



PROBABILISTIC COLLOCATION METHOD FOR EVALUATION AVAILABLE TRANSFER CAPABILITY HYBRID WIND POWER SYSTEM

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ABSTRACT

Available Transfer Capability (ATC) computation of power systems including an excessive wind power penetration became a vital issue for system operation and planning due to the uncertainty and fast variation of wind speed. This paper proposes Probabilistic Collocation Method (PCM) for ATC determination with a grid connected hybrid wind power system. The low-order orthogonal polynomials are utilized to generate the approximation of the random variable of interest as the function of uncertainty parameters. The proposed method is a computational effectual solution to specify quite a precise approximation for the given probability distribution (PD) of system response. Consequently the method can significantly lessen the computational time compared to the traditional Monte Carlo (MC) approach. Examples are given on the IEEE 39 bus system to illustrate the effectiveness of the proposed method.

Keywords: probabilistic collocation method, available transfer capability, wind energy, uncertainty analysis.

1. INTRODUCTION

Contemporary and ongoing developments in Global Energy Technology (GET) indicate a tendency towards utilization of solar and wind energy as foremost substitutes to conventional energy derived from fossil fuels due to its non-sustainability and public environmental concerns.

In European countries there will be 20% electricity generated from wind turbines by 2020 [1]. High wind power penetration has had significant impact on power system stability, security, and reliability due to random and discontinuous characteristics of wind energy. The uncertainty in power networks has been significantly increased by the great expansion of renewable energy. It is important to evaluate the impact of the uncertainty of renewable energy on the security assessment for such a varied generation and transmission system. This hints to utmost needs of probabilistic analysis tools for power system planning and operation. ATC calculations are such an analysis tool to provide the quantitative assessment of uncertainty's impact on reliability of power flow system. It can be used to assess adequacy indexes, such as the probability of a nodal voltage being outside acceptable levels and the probability of a line flow being greater than its thermal rating, under load uncertainties and random contingencies.

Stability also is an important factor of power system security, which is closely dependent on the ATC calculations value, in fact, load model parameters, in particular, are often a source of perceivable error. The majority of past studies into power system uncertainties have focused primarily on reliability [2]. The deterministic approach is not sufficient for the analysis of modern power systems, and the results from Deterministic Load Flow

(DLF) may give an unrealistic assessment of the system performance. In order to take the uncertainties, e.g. inform of e.g. the outage rate of generators, the modification of network configurations and the variation of load demands. Furthermore, modern power systems with integration of DG units, such as WTs and photovoltaic systems, introduce additional power fluctuations into the system due to their uncontrollable prime sources. The PLF was first proposed in 1974 and has been further developed and applied into power system normal operation, short-term/long-term planning as well as other areas [3] [4] [5]. The PLF requires inputs with Probabilistic Density Function (PDF) or Cumulative Distribution Function (CDF) to obtain system states and power flows in terms of PDF or CDF, so that the system uncertainties can be included and reflected in the outcome. The PLF can be solved numerically, i.e. using a MC method, or analytically, e.g. using a convolution method, or a combination of them [6] [7] [8]. The main concern about the MC method is the need of large number of simulations, which is very time-consuming; whereas the main concerns about the analytical approach are the complicated mathematical computation and the accuracy due to different approximations.

The parameters of many dynamic models are quite uncertain, it is difficult to obtain certainty limits by employing conventional statistics, also in case of high penetration of renewable energy on power system a problem may occur in term of significant impact on the value of Available Transfer Capacity (ATC) in the power systems, NERC defined ATC as "a measure of the transfer capability remaining in the physical transmission network for further commercial activity over and above already committed uses", derived in Equation (1) as:



$$ATC = TTC - TRM - CBM - ETC \quad (1)$$

Where TTC is the Total Transfer Capability, TRM is the Transmission Reliability Margin, CBM is the Capacity Benefit Margin, ETC is the Existing Transmission Commitments [9], limitations incorporated ATC related parameters as illustrated in Figure-1.

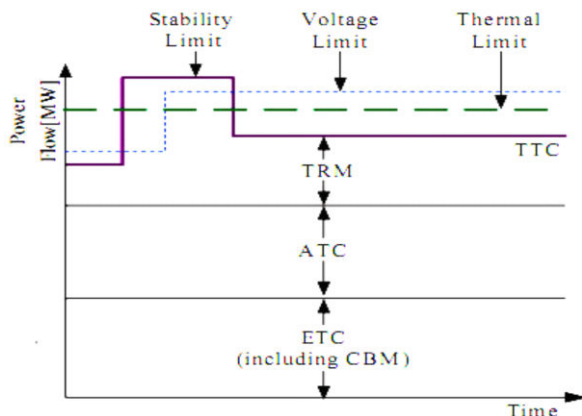


Figure-1. Limitations incorporated ATC related parameters.

Several uncertainties correlating with the parameters and forecasting quantities used in the ATC evaluation, several techniques are used scarcity to accuracy and speed. This paper propose a solution of ATC determination involving diversity uncertainties of grid connected hybrid wind power station with voltage stability constrained using PCM technique, where vast saving time and facilitates towards ATC stability online studies. The MC method is widely used in stochastic modelling, and is independent of the number of random dimensions. However, if the process encompassed within the MC approach is complex, the full study can require vast computational effort and time, resigning the analysis to the offline or planning stage [10]. Furthermore, for complex systems a great number of simulations may be required in order to be confident that full system variation in the parameters is captured.

It is necessary to construct a rational ATC model and effectual calculation method for power systems containing uncertainties. Based on that, it is expected that PCM can be a method to determine ATC with the uncertainty voltage stability. The PCM by Tatang in [11], is a technique significantly reduce the number of simulations that are required and still accurately produce the probability distributions for system parameters, but was first introduced as a tool for dynamic power system studies in [12], is the method has been applied in several field [13], and later for investigating the effects of load models involving very few uncertain parameters (no more than two) in [14] [15].

The main purpose of applying PCM method is to approximate the system response as a polynomial function

of the uncertain parameters, replacing the computationally intensive study with a simple deterministic algorithm; also it provides two clear benefits essentially suitable to the application of power system uncertainty studies. Firstly, the method is designed to provide a statistical distribution for the system output, this is of great value when considering statistical uncertainty in future operating points and trying to calculate risk indices with system uncertainty likely to increase, methods that present the probabilistic nature of the network response are vital in ensuring accurate system representation. Secondly, the method affords significant savings with respect to the number of full system studies that are required [16], this leads to a decline in computational time when performing these types of analysis and could permit the incorporation of uncertainty into online studies which assist a move towards a greater use of probabilistic studies in power systems analysis. It is necessary to construct a rational ATC model and effectual calculation method for power system containing with wind generation.

Conventional MC method suggested to simulate collected uncertainties output responses, expected numerical simulation results will be simulated using the IEEE 39-bus system; validation step would be performing via a comparative analysis of the results with the proper outputs and with the traditional MC model, initial results shows feasible and effective. The impact of wind turbines connected to power system on ATC should be analysed with help of MATLAB calculation technique and NEPLAN software for validity analysis. The suggested model will developed for National Electric Power Grid (NEPG), proposed integrated method would be implemented.

The results provide a great opportunity to have a clear evaluation of hybrid system ability. Rest of this paper will be organized as follows; Second section will give a brief review on Probabilistic Method; Third section will introduce a review of previous PCM method applications. PCM theoretical explanation, verifies the applicability of ATC computation using proposed method, in section four. Analysis the effects of uncertain parameters on system transient stability and dynamic simulation applied on IEEE 39-bus discussed in section five.

2. PROBABILISTIC METHODS

The Monte Carlo Simulation (MCS) is an exact and robust method, however, its excessive computational cost makes it intractable a simulation since thousands of solves are needed to obtain a satisfactory accuracy. Moreover, each solve on itself is already computationally intensive, making this method not efficient.

To overcome the drawback of the MCS method on computational efficiency, a number of analytical techniques and methods are proposed to solve ATC calculation problems, here are four uncertainty propagation methods, the Generalized Polynomial Chaos method (GPCM), Non- Intrusive Polynomial Chaos



method (NIPCM), Stochastic Collocation method (SCM), and PCM, important properties such as intrusiveness, exponential convergence with respect to polynomial order, and efficiency of handling multiple uncertain parameters are compared in Table-1. Method is non-intrusive if the set of equations to be solved is identical to the deterministic set. This is advantageous because it means commercial solvers can be used without adaptation. The efficiency of handling multiple uncertain parameters is not straightforward to compare since the amount of work is problem related. NIPCM is about twice as much work as GPCM. Furthermore, only approximations of the

coefficients are obtained by NIPCM. Since SCM converges only exponential for uniform input distributions, this method is considered least efficient. Although there is no distinct threshold, PCM is preferred for problems with less than 5-10 random variables. Over 10 random variables, GPCM is preferred at the cost of intrusiveness. In this thesis, however, only one-dimensional stochastic cases are treated, i.e. the random variables are assumed uncertain one at the time. Therefore, the efficiency at handling multiple uncertain parameters is not important in this study [17].

Table-1. Comparison between uncertainty propagation methods (GPCM): Generalized Polynomial Chaos method, (NIPCM): Non-Intrusive Polynomial Chaos method, (SCM): Stochastic Collocation method, (PCM): Probabilistic Collocation method.

| Property | GPCM | NIPCM | SCM | PCM |
|--|------|-------|-----|-----|
| Non-intrusive | | ✓ | ✓ | ✓ |
| Exponential convergence for all type of type of input distribution | ✓ | | | ✓ |
| Efficient at handling multiple uncertain parameters | + | 0 | -- | - |

Because of the necessity of considering several limitations in ATC computations, different optimization methods are used; most important of them are voltage, thermal and satiability limitations which are considered in most of ATC computations. ATC calculation are tremendously time consuming and cannot be executed for online power systems, therefore more efficient method which:

- Can consider almost of constraints.
- It is possible for predicting ATC by the effect of uncertainties.
- Its computational time is low [18]. Generally, the influence of system uncertainties on the ATC has been assessed using probabilistic methods.

Yuan and his colleague introduced a novel algorithm by incorporates linear distribution factors and AC load flow sensitivity-based method in order to calculate ATC values efficiently and speedily considering line outages, distribution factors still unsatisfactory covered [19]. Kulyos Audomvongseeree and Akihiko Yokoyamaa, propose a method by using probability density function (PDF) to provide an alternative choice for transmission providers (TPs) or ISOs. It seems a solution to decrease error, but still time consumed [20]. Shaaban and his colleagues, with steady-state security constraints using Benders decomposition method, the results improved rather other technique mentioned [21].

Leite da Silva *et al.* proposed a new methodology to determine the best points in the system, in order to maximize the ATC, without violating a pre-established

reliability level. In the process, the ATC probability density function is determined considering uncertainties from equipment un-availabilities. Proposed algorithm uses Monte Carlo simulation, this means time consumed needed [22]. The most important methods used are artificial, Cubic Spline, Stochastic programming and Monte Carlo methods. Deqiang G, *et al.*, using new method called Min-Max transfers capabilities interfaces, because of too much data needed and calculations cons of this method [23]. The Monte Carlo method is a known method, used to obtain the solution of the stochastic power flow problem, often characterized by a large computation time. Nonetheless, the Monte Carlo has been used in many general engineering applications [24], [25].

To enhance disadvantages of MC, a stochastic-algebraic method employed for the first time by assuming the system is linear with applying just thermal limitation [26]. Stahlhut and his colleague, propose Stochastic-algebraic method with voltage and thermal constraints, improved by considering two uncertainties, bus loading and transmission element status in [27]. To deal with uncertainties, several techniques aims to define the points to evaluate the problem and calculate the statistics of the outputs using different methods as: methods as:

1. Multi-point Criteria:
Select regularly spread values within the range.
No statistical criteria is used.
2. Monte-Carlo (MC) method:
Statistical definition of the set of samples.



Values defined according the Probability Density Function (PDF).

3. Latin Hypercube method:
Statistical definition based on MC.
Improves the variance convergence.
Based on Latin square criteria.
4. Probabilistic collocation method:
Based on collocation techniques.
Can be considered a multi-point method which account with probabilistic information [14].

Khairuddin, has proposed a novel solution using Artificial Neural Network (ANN) method requires a large input vector so that it has to oversimplify determination of ATC [33]. Kumar *et al.*, present a real time ATC calculation using three different techniques: Back propagation Algorithm, Radial Basis Function Neural Network and Adaptive Neuro Fuzzy inference system [34]. All of these methods follow all the paths between the transfer buses of the ATC. The path with the least impedance is chosen as a basis of sample generation in the intelligent programs. They applied these methods on a 24 bus system considering only line thermal ratings and a few selected line outages. Othman *et al.*, 2004 a cubic spline interpolation curve fitting that reduces the computation time of the power flow computations [35], the ATC limited by voltage and power flow limits, the results show

that this method is accurate and faster than an iterative AC power flow. The MC method is a known method used to obtain the solution of the stochastic power flow problem. This method utilizes repeated distributions of the nodal powers, line flows and losses. Since the accuracy of the probability distribution of line flows, voltages and losses is assumed to be better when modelling all stochastic inputs over a large number of trials, MC method is often characterized by a large computation time. Nonetheless, MC has been used in many general engineering applications [36], [37], [38]. This method has an appeal that a wide range of stochastic phenomena can be modelled, thus proposing accuracy in the results. MC approach does not rely on any required system characteristics: e.g. nonlinear systems are just as readily studied as linear systems. But computational burden is a disadvantage of the MC approach and researchers have sought faster methods to calculate the probability distributions. The MC method can be used to verify and validate these faster methods. Sequential MC Simulation utilized with considering the time-varying load and the fault and repair of equipments (generator, transformers and lines). a new semi smooth ATC model is constructed based on the so-called point wise maximum function. Various methods can be utilized to evaluate transmission transfer capability, comparison between calculations techniques used over decades illustrating in Table-2.

Table-2. Comparison between calculations techniques used over decades [15].

| Constraints and uncertainties | Methods | | | | |
|-------------------------------|-----------------------|-------------|--------------|--------------------|-------------------|
| | Stochastic programing | Monte carlo | Cubic spline | Artificial methods | Linear Sensitivit |
| V | ✓ | ✓ | ✓ | ✓ | ✓ |
| P _{max} | ✓ | ✓ | ✓ | ✓ | ✓ |
| P _g | | ✓ | | ✓ | |
| Q _g | | ✓ | | ✓ | |
| P _l | | ✓ | | ✓ | |
| V. Stability | ✓ | | | | |
| Outage | ✓ | ✓ | ✓ | ✓ | |
| L. Variation | ✓ | ✓ | | | ✓ |

2.1 ATC WIND GENERATION COMPUTATIONS

The impact of wind power generation on the ATC value explained by Luis Luna *et al.*, and the use of linear approximations to update for estimating the capacity of transmission is widespread [18]. Reference [19] proposed the improved PQ model, and presented RX model for modelling of wind farms in load flow analysis, and applied the impedance containing slip to be the equivalent of wind turbines, which seems effective approach for final approximation of ATC value.

Karki R, Po Hu, Billinton, confirmed that sequential Monte Carlo simulation or a multistate wind farm representation approach is often used to evaluate wind sources integration reliability, and presents a simplified method for reliability evaluation of power systems with wind power. The development of a common wind speed model applicable to multiple wind farm locations to gain high level reliable system [20]. Yajing Gao, *et al.* Presents a new ATC computation model with offshore wind farms connecting to the grid via VSC-HVDC is proposed, in this model the characteristics of



offshore wind farms and VSC-HVDC control is considered, CPF algorithm is presented to solve this model and then the ATC value of power system can be gained [24]. ATC computation related to dynamic voltage stability constrained, is one of the significant issues must be taken in account in large GCHWPS, Paensuwan and Yokoyama present a method suitable to exam such impacts on TTC, using Monte Carlo simulation in [15]. Reference construct static security based smoothing ATC model by using the pointwise maximum function. Along the work, derives the power flow model including wind turbines where the slip is as a new state variable, then, by integrating power flow model combining with wind turbines and conventional static security constraints.

3. PCM TECHNIQUE APPLICATIONS

Lesieutre, B.C. and J. Hockenberry, was the first introduce treating uncertainty analysis of power system simulations and ATC calculations using the probabilistic collocation method [32]. Yan Wan *et al.*, present realistic comparison between PCM technique and Monte Carlo in uncertainty evaluation through mapping identification in intensive dynamic simulations testing just one or two uncertainties [28]. Webster *et al.*, presents the probabilistic collocation method as a computationally efficient method for performing uncertainty analysis on large complex models such as those used in global climate change research [29]. Ce Zheng, Mladen Kezunovic preform uncertainty analysis related impact of wind generation variation, on the small disturbance voltage stability, using new method PCM, comparison the results with conventional methods to realize the accuracy, saving time consumed computations [30]. Preece. R, Milan apply PCM Method to model operational uncertainties in small disturbance power system studies included distributed generation and renewable energy sources, the significant computational time savings achieved could allow such analyses to be performed in online scenarios [31]. Hockenberry, Lesieutre in 2004 was first introduce of PCM method studding uncertainties of dynamic simulation of power system models, using only a handful of simulations, The larger system example demonstrates the time savings and enabling aspect of PCM in conjunction with the associated methods. The results presented here would take months to gather using other standard methods [12].

4. PROBABILISTIC COLLATION METHOD

The elementary idea of PCM is to construct an approximate bond between uncertain parameters and outputs of interest through polynomial models. The quantified probability density functions of uncertain parameters are employed to create the orthogonal polynomials. Collocation methods choose the collocation points to solve the model coefficients. In order to simplify notation, the case of distinct uncertain parameter is applied. Note that PCM is able to handle multi-parameters, however with increased computations.

A. Polynomial approximation

In evaluating the impact of parameter uncertainty x , it is assumed that the parameters of interest satisfy given probability density function is $f(x)$. The output of interest $Y = g(Yx)$ is the function of random parameters, which can be represented by a complex, high-ordered, or even "black-box" model. PCM essentially creates polynomial models Y to approximate \bar{Y} :

$$\begin{aligned}\bar{Y} &= \bar{g}(x) \\ &= \sum_{i=0}^{n-1} c_i H_i(x) \\ &= c_0 H_0(x) + \dots + c_{n-1} H_{n-1}(x)\end{aligned}\quad (2)$$

where $H_i(x)$ are the orthogonal polynomials in terms of the uncertain parameter x , c_i are the model coefficients, and n is the order of PCM models.

B. Orthogonal polynomials

An orthogonal polynomial sequence is a family of polynomials such that any two different polynomials in the sequence are orthogonal to each other under some inner product. The orthogonal polynomial is defined as follows:

$$\begin{aligned}\langle H_j, H_k \rangle &= \int_A f(x) H_j(x) H_k(x) dx \\ &= \begin{cases} 0 & \text{if } j \neq k; \\ A_{jk>0} & \text{if } j = k. \end{cases}\end{aligned}\quad (3)$$

where $f(x)$ is any nonnegative weighting function defined everywhere in an area A . In our case, $f(x)$ is a probability density function describing the uncertainty in a system parameter. The inner product of A_{jk} usually is normalized into one.

C. Gaussian quadrature integration

Rendering to Gaussian quadrature integration, if the probability density functions of uncertainty parameter x is described by $f(x)$, the expected value of approximation \bar{Y} can be described as:

$$E[\bar{Y}] = \int_A f(x) \bar{g}(x) = \sum_{i=1}^m f(x_i) \bar{g}(x_i) \quad (4)$$

where constants x_i and $f(x_i)$ are the nodes and weights of Gaussian quadrature rules. The evaluation points $x_i \in A$ are just the roots of a polynomial belonging to a class of orthogonal polynomials. Therefore the evaluation points x_i can be determined by constructing orthogonal polynomials.

D. Collocation methods

In order to determine the coefficients in (2), evaluation points are selected in the probability space of



random system parameters. In the context of PCM, this important procedure is called collocation point's selection. Then the deterministic simulations on the studied system (i.e. nonlinear load flow equations) are launched at each collocation points.

The selection of collocation points has significant impact on the accuracy of the approximation of PCM model. In the collocation method, the roots of the next higher order orthogonal polynomial $H_{n+1}(x)$ for the uncertain parameter are selected as collocation points for n th order PCM model. According to (2), a linear set of equations can be yielded by evaluating $g(x)$ at the collocation points:

$$\begin{bmatrix} g(x_1) \\ \vdots \\ g(x_n) \end{bmatrix} = \begin{bmatrix} H_{n-1}(x_1) & \dots & H_0(x_1) \\ \vdots & \ddots & \vdots \\ H_{n-1}(x_n) & \dots & H_0(x_n) \end{bmatrix} \begin{bmatrix} c_{n-1} \\ \vdots \\ c_0 \end{bmatrix} \quad (5)$$

where x_1, \dots, x_n are collocation points, $g(x_1), \dots, g(x_n)$ are the corresponding responses at each collocation point, and $H_0(x), \dots, H_{n-1}(x)$ are orthogonal polynomials evaluated at each collocation point. Only the coefficients c_i are unknown variables. Therefore, the coefficients c_0, \dots, c_{n-1} can be obtained to solve the linear equation of (5):

$$\begin{bmatrix} c_{n-1} \\ \vdots \\ c_0 \end{bmatrix} = \begin{bmatrix} H_{n-1}(x_1) & \dots & H_0(x_1) \\ \vdots & \ddots & \vdots \\ H_{n-1}(x_n) & \dots & H_0(x_n) \end{bmatrix}^{-1} \begin{bmatrix} g(x_1) \\ \vdots \\ g(x_n) \end{bmatrix} \quad (6)$$

E. Statistics calculation

Due to the fact that the orthogonal polynomials are generated based on the probability density function of uncertain parameters, the statistics of the system responses are easily to obtain. For example, the mean and variance of output values are given by:

$$E[\bar{g}(x)] = \int f(x) \bar{g}(x) dx = c_0 \quad (7)$$

$$\begin{aligned} E[\bar{g}(x)] &= \int f(x) [\bar{g}(x) - E[\bar{g}(x)]]^2 dx \\ &= \sum_{i=1}^{n-1} c_i^2 \langle H_i, H_i \rangle \end{aligned} \quad (8)$$

F. Error check

The approximation error of PCM model should be evaluated to guarantee the accuracy of estimation. The sum-square-root (ssr) error and the relative ssr (rssr) are defined as follows:

$$ssr = \sqrt{\frac{\sum_{i=1}^m [(\bar{Y}_i - Y_i)^2 \cdot f(x_i)]}{m \cdot f(\bar{x}_c)}} \quad (9)$$

$$rssr = \frac{ssr}{E(\bar{Y})} \cdot 100\% \quad (10)$$

where Y_i and \bar{Y}_i are the real response and the approximation at the i th collocation point x_i respectively,

m is the number of collocation points, $f(x_i)$ is the probability density at x_i , and \bar{x}_c is the collocation point with highest probability in A . In the case of single uncertainty parameter, the number of collocation points equals the order of PCM model plus one. In other words, $m = n + 1$. Note ssr is dependent on the typical Y , the normalized rssr is more useful measure of the error.

5. PROPOSED SOLUTION FOR ATC CALCULATION BASED ON PCM

In this section, the PCM method is applied to power flow analysis in presence of wind farms. The non-Gaussian variables are wind speed and wind power output. The orthogonal polynomials can be constructed correspondingly for given distributions. The collocation points are selected to calculate coefficients of the polynomial expansion. The detailed procedure is illustrated as follows.

A. Wind model

Weibull distributions have been widely used to model the wind speed, which can be described as:

$$f(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right], v \geq 0 \quad (11)$$

where $f(v)$ is PDF of wind speed v , k and c are the shape parameter and scale parameter of the distributions respectively. The parameters k and c can be derived from the mean μ and standard deviation σ of wind speed, satisfying the following equations:

$$\mu = c\Gamma(1 + k^{-1}) \quad (12)$$

$$\sigma^2 = c^2 \Gamma(1 + 2k^{-1}) - \mu^2 \quad (13)$$

where the gamma function is:

$$\Gamma(x) = \int_0^\infty u^{x-1} \exp(-u) du,$$

$u > 0$. The power output characteristics of a wind turbine can be modelled as the function of wind speed, described by the following equation:

$$P(v) = \begin{cases} 0 & v \leq v_c \\ P_r \frac{v - v_c}{v_r - v_c} & v_c < v \leq v_r \\ P_r & v_r < v \leq v_f \\ 0 & v_f < v \end{cases} \quad (14)$$

where v_i , v_r and v_c are the cut-in, rated and cut-out wind speed, and P_r is the rated power of a single wind generator.



B. Orthogonal polynomial generation

Next the orthogonal polynomials for Weibull distribution will be derived. The standard Gram-Schmidt orthogonalization process is used to generate the polynomials [39]. This Gram-Schmidt orthogonalization is an iterative process and can generate any high order polynomials. We illustrate as follows, starting with:

$H_0(x) = 1$ and $H_1(x) = x - a_1$, the recursive process to construct the orthogonal polynomials give:

$$H_j(x) = (x - a_j)H_{j-1}(x) - b_j H_{j-2}(x), \quad (15)$$

where $j = 2, \dots, n$ and

$$a_j = \frac{\langle xH_{j-1}, H_{j-1} \rangle}{\langle H_{j-1}, H_{j-1} \rangle} \quad (16)$$

$$b_j = \frac{\langle xH_{j-1}, H_{j-2} \rangle}{\langle H_{j-2}, H_{j-2} \rangle} \quad (17)$$

In our case, the wind speed has the mean value $\mu = 5$, and standard deviation $\sigma = 2.7$. The shape and scale factors of the Weibull distribution are calculated based on equations (11)-(12): $k = 1.9526$, $c = 5.6390$. According to (14)-(16), the first four polynomials for Weibull distribution can be derived as follows:

$$H_0(x) = 1 \quad (18)$$

$$H_1(x) = x - 5 \quad (19)$$

$$H_2(x) = x^2 - 11.7719x + 26.7273 \quad (20)$$

$$H_3(x) = x^3 - 20.0350x^2 + 110.6824x - 154.2638 \quad (21)$$

The polynomials have been normalized as shown in Figure-2.

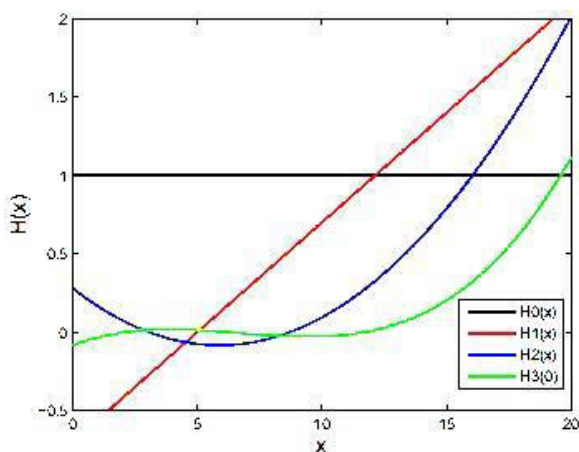


Figure-2. Orthogonal polynomials for Weibull distribution.

C. Collocation points selection

The collocation points must be selected carefully to provide accurate approximation for the system response with the given order of models. In the case of a single uncertain parameter, the number of simulations necessary to fit the model is equal to one plus the order of the polynomials. In the collocation method, the roots of the next higher order orthogonal polynomial $H_{n+1}(x)$ for the uncertain parameter are selected as collocation points for n th order PCM model. In our case, the collocation points which are found from polynomials (18)-(21) are shown in Table-3.

Table-3. Collocation points for weibull distribution of wind speed.

| Order | Collocation points wind speed (m/s) |
|-------|-------------------------------------|
| 1st | 5 |
| 2nd | 3.0722, 8.6997 |
| 3rd | 2.1234, 6.2067, 11.7049 |

It is shown in Figure-2 that these points are zeros (roots) of the corresponding polynomials. These collocation points represent wind speeds with high probability for the given Weibull distribution, which are used to generate the wind power output and then launch load flow analysis at each point.

D. Load flow analyses

The load flow equations for a power system can be written as:

$$0 = P_i^{\text{inj}} - V_i \sum_{j=1}^N V_j Y_{ij} \cos(\theta_i - \theta_j - \phi_{ij}) \quad (22)$$

$$0 = Q_i^{\text{inj}} - V_i \sum_{j=1}^N V_j Y_{ij} \sin(\theta_i - \theta_j - \phi_{ij}) \quad (23)$$

where P_i^{inj} and Q_i^{inj} are real and reactive power injections, and V and θ are the nodal voltages and angles; $Y_{ij} - \phi_{ij}$ is the (i, j) th element of the admittance matrix; N is the number of nodes in the power network. The load flow equations (22)-(23) can be solved using Newton-Raphson iteration. The random generation of power injections makes the original deterministic problems into probabilistic (stochastic) problems. MC simulations and PCM follow a similar procedure for uncertainty analysis but the number of simulations for PCM is much smaller. The procedure of PCM-based probabilistic load flow analysis consists of the following steps:

step1. Specify the probability density function of wind speed.



- step2.** Generate the orthogonal polynomials: $H_k(x)$, $k = 0, \dots, n$.
- step3.** Find the roots of $H_{n+1}(x)$ as the collocation points.
- step4.** Substitute the injection wind power at the collocation points into (22)-(23) and solve the deterministic load flow equations.
- step5.** Compute the statistics of Y using the corresponding quadrature rule (7)-(8).
- step6.** Check if error is less than the threshold, otherwise increase the polynomials order and repeat the procedure. Flow chart of uncertainty analysis procedures PCM method calculating ATC shown in Figure-3.

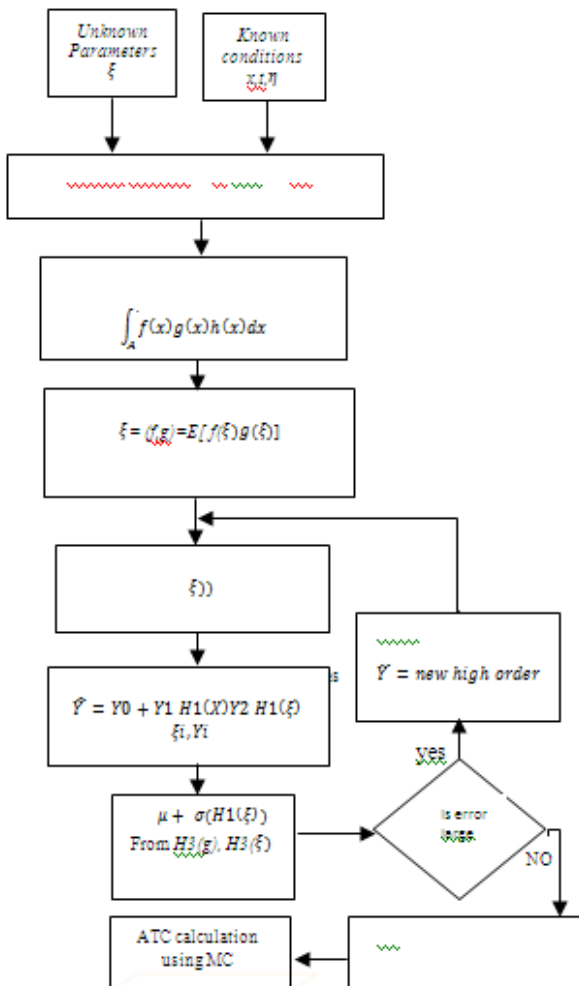


Figure-3. Flow chart of uncertainty analysis procedures PCM method calculating ATC.

6. SYSTEM IMPLEMENTATION

The IEEE 39 Bus test system, shown in Figure-4, has been modified to illustrate the proposed models and techniques. The system contains 39 buses with 10 generators.

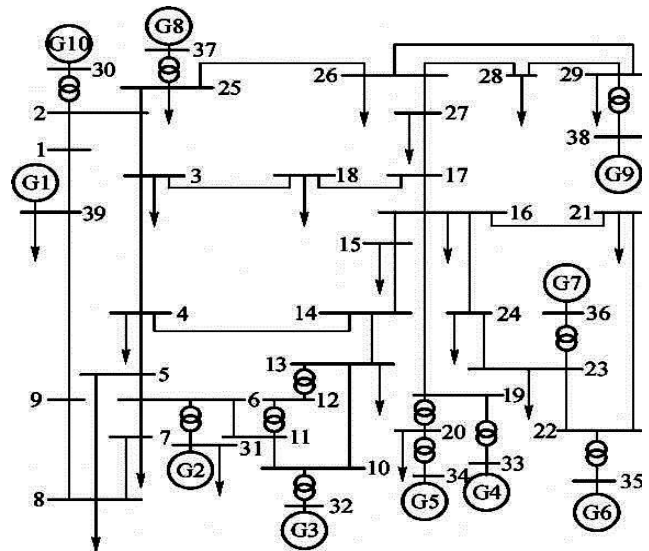
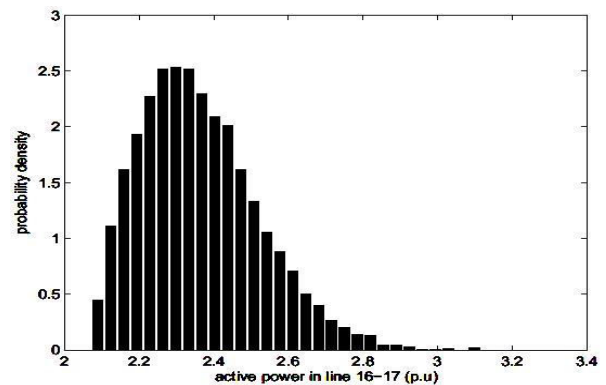
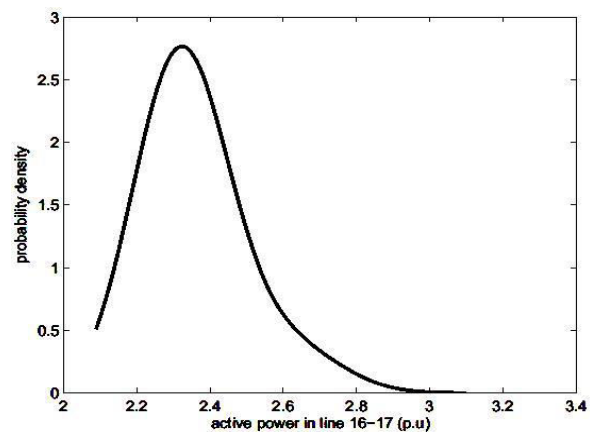


Figure-4. IEEE 39 bus system

The total load are 6097.1 MW and 1409.1 Mvar. The detail system data can be found in Matlab-based Power System Toolbox (PST) [40].



a) MCS



b) PCM

Figure-5. Comparison of power flow distributions in line 16-17: (a) Monte Carlo Simulation (MCS) (b) Probabilistic Collocation Method (PCM).



The wind farm consists of 100 Vestas V112-3MW WTGs. The cut-in, rated and cut-out wind speeds are 3, 12.5 and 25 m/s, respectively [41]. The rated power of each WTG is 3MW. The total wind capacity is 300 MW, which is about 5% of wind penetration. The Weibull distribution is applied to describe the wind speed probability distribution. Note that if the parameters of Weibull distribution change, the polynomials and collocation points should be reevaluated correspondingly.

The wind farm is located at bus 15. The distributions of active power flow in line 16-17 are compared between MCS and PCM. The distribution for MCS is estimated by histograms. The order of PCM is two. The results of the two approaches are found to be consistent as shown in Figure-5. The line flows in a number of lines around the wind farm are also investigated. The means and standard deviations of transmitted power are shown in Table-4. The results show PCM and MCS both provide accurate statistic information (means and standard deviation). However, PCM has better performance than Monte Carlo approach in terms of the number of simulations and computational efficiency. The computational time and the number of the simulations by MCS and PCM are listed in Table 5. The CPU times are measured based on Matlab scripts running in a processor Intel Core (TM) i5 of 2.5 GHz with 4 Gb of RAM.

Table-4. Means and standard deviations of transmitted power in lines.

| Line | Monte Carlo | | PCM | |
|-------|-------------|--------|----------|--------|
| | σ | μ | σ | μ |
| 4-14 | 2.752 | 0.108 | 2.750 | 0.107 |
| 3-4 | 0.972 | 0.114 | 0.971 | 0.113 |
| 3-18 | 0.570 | 0.120 | 0.569 | 0.120 |
| 15-16 | 3.098 | 0.158 | 3.100 | 0.157 |
| 14-15 | 0.289 | 0.289 | 0.289 | 0.289 |
| 16-24 | 0.427 | 4.6E-6 | 0.427 | 4.6E-6 |
| 16-17 | 2.353 | 0.156 | 2.354 | 0.156 |

μ : mean; σ : standard deviation

Table-5. Comparative computational efficiency of PCM and Monte Carlo method.

| Monte Carlo | | PCM | |
|--------------|--------------|--------------|--------------|
| N of samples | CPU time (s) | N of samples | CPU time (s) |
| 5000 | 34 | 3 | 0.05 |

7. CONCLUSIONS

Probabilistic collocation method one of the efficient probabilistic methods it could be used for uncertainty analysis in load flow problems significantly

with a penetration of wind energy sources. PCM is an effective numerical method to compute non-Gaussian uncertainty in complex nonlinear system phenomena. As compared with the traditional Monte Carlo simulation approaches, the collocation method could provide an precise approximation for the quantitative measures of system uncertainty with a much smaller number of model runs. Future research will focus on including multiple uncertainty parameters, convergence and collocation points' selection. And the key issues for PCM models, dependent parameters,

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