



## FEKF ESTIMATION FOR MOBILE ROBOT LOCALIZATION AND MAPPING CONSIDERING NOISE DIVERGENCE

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### ABSTRACT

This paper proposed an approach of Fuzzy-Extended Kalman Filter (FEKF) for mobile robot localization and mapping considering unknown noise characteristics. The techniques apply the information extracted from EKF measurement innovation to derive the best output for mobile robot estimation during its observations. This information is then fuzzified using Fuzzy Logic technique, designed with very few design rules to control the information. The method can further reduced measurement error and as a result provides better localization and mapping. Simulation results are also presented to describe the efficiency of the proposed method in comparison with the normal EKF estimation. Preliminary results emphasize that FEKF has exceeds the estimation results performance of normal EKF in non-Gaussian noise environment.

**Keywords:** fuzzy logic, kalman filter, mobile robot localization, mapping.

### INTRODUCTION

Mobile robot localization and mapping problem or known as the Simultaneous Localization and Mapping (SLAM) problem addressed a condition where a mobile robot attempts to infer its position relatively to any observed landmarks and then concurrently build a map based on what it has measured (Ahmad *et al.*, 2013, 2015). The problem has different types of solution categories including the mathematical analysis, behavioral techniques or the probabilistics approaches (Thrun *et al.*, 2000, 2005). One of the famously used method is the probabilistics as it offers easier modeling and has less computational cost.

Extended Kalman Filter (EKF) is the mostly applied approach to deal with the SLAM problem especially when uncertainties such as the mobile robot kinematic model, sensor errors are concerned. However, it has shortcomings that could not effectively tolerate in a condition where a non-Gaussian noise characteristic is available. Due to this disadvantage, researcher explores other possible solution such as the Particle Filter, Graph-SLAM, Topological SLAM and others, but each of them is facing the computational cost. Besides, these methods cannot be fully realized in real-time application as what EKF is capable of. Hence, EKF is still celebrated as the ultimate selection in real SLAM application.

Observing an environment with a sensors or sensor-array with unknown surface and mobile robot motions requires a good modeling to represent the uncertainties. To aid the system reliability, Kramer *et al* (Kramer and Kandel, 2011) investigates four techniques which includes the FEKF to identify their strength and weaknesses on different situations for mobile robot localization problem. It was found that FEKF has better results than EKF and can be further improved if better rules designs are provided. In fact, fuzzy logic is the only recognized method to be successfully adopted by EKF (Asadian *et al.*, 2005). FEKF unlike others such as the

neural network technologies which requires less computations. Works on the mobile robot with FEKF was also successfully demonstrated by Raimondi *et al.* (Raimondi *et al.*, 2006) to control the disturbance during the mobile robot motions.

Most of the approaches and study in FEKF have focused on the state covariance,  $P$ , the process noise covariance  $Q$  and measurement noise covariance,  $R$  (Kobayashi *et al.*, 1998), (Abdelnour *et al.*, 2003). This is motivated by the means that above parameters are showing the amount of uncertainties exists during mobile robot measurements. These approaches attempts to tune the system output based on  $P$ ,  $Q$  and  $R$  covariances to obtain smaller error. The study on FEKF that considered the inputs from innovation and past information to deal with uncertain noises has also been carried. Yet the output is only based on a singleton decisions which may accidentally neglects some important information (Ip *et al.*, 2010). Wang *et al* (Wang *et al.*, 2014) examines further the fuzzy logic competencies in EKF by taking into account the error of angle, distance and innovations as the inputs to lower the state covariance update,  $P$  in each process. However they did not clearly indicates about the noise characteristics being considered. Interestingly, bringing three parameters to be calculated simultaneously during observations leads to higher processing time and complexity. Moreover, if more rules are designed for the system, it will require more time.

Motivated by the above conditions, this paper deals with FEKF to design and control the measurement innovation information in achieving better estimation results. The technique also look into the estimation results in a non-Gaussian environment to analyze the system reliability. The designed rules are also few to reduce the processing time where only three number of fuzzy sets for each input are recognized for analysis.



The remaining of this paper is organized as follows. Next section describes the mathematical formulations of normal EKF and Fuzzy Logic design. The explanation will also include the methodology of FEKF technique that integrates both methods in estimating the mobile robot and its surrounding landmarks. This is then followed by the simulation and analysis section. Finally, the paper is summarized.

## MATHEMATICAL FORMULATIONS

### EKF algorithm

SLAM consists of two distinguish models which are the process model that calculates the kinematic movement or mobile robot motions, and the measurement model that measures the relative angle and distance between mobile robot and any landmarks observed. The general model of both stages are presented as follows.

#### Process model

$$\begin{aligned} x_{k+1} &= x_k + (v_k + \delta v)T \cos \theta_k \\ y_{k+1} &= y_k + (v_k + \delta v)T \sin \theta_k \\ \theta_{k+1} &= \theta_k + f(\omega_k, v_k + \delta \omega + \delta v) \\ L_{k+1} &= L_k \end{aligned} \quad (1)$$

where  $\theta$  is the mobile robot pose angle,  $\omega, v$  are the mobile robot turning rate, velocity respectively.  $x, y, L$  ( $x_i, y_i$ ) are the mobile robot  $x, y$  positions and landmarks location respectively.  $T$  defines the sampling time. Note that the landmarks are point landmarks and is stationary at all time during mobile robot observations.

#### The measurement model

$$z_i = \begin{bmatrix} r_i \\ \phi_i \end{bmatrix} = \begin{bmatrix} \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2} + v_r \\ \arctan \frac{y_i - y_k}{x_i - x_k} - \phi_{k+1} + v_\phi \end{bmatrix} \quad (2)$$

$$= H_i X_{k+1} + v$$

$r, \phi$  = relative distance and angle between mobile robot and landmarks,

$v_r, v_\phi$  = associated noise to both distance and angle measurements.

EKF has a prediction and an update stages as shown below.

#### Predicted stage

$$\hat{X}_{k+1} = f(\hat{X}_k, \omega_k, v_k, 0, 0) \quad (3)$$

$$P_{k+1|k} = f P_{k|k} f^T + g \Sigma_{k|k} g^T \quad (4)$$

$f$  = jacobian of mobile robot motion,  
 $\Sigma$  = control noise covariance,  
 $g$  = jacobian of the control noise,  
 $P$  = state covariance.

#### Update stage

$$P_{k+1|k+1} = P_{k+1|k} - K H_i P_{k+1|k} \quad (5)$$

$$K = P_{k+1|k} H_i^T (H_i P_{k+1|k} H_i^T + R)^{-1} \quad (6)$$

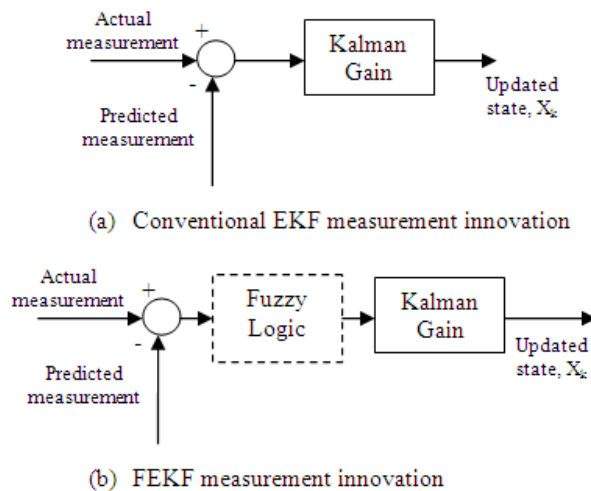
$$\hat{X}_{k+1|k+1} = f \hat{X}_{k+1|k} + K H_i (X_{k+1|k} - \hat{X}_{k+1|k}) \quad (7)$$

$K$  = Kalman gain of EKF.

Being the research objectives, the noises are assumed to be non-Gaussian noise that holds the following expression.

$$\begin{bmatrix} w_k & 0 \\ 0 & v_k \end{bmatrix} \begin{bmatrix} w_k & 0 \\ 0 & v_k \end{bmatrix}^T = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix} \quad (8)$$

Remark that  $Q_k \geq 0, R_k > 0$  are both the process and measurement noise covariances. Our interest is in equation (7), where the updated state is being calculated based on the previous state, kalman gain and the innovation. The equation inherently expressed that the innovation or the measurement obtained by the mobile robot is important to infer the updated states. If Kalman gain has smaller changes or similar from time  $k$  to  $k+1$  as well as the innovation, then the estimation error is decreasing. These characteristics are what actually this research is suggesting i.e to improve the estimation performances by using Fuzzy Logic directly to the measurement innovation. The concept of design is presented in the following Figure-1.



**Figure-1.** Proposed method of FEKF for mobile robot localization and mapping.

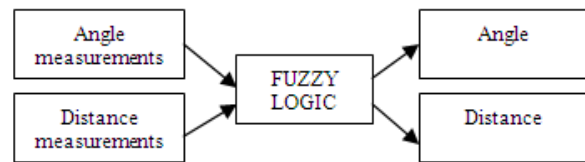
#### Fuzzy logic design

The measurement innovation will be referred as the main reference in designing the Fuzzy logic. The inputs to the Fuzzy logic are the angle and distance errors. The outputs are also the same as the Fuzzy logic tend to decrease the error of both parameters affected by the unknown measurement noise. By choosing the output appropriately, the effects or measurement error due to sensor inaccuracies can be minimized further.

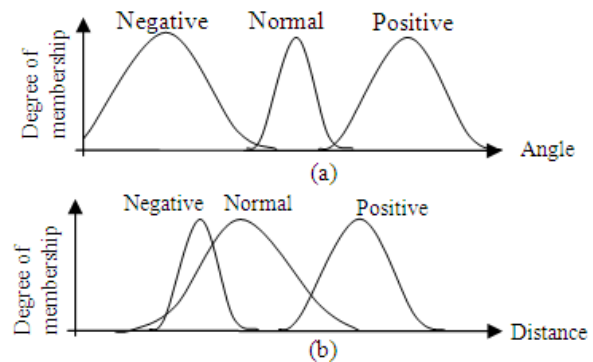
To describe in details, again refer to equation (7). Normally,  $K$  intend to make the error smaller and its calculation is depends to the measurement matrix that defines the effectiveness of measurement. If the mobile robot is stationary and observing a specific landmark many times, then it has been proven that as long as its exteroceptive sensors are working well, the measurement will yield smaller error (Huang *et al.*, 2007). In a non-Gaussian noise, these properties are still not investigated and left with undefined conditions. As the noises characteristic are unknown, the sensors reading can be interfered and consequently exhibits bigger error; and hence bigger  $R$ . If  $K$  is recursively updated without any modifications and controlled, the EKF results in non-Gaussian noise can be erroneous. To overcome this, before updating the states in equation (7), fuzzy logic aims to find the best value of measurement innovation to pursue lower error. This was also inherently described by Kobayashi *et al.* (Kobayashi *et al.*, 1998) whose proposed that by selecting the  $P$ ,  $Q$ , and  $R$  from Fuzzy logic, smaller uncertainties is achieved. This is also exactly what has Wang *et al.* (Wang *et al.*, 2014) identified as Kalman gain is absolutely related to the measurement noises.

The proposed design used the Mamdani technique for analysis purposes. The general design is illustrated in Figure-2-4. The following describes the rules of Fuzzy logic that are used to define the output of the measurement innovation.

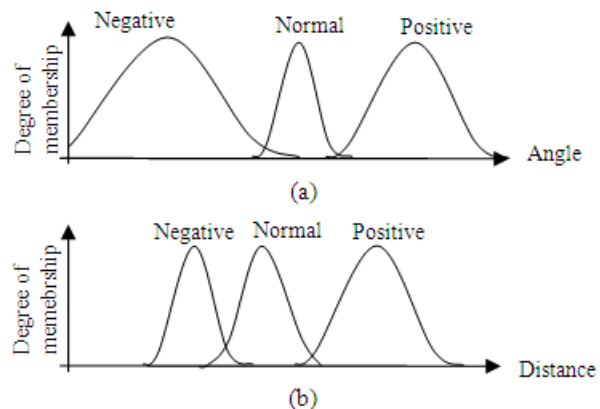
- IF angle error is negative and distance error is negative, THEN angle is negative
- IF angle error is negative and distance error is normal, THEN angle is normal
- IF angle error is negative and distance error is positive, THEN angle is negative, distance is normal
- IF angle error is positive and distance error is normal, THEN angle is negative
- IF angle error is positive and distance error is negative, THEN distance is normal
- IF angle error is positive and distance error is positive, THEN angle is negative, distance is normal



**Figure-2.** Fuzzy logic with inputs and outputs.



**Figure-3.** (a) Angle measurement (b) Distance measurement.



**Figure-4.** (a) Fuzzified angle (b) Fuzzified distance measurement.

The fuzzy sets are designed based on the Gaussian membership functions. Only three fuzzy sets is defined; can be seen through the rules, which ranging from



the negative, normal and positive regions. The value differs to each of the fuzzy sets and the designed has been tuned several times to obtain the best estimation results.

To highlight the differences between this fuzzy membership function and what have Wang *et al.*, (Wang *et al.*, 2014) investigated, all the membership functions are including positive and negative range. This can be further assessed if previous works are referred where the membership function of the angle error is positive all the time. In contrast to this arrangement, the range is now change in an intuitive way to demonstrate the possibility of the value to be either positive or negative.

### SIMULATION RESULTS AND DISCUSSIONS

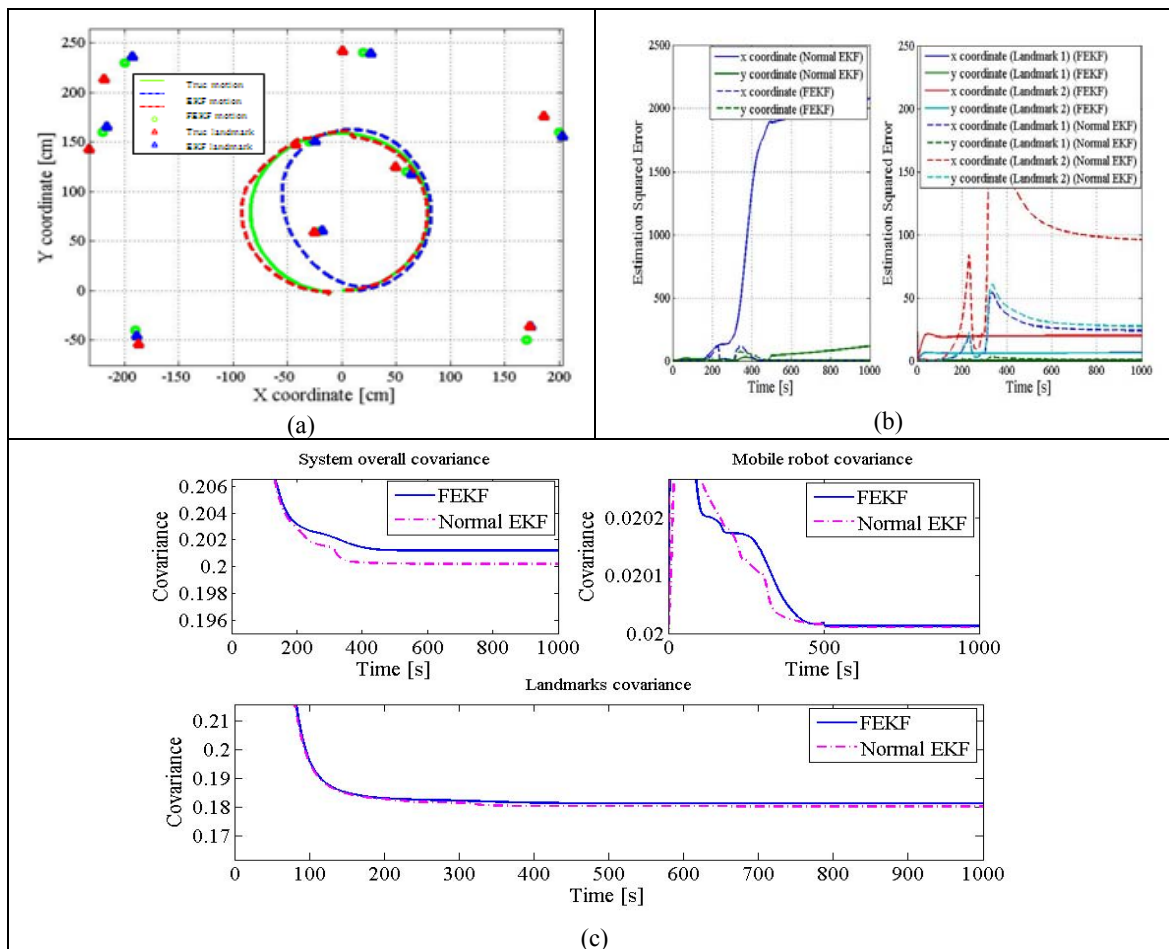
There are some assumptions being made to evaluate the proposed technique as mentioned below:

- Data association is expected to be available at all time
- Landmarks are point landmark and stationary

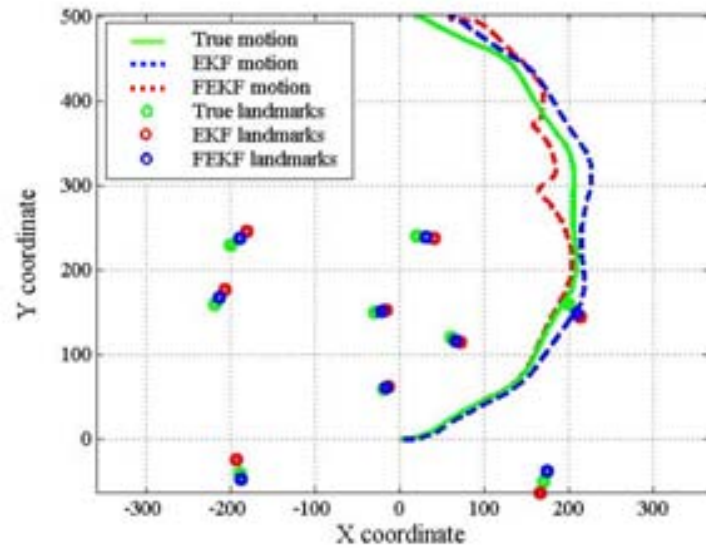
Simulations are carried in MATLAB Simulink for 5000[s] to ensure that the results are consistent and reliable. All the parameters are based on Table-1. These parameters are selected to model the real mobile robot which equipped with at least one sensor for measurement. The estimation results are shown in the following page for two different mobile robot motions between normal EKF and FEKF performances.

**Table-1.** Simulation parameters.

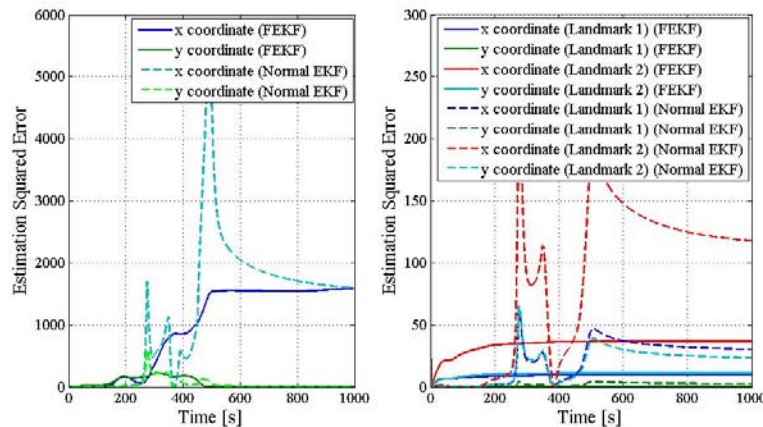
Variables	Parameter values
<b>Process noise;</b> $Q_{min}, Q_{max}$	-0.002, 0.001
<b>Measurement noise;</b> $R_{\theta min}, R_{\theta max}$ $R_{dist-min}, R_{dist-max}$	-0.04, 0.01 -0.15, 0.3
<b>Initial covariance;</b> $P_{robot}, P_{landmark}$	0.001, 100
<b>Simulation time</b>	1000[s]



**Figure-5.** (a) Comparison between EKF and FEKF estimation (b) Squared error comparison (c) State covariance comparison.



(a)



(b)

**Figure-6.** (a) Comparison between EKF and FEKF estimation with different movements  
(b) Squared error comparison.

Figures 5-6 illustrates the performance comparison between normal EKF and FEKF in non-Gaussian noise environment where the characteristics have been shown in Table-1. Figure-5 has described that the estimation of mobile robot for normal EKF is weaker than FEKF. Notice that the landmark estimation also shows consistent results, where FEKF outperforms normal EKF. This is expected as the measurement innovation attempts to correct the measured distance and angle of the landmarks. The squared error analysis presented in details the error exhibits by normal EKF about the landmarks estimation. Looking on the state covariance update aspects, the normal EKF depicts lower uncertainties than FEKF. The possible reason to this behavior is because of the fuzziness is now included in the system which in turn results in more uncertainties;  $\pm 0.01$  error than the normal

EKF. Nevertheless, the estimation is improving for landmarks estimation.

Through observations of different motions of mobile robot, FEKF capabilities to improve estimation are undeniable. Figure-6 explains that now both the mobile robot and landmarks estimations of FEKF has surpassed the normal EKF performance. The error is also reduced and smaller than normal EKF. Hence, it can be concluded that FEKF is more robust and capable to infer the positions of mobile robot especially landmarks if the Fuzzy logic is designed properly.

The outcomes presented and discussed above have agreed with the preceding works and suggest that FEKF can be a solution for normal EKF to operate in non-Gaussian noise condition. The proposed technique is also





posses less computational time as fewer rules have been designed in correspond to the defined two inputs.

## CONCLUSIONS

FEKF is one of the possible solutions for mobile robot localization and mapping especially when the mobile robot motions are uncertain, sensors limitation and for robust conditions. In this context, to overcome an issue that EKF suffer to provide good estimation results, FEKF method is proposed. Even though EKF can work in the non-Gaussian noise with acceptable estimations, FEKF offers better solution for robust conditions. This can be achieved if at least the measurement innovation information is processed and observed appropriately about its characteristics before designing the fuzzy rules and its membership functions. The behavior is made as references to define the fuzzy sets and membership function accordingly.

This paper also point out that by only using the measurement innovation information as an input to the Fuzzy logic, it is possible to gain better estimation results. Thanks to this, the computational cost and processing time can be further reduced compared to a case of using the distance, angle and measurement innovation information concurrently.

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