as:

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{1}$$

where $c\alpha$ denotes cos α and $s\alpha$ is sin α.

X4-AUV DYNAMIC MODEL

Following a Lagrangian method, this section describes the dynamic model of the X4-AUV with the assumption of balance between buoyancy and gravity. The kinetic energy formula is:

$$T = T_{trans} + T_{rot} \tag{2}$$

where T_{trans} and T_{rot} are the translational kinetic energy and the rotational kinetic energy is defined by:

$$T_{trans} = \frac{1}{2}\dot{\boldsymbol{\xi}}^T M \dot{\boldsymbol{\xi}} \tag{3}$$

$$T_{rot} = \frac{1}{2}\dot{\boldsymbol{\eta}}^T J \dot{\boldsymbol{\eta}} \tag{4}$$

in which M is the total mass matrix of the body, and J is the total inertia matrix of the body. From the characteristics of added mass, it can be written as:

$$M = \mathrm{diag}(m_1, m_2, m_3) = m_b I + M_f \tag{5}$$

$$J = \mathrm{diag}(I_x, I_y, I_z) = J_b + J_f \tag{6}$$

Here, m_b is a mass of the vehicle, J_b is an inertia matrix of the vehicle and I is a 3×3 identity matrix.

Letting ρ denote a density of the fluid and using the formulation of the added mass and inertia under the assumption of $r_1 = 5r_2$ and $r_2 = r_3 = r$, where r_1, r_2 and r_3 the added mass matrix M_f and the added inertia matrix J_f are:

$$M_f = \mathrm{diag}(0.394\rho\pi r^3, 5.96\rho\pi r^3, 5.96\rho\pi r^3) \tag{7}$$

$$J_f = \mathrm{diag}\,(0, 24.2648\rho\pi r^5, 24.2648\rho\pi r^5) \tag{8}$$

From the assumption of the balance between the buoyancy and the gravity, i.e., the potential energy $U = 0$, the Lagrangian can be written as:

$$\begin{aligned} L &= T - U \\ &= T_{trans} + T_{rot} \end{aligned} \tag{9}$$

The dynamic model of X4-AUV summarized as:

$$\begin{aligned} m_1\ddot{x} &= \cos\theta\cos\psi u_1 \\ m_2\ddot{y} &= \cos\theta\sin\psi u_1 \\ m_3\ddot{z} &= -\sin\theta u_1 \\ I_x\ddot{\phi} &= \dot{\theta}\dot{\psi}(I_y - I_Z) + u_2 \\ I_y\ddot{\theta} &= \dot{\phi}\dot{\psi}(I_z - I_x) - J_t\dot{\psi}\Omega + lu_3 \\ I_z\ddot{\psi} &= \dot{\phi}\dot{\theta}(I_x - I_y) - J_t\dot{\theta}\Omega + lu_4 \end{aligned} \tag{10}$$

Where u_1, u_2, u_3, and u_4 are the control inputs for the translational (x, y, and z-axis) motion, the roll (ϕ-axis) motion, the pitch (θ-axis) motion, and yaw (ψ-axis) motion, respectively. A detailed derivation for dynamics model (10) given in (Zain *et al.*, 2010).

Defining that b is a thrust factor, d is a drag factor, taken from $\tau_{Mi} = d\omega_i^2$ then Ω, u_1, u_2, u_3, and u_4 are given by:

$$\begin{aligned} \Omega &= (\omega_2 + \omega_4 - \omega_1 - \omega_3) \\ u_1 &= f_1 + f_2 + f_3 + f_4 \\ &= b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ u_2 &= d(-\omega_2^2 - \omega_4^2 + \omega_1^2 + \omega_3^2) \\ u_3 &= f_1 - f_3 = b(\omega_1^2 - \omega_3^2) \\ u_4 &= f_2 - f_4 = b(\omega_2^2 - \omega_4^2) \end{aligned} \tag{11}$$

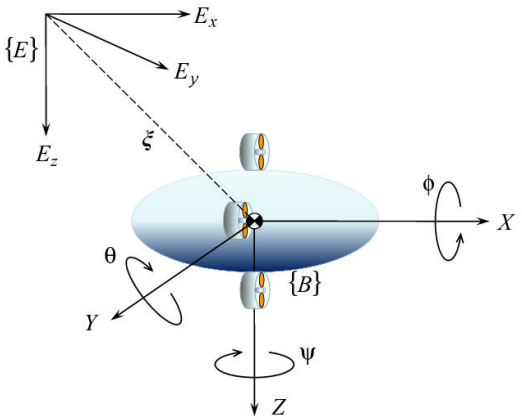


Figure-2. Coordinate systems of AUV.

The dynamic model (10) can be rewritten in a state-space form $\dot{X} = f(X,U)$ by introducing $X = (x_1 \cdots x_{12})^T \epsilon \Re^{12}$ as state vector of the system as follows:

$$f(X,U) = \begin{pmatrix} x_2 \\ \cos\theta\cos\psi \dfrac{1}{m_1} u_1 \\ x_4 \\ u_y \dfrac{1}{m_2} u_1 \\ x_6 \\ u_z \dfrac{1}{m_3} u_1 \\ x_8 \\ x_{10} x_{12} a_1 + b_1 u_2 \\ x_{10} \\ x_8 x_{12} a_2 - a_3 x_{12} \Omega + b_2 u_3 \\ x_{12} \\ x_8 x_{10} a_5 + a_4 x_{10} \Omega + b_3 u_4 \end{pmatrix} \tag{12}$$

with,

$$a_1 = \frac{I_y - I_z}{I_x}, a_2 = \frac{I_z - I_x}{I_y}, a_3 = \frac{J_t}{I_y}, a_4 = \frac{J_t}{I_z}, a_5 = \frac{I_x - I_y}{I_z},$$

$$b_1 = \frac{1}{I_x}, b_2 = \frac{l}{I_y}, b_3 = \frac{l}{I_z}, u_y = \cos\theta\sin\psi, u_z = -\sin\theta$$

INTEGRAL BACKSTEPPING CONTROLLER DESIGN

The rotational system and its derivatives do not rely on the translational system; however the translations rely on the rotational system. The complete system described by equation. (12) composed of two subsystems, the angular rotations and the linear translations as shown in Figure-3.

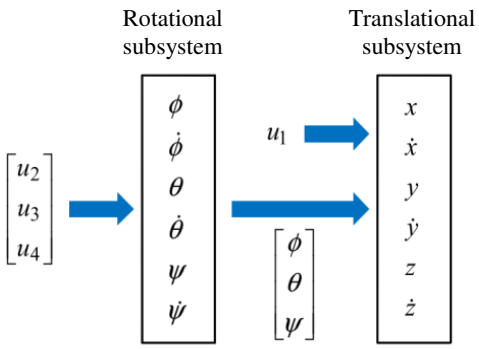


Figure-3. Connection of rotational and translational subsystem.

Rotational control

The control of rotational subsystem is considered first due to its complete independence compare than translational. For the first step, consider the roll tracking error $e_1 = \phi_d - \phi$ and its dynamics:

$$\dot{e}_1 = \dot{\phi}_d - \omega_x \tag{13}$$

The angular speed ω_x is not an input and has its own dynamics. So, set desired behaviour and consider it as virtual control.

$$\omega_{xd} = c_1 e_1 + \dot{\phi}_d + \lambda_1 X_1 \tag{14}$$

With c_1 and λ_1 a positive constant and $X_1 = \int_0^t e_1(\tau) d_t$ isan integral of the roll tracking error.

Since ω_x has its own error e_2, compute its dynamic equation. (14) as follows:

$$\dot{e}_2 = c_1(\dot{\phi}_d - \omega_x) + \ddot{\phi}_d + \lambda_1 e_1 - \ddot{\phi} \tag{15}$$

where the angular velocity tracking error, e_2 defined by:

$$e_2 = \omega_{xd} - \omega_x \tag{16}$$

Using equation. (13) and equation. (15), rewrite the roll tracking error dynamics as:

$$\dot{e}_1 = -c_1 e_1 - \lambda_1 X_1 + e_2 \tag{17}$$

By substitute $\ddot{\phi}$ in equation. (14) by its corresponding expression model equation. (9), control input u_2 appears as follows:

$$\dot{e}_2 = c_1(\dot{\phi}_d - \omega_x) + \ddot{\phi}_d + \lambda_1 e_1 - x_{10}x_{12}\alpha_1 + b_1 u_2 \quad (18)$$

The desirable dynamics for the angular speed tracking error is:

$$\dot{e}_2 = -c_2 e_2 - e_1 \quad (19)$$

By combining equation. (12) and dynamics model equation. (10), control input u_2 given by:

$$u_2 = \frac{1}{b_1}(1 - c_1^2 + \lambda_1)e_1 + (c_1 + c_2)e_2 - c_1\lambda_1\chi_1 \quad (20)$$

wherec_2 is a positive constant which determine the convergence speed of the angular speed loop.

Similarly, the same steps are followed to extract $u_3$andu_4

$$u_3 = \frac{1}{b_2}(1 - c_3^2 + \lambda_2)e_3 + (c_3 + c_4)e_4 - c_3\lambda_2\chi_2 + \ddot{\theta}_d - x_9 x_{12}\alpha_2 - \alpha_3 x_{12}\Omega \quad (21)$$

$$u_4 = \frac{1}{b_3}(1 - c_5^2 + \lambda_3)e_5 + (c_5 + c_6)e_6 - c_5\lambda_3\chi_3 + \ddot{\psi}_d - x_9 x_{10}\alpha_5 - \alpha_4 x_{10}\Omega \quad (22)$$

with:

$$\begin{cases} e_3 = \theta_d - \theta \\ e_4 = c_2 e_2 + \dot{\theta}_d + \lambda_2 X_2 - \dot{\theta} \\ e_5 = \psi_d - \psi \\ e_6 = c_3 e_3 + \dot{\psi}_d + \lambda_3 X_3 - \dot{\psi} \end{cases} \quad (23)$$

where$(c_3, c_4, c_5, c_6, \lambda_2, \lambda_3) > 0$ andχ_2, χ_3 is an integral of tracking error of e_2 and e_3.

Translational Control

The altitude control keeps the X4-AUV stabilized in desired point. By used same approach described in subsection rotational control, the control law for altitude controller is:

$$u_1 = \frac{1}{\cos\theta\cos\psi}(1 - c_7^2 + \lambda_4)e_7 + (c_7 + c_9)e_9 - c_7\lambda_4\chi_4 \quad (24)$$

with:

$$\begin{cases} e_7 = x_d - x \\ e_8 = c_4 e_4 + \dot{x}_d + \lambda_4 X_4 - \dot{x} \end{cases} \quad (25)$$

where c_7,c_8 and λ_4 is a positive constant while χ_4 is an integral of tracking error for x-position.

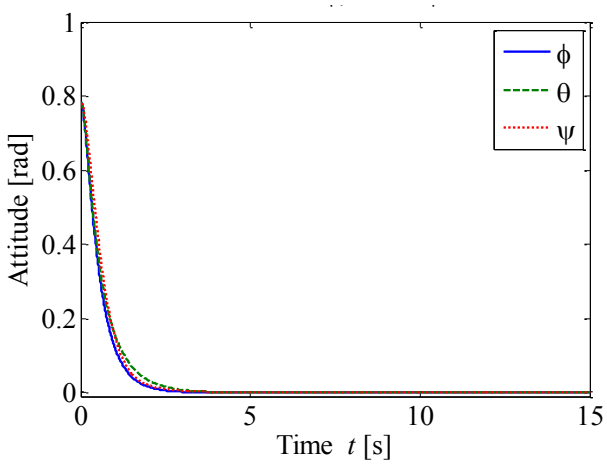
RESULTS AND DISCUSSIONS

In order to stabilize the position and angles of the X4-AUV, a nonlinear control strategy; integral backstepping is implemented. The simulation is conducted to verify the proposed control method by using u_1, u_2, u_3andu_4 respectively as control input. The system started with an initial state $X_0 = \left(0, 0, 0, 0, 0, 0, \frac{\pi}{4}, 0, \frac{\pi}{4}, 0, \frac{\pi}{4}, 0\right)^T$ and desired value x–position is setting at 3m with all orientation angles is zero. Parameters used as follows: $c_1 = 8$, $c_2 = 2$, $c_3 = 8$, $c_4 = 2$, $c_5 = 4$, $c_6 = 2$, $c_6 = 2$, $c_7 = 3$, $c_8 = 1$ and $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, $\lambda_3 = 0.5$, $\lambda_4 = 0.5$. Note that this integral backstepping technique widely used for Unmanned Aerial Vehicles (UAV) studied in [4-5]. The physical parameters shown in Table-1 were used for simulating X4-AUV.

Rotational controller responsible for stabilizing and maintaining all angles (ϕ, θ and ψ) to zero during X4-AUV cruising. Figure-4 show the response of integral backstepping controller succeeded in stabilizing roll, pitch and yaw angles of X4-AUV less than 5s. The position control and rate response of X4-AUV in Figure-5 indicate the controller effectively stabilized x-position at desired point 3m. Figure-6 illustrate the rotor speed response and inputs for controlling X4-AUV using an integral backstepping controller where u_1, u_2, u_3andu_4 denote command signal for a position and angles. Note that this simulation only stabilize x-position and angles (ϕ, θ and ψ).

Table-1. Physical parameters for X4-AUV.

Parameter	Description	Value	Unit
m_b	**Mass**	**21.43**	kg
ρ	Fluid density	1023.0	kg/m^3
l	Distance	0.1	m
r	Radius	0.1	m
b	Thrust factor	0.068	N·s^2
d	Drag factor	3.617e^{-4}	N·m·s^{-2}
J_{bx}	Roll inertia	0.0857	kg·m^2
J_{by}	Pitch inertia	1.1143	kg·m^2
J_{bz}	Yaw inertia	1.1143	kg·m^2
J_t	Thruster inertia	1.1941e^{-4}	N·m·s^{-2}



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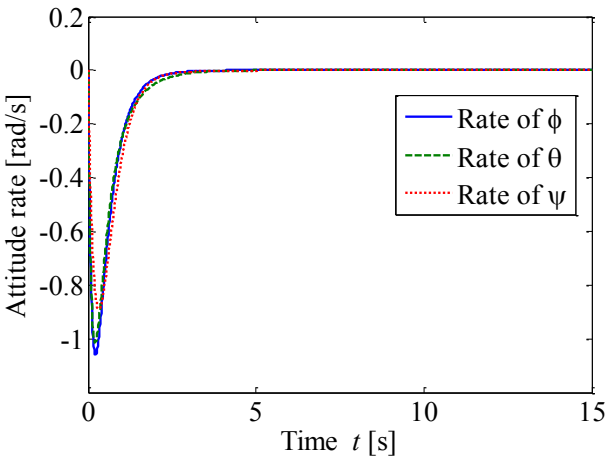
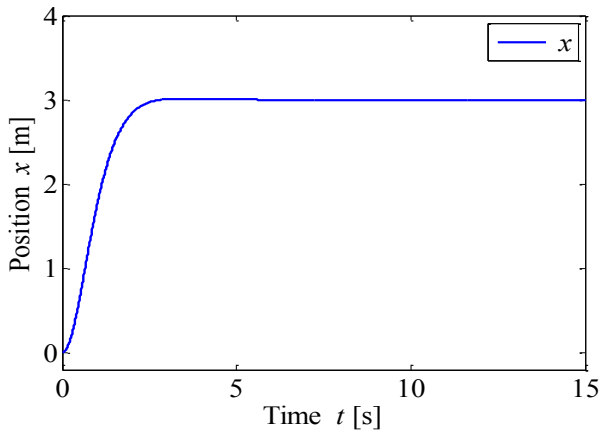


(b)

Figure-4. (a)Attitude control (b) Attitude rate control.



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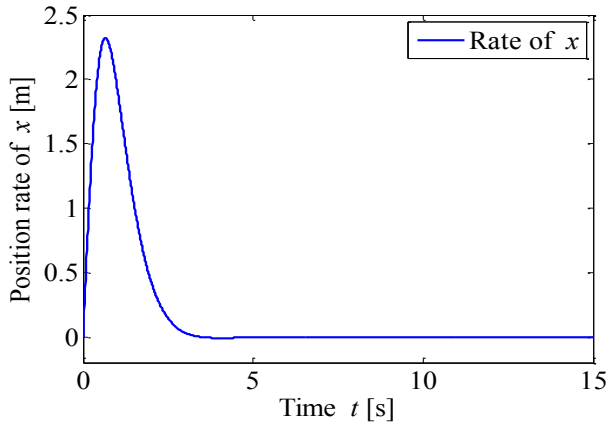
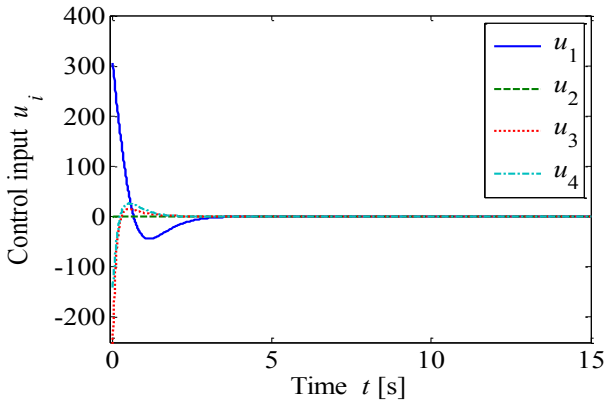
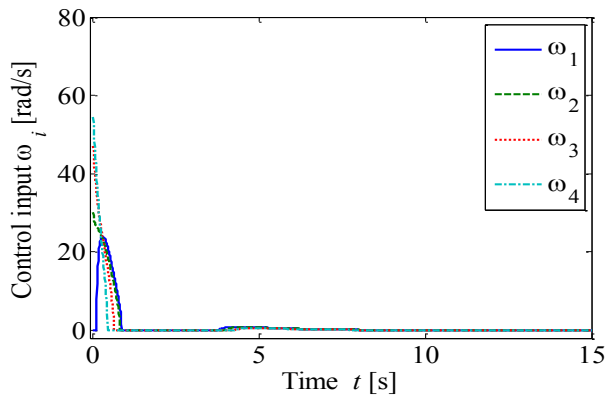


(b)

Figure-5. (a)Position control (b) position rate control.



(a)



(b)

Figure-6. (a)A control inputs (b) Control inputs in rotations.

CONCLUSIONS

This article presented an integral backstepping control method in stabilizing attitudes and *x*-position for an underactuated X4-AUV with four inputs and six DOFs. The controller designs are executed by separating the system into two parts subsystem which is translational and rotational subsystems. Integral backstepping control is

applied for translational and rotational subsystem. Simulation results demostrate effectiveness of the proposed control method in stabilizing *x*-position and all attitude into desired point.

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