



THE EFFECT OF ROCKER RATIO AND CONNECTING ROD LENGTH ON THE PERFORMANCE OF A CRANK-ROCKER MECHANISM WITH AN EXTENDED ROCKER INPUT LINK

Salah E. Mohammed, M. B. Baharom and A. Rashid A. Aziz

Mechanical Engineering Department, Universiti Teknologi Petronas, Bandar Seri Iskandar, Tronoh, Perak, Malaysia

E-Mail: Salah_7888@yahoo.com

ABSTRACT

The effect of the rocker ratio, and the distance between the crank and rocker extreme positions, of a general four-bar mechanism with an extended rocker arm length are analyzed. The crankshaft rotation was set at a constant rate of 2000rev/min, and all mechanisms had the same strokes. A constant input force of 100N was applied at the rocker tips of all the mechanisms having a unit ratio of 1. A computer program using MATLAB code was developed for the kinematic and dynamic analysis of the crank-rocker mechanism to solve the governing equations. The results obtained from aforementioned analysis were plotted, compared and verified with the results obtained from ADAMS Software. It is concluded that the variation of rocker ratio did not have any effect on the rocker angular acceleration and torque output. However, a shorter distance, i.e. shorter connecting rod, is better than a longer connecting rod in terms of the peak values of angular acceleration and torque output. The results of the analytical solution were in good agreement with those obtained from ADAMS within 2% error. The findings are very significant for the improved design of the mechanism in term of power and packaging advantages.

Keywords: Crank-rocker mechanism, extended rocker link, rocker ratio, connecting rod, extreme positions.

INTRODUCTION

A four-bar mechanism has been used for a long time, and it has numerous applications in the industrial and automotive engineering fields. Well-known applications include the windshield wiper, vehicle suspension subsystems and quick-return mechanisms. The concepts of a four-bar mechanism have been widely used in other various applications such as manipulators, flapping wings, cutting mechanisms and internal combustion engines [1-4].

In general, the crank-rocker mechanisms are used to transform rotational motion of the crank into oscillating motion of the rocker and also to transmit the desired output torque from a given input torque. The mechanism has advantages of generating complex motion from a simple input rotational motion just using four linkages.

Figure-1 shows the crank rocker mechanism where the torque, T_2 is transmitted from the crank to the rocker, T_4 . The torque applied by link L_2 (crank) produces a force at point B which is then transmitted to link L_4 at point C through link L_3 (connecting rod). This force can be divided into radial and tangential components using an axis along and perpendicular to link L_4 . However, the tangential force applied to joint C is the only one that creates torque around the rocker pivot D. Neglecting the inertia forces, the torque output can therefore be calculated as follows [5]:

$$T_4 = F_{34} * \cos\left(\frac{\pi}{2} - \mu\right) * L_4 \quad (1)$$

Where μ is the transmission angle

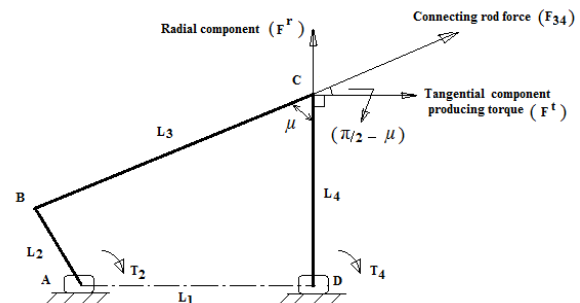


Figure-1. The torque transmitted from crank-rocker mechanism.

The performance of a crank rocker mechanism can be defined as the effective transmission of motion and force from the crank input-link to the rocker output-link. For a constant input torque, a well-designed mechanism produces a high torque output [5, 6].

In the development of mechanisms, the kinematic synthesis is an important approach and must be considered in the design process. Many researchers have conducted and studied the optimal synthesis of the crank-rocker mechanism using different optimization approaches [6-11]. Based on their findings, it was concluded that the optimal design and synthesis of the crank rocker mechanism depended on the linkage ratio, mechanism time ratio and transmission angle. Also, most of the previous studies have analyzed the kinematics and kinetics separately [7, 12].

Most of the applications of the four bar mechanisms are in the area of quick return mechanisms where the crank is used as the input driver while the rocker as the output. In this research, the analysis was conducted



for the cases where the rocker is used as the input while the crank is for the output. The configuration of the four-bar-mechanism for analysis and power optimization in this paper is shown in figure 2. The input motion comes from a force acting perpendicularly at the rocker tip. The force could be represented by a combustion force which decays at the end of the rocker stroke. The time taken for the rocker to move from either extreme position to the other should be equal to the time taken for the crank to rotate 180 degrees. Similar to the slider-crank engine, the gases pressure produced from the combustion of fuel applies a direct force to the piston which is attached to the rocker tip. This force is transferred to the crank along the connecting rod, and generates useful mechanical power.

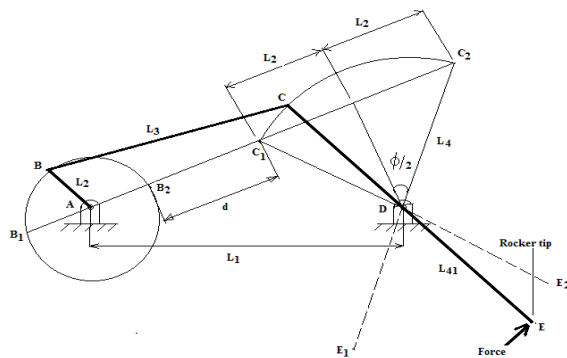


Figure-2. Crank-rocker mechanism with unit time ratio.

The main purpose of the analysis was to determine the optimized dimensions of the rocker and connecting rod links which would yield the maximum torque and power outputs with better packaging advantages. The effect of the rocker ratio and the length of the connecting rod in obtaining the optimal results which meet the design requirement were investigated. The optimization was done using analytical solutions of a general four-bar mechanism. For validation purposes, different types of crank rocker mechanisms were modeled and analyzed using ADAMS software.

ANALYTICAL METHOD

In this section, the analytical method is discussed in details for the kinematic and dynamic analysis of the crank-rocker mechanism. The calculations of the kinematic and dynamic properties of the crank-rocker mechanism such as link position, angular velocity, angular acceleration, and torque outputs from the crank are presented.

Kinematic analysis of the crank-rocker mechanism

The kinematic analysis of the crank-rocker mechanism (Figure-2) is performed in order to obtain the values of position, velocity and acceleration. For a given input velocity and crank angle, the angular velocity and acceleration of the coupler and rocker can be analyzed using the first and second derivative of the closure loop equations which can be found in [13, 14].

The challenge associated with the synthesis of a crank-rocker mechanism is to design appropriate movements of the linkage for a given oscillation angle and time ratio. In addition, understanding and analyzing the motion is important for proper operation.

In order to generate a smooth motion and good transmissive force in a design, the angle between coupler and rocker (transmission angle) should be as close as possible to 90° [6]. Most researchers and designers recommended that the minimum transmission angle must be greater than 40° for best results [5, 15-17].

If the link lengths, time ratio and the rocker throw angle are specified, it is possible to determine the relationship between L_2 , L_4 , ϕ and S . The relationship can be written in the following form:

$$\phi = \left(2 * \sin^{-1} \left(\frac{L_2}{L_4} \right) \right) \quad (2)$$

The relationship between the crank-rocker stroke and throw angle can be defined as follows:

$$S = L_{41} * \phi \quad (3)$$

Substituting equation (2) into equation (3), the crank-rocker stroke is then given by:

$$S = L_{41} * \left(2 * \sin^{-1} \left(\frac{L_2}{L_4} \right) \right) \quad (4)$$

Where L_{41} is the extended rocker- link.

Dynamic analysis of the crank-rocker mechanism

In addition to the kinematic requirements of the crank-rocker mechanism, it is important to make sure that the mechanism is able to transmit output torques from the rocker to the crankshaft smoothly. Also, in order to carry out the optimum design of a crank rocker mechanism, it is very important to understand the dynamic performance of the mechanism.

In figure-3, the rocker length, L_4 has been extended (L_{41}) to a specified constant length. A constant force is applied at the tip and it is always perpendicular to the rocker length.

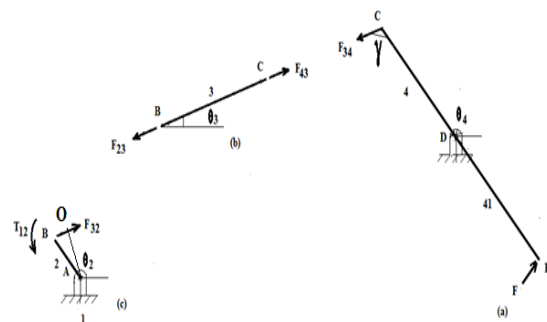


Figure-3. (a-c) Crank-rocker mechanism free-body diagram.

In order to calculate the transferred force from the rocker arm tip to the crank via the connecting rod,



dynamic force analysis has to be performed on the crank-rocker mechanism. The free-body diagram method is used to derive the equation of motion [5]. The free-body diagram of link 4, 3, and 2 are shown in Figure-3a, Figure-3b, and Figure-3c respectively.

The crank-rocker mechanism in this analysis is assumed to be balanced and for each case the value of moment of inertia is assumed to be the same. Therefore, the centre of mass of the mechanism is assumed to be concentrated at the rocker pivot. By taking the moment about D from Figure-3a, the dynamic equilibrium equation for the link is given below.

$$+\circlearrowleft \sum M_D = I_4 \alpha_4$$

$$F * L_{41} + F_{34} * \sin \gamma * L_4 = I_4 \alpha_4$$

$$F_{34} = \frac{(I_4 \alpha_4 - F * L_{41})}{\sin \gamma * L_4} \quad (5)$$

From figure 3b, the force magnitude at point B must be equal to the force at point C, thus

$$F_{23} = F_{34} \quad (6)$$

The force transmitted from the rocker to crank pin B is equal to

$$F_{32} = -F_{34} \quad (7)$$

However, only the tangential force (F_{32}) applied at joint B creates torque around point A on Link 2. Finally, from Figure-3c the torque output T_{12} can be calculated as follows:

$$T_{12} = F_{32} * AO \quad (8)$$

Where

$$AO = L_2 * \sin(\theta_2 - \theta_3) \quad (9)$$

Substituting (7) and (9) into (8) and simplifying:

$$T_{12} = \frac{(F * L_{41} - I_4 \alpha_4)}{\sin \gamma * L_4} * L_2 * \sin(\theta_2 - \theta_3) \quad (10)$$

Which upon Substitution $L_2 = L_4 * \sin\left(\frac{\theta}{2}\right)$ in equation (10), finally, the torque output will be

$$T_{12} = \frac{(F * L_{41} - I_4 \alpha_4)}{\sin \gamma * L_4} * L_4 * \sin\left(\frac{\theta}{2}\right) * \sin(\theta_2 - \theta_3) \quad (11)$$

The extended rocker length L_{41} and the amplitude of oscillation angle have been fixed to constant values. The ratio k is defined as follows:

$$k = L_{41} / L_4 \quad (12)$$

Substituting the value of $L_2 = L_{41}/k$ into equation (11) yields,

$$T_{12} = \left(\frac{F}{\sin \gamma} - \frac{I_4 \alpha_4}{L_{41} * \sin \gamma} \right) * L_{41} * \sin\left(\frac{\theta}{2}\right) * \sin(\theta_2 - \theta_3) \quad (13)$$

For the given link lengths L_1 , L_2 , L_3 , and L_4 at certain crank angle (θ_2), the interior joint angles (θ_3 , θ_4 , and γ) and the angular velocity and acceleration for the rocker arm can be calculated, and the equations can be summarized as follows [14]:

$$BD = \sqrt{L_1^2 + (H)^2 - 2 * L_1 * (H) \cos \theta_2} \quad (14)$$

The values for L_3 were set to be equal $3.5 * L_2$ as suggested by [18] while the values for L_1 was calculated based on the value of the throw angle and upon substitution of L_2 and L_4 is given by:

$$L_1 = \sqrt{(3.5 * H)^2 + (H_1)^2} \quad (15)$$

Where H and H_1 are:

$$H = \frac{L_{41}}{k} * \sin\left(\frac{\theta}{2}\right)$$

$$H_1 = \frac{L_{41}}{k} * \cos\left(\frac{\theta}{2}\right)$$

The angle between the connecting rod and the rocker γ and the rocker angle θ_4 are given by:

$$\gamma = \cos^{-1} \left(\frac{(3.5 * H)^2 + \left(\frac{L_{41}}{k}\right)^2 - BD^2}{2 * (3.5 * H) * \frac{L_{41}}{k}} \right) \quad (16)$$

$$\theta_4 = 180 - \cos^{-1} \left(\frac{L_1^2 - (H)^2 + BD^2}{2 * L_1 * BD} \right) - \cos^{-1} \left(\frac{\left(\frac{L_{41}}{k}\right)^2 - (3.5 * H)^2 + BD^2}{2 * \frac{L_{41}}{k} * BD} \right) \quad (17)$$

The connecting rod inclination angle is given by

$$\theta_3 = \theta_4 - \gamma \quad (18)$$

The angular velocity and acceleration can be calculated as follows:

$$\omega_2 = \omega_1 \left(\frac{(H) * \sin(\theta_2 - \theta_4)}{(3.5 * H) * \sin(\theta_4 - \theta_3)} \right) \quad (19)$$

$$\omega_4 = \omega_2 \left(\frac{(H) * \sin(\theta_2 - \theta_3)}{\frac{L_{41}}{k} * \sin(\theta_4 - \theta_3)} \right) \quad (20)$$



$$\alpha_4 = \left(\frac{(Q)}{L_{41}} \sin(\theta_4 - \theta_3) \right) \quad (21)$$

Where Q is:

$$Q = \frac{\left((H)^2 \omega_2^2 \cos(\theta_2 - \theta_3) - \frac{L_{41}}{k} \right)}{\omega_4^2 \cos(\theta_4 - \theta_3) + (3.5 * H) * \omega_3^2}$$

All of the previous equations have been derived to be dependent on variables k and θ_2 while the other parameters are constants. Due to the complexity of simplifying the previous equations, the characteristics of angular accelerations and torque output as functions of k and θ_2 were analyzed using graphical plots.

In this analysis, ω_2 is kept constant for all the cases. Therefore, the linkage performance is purely measured based on the output torque T_{12} . The constant speed applied at the crank acts like a dynamometer which provides brake torques to the system.

The effect of variation in geometry

In this paper, analysis was conducted in order to determine the best crank-rocker mechanism which will give the highest delivered torque, leading to the highest power output (angular speeds are kept constant).

The analysis is divided into two parts:

- Part 1 - Investigation of the relationship between rocker ratio k , on the angular acceleration and torque output.
- Part 2 - Observation of the effect of changing the distance between crank and rocker extreme position (d) or connecting rod lengths on the angular acceleration and torque output.

The results from the analysis were validated using ADAMS software.

Effect of rocker length ratio on the rocker angular acceleration and torque output

In order to study the effect of changing the ratio k on the angular acceleration and torque output of the crank-rocker mechanisms shown in figure 2, the stroke S , was fixed. This was done in order to keep all the input energy to all the systems the same. Therefore, link L_{41} and throw angle are maintained with the same values for all the mechanisms, while Link L_3 , is set to be 3.5 times the crank radius. The crank speed for this analysis was fixed at 2000 rpm.

The analysis was conducted by plotting the derived equations using MATLAB. The minimum k was set to 0.1 (must be non-zero) with increments of 0.1, while the maximum ratio was 3. A MATLAB program code was developed to compute the kinematic and dynamic analysis of the mechanisms in order to obtain the values of angular acceleration of the rocker, and from the crank at different crank angle positions.

For validation purposes of each case, ADAMS models were constructed using the dimensions specified in the previous equations with the values of k to be 0.8, 1 and 1.2. All the three values were selected and tested based on the following conditions:

$$L_{41} < L_4, L_{41} = L_4 \text{ and } L_{41} > L_4$$

The input forces acting on the mechanisms and their characteristics were kept the same as specified in the analysis.

Effect of changing the distance D , on the rocker angular acceleration and torque output

The connecting rod is responsible for the transmission of torque from the rocker to the crank shaft. In the conventional slider crank engine, the connecting rod cannot be designed to be very short because it can create high friction on the piston skirts and cylinder walls. Also, it cannot be too long because of the weight and the size of the engine will become very tall.

In the crank rocker mechanism, the connecting rod length can be calculated using the following equation:

$$L_3 = d + 2 * L_2 \quad (22)$$

Where d is the distance between the crank and rocker extreme positions (see Figure-2).

For analysis, the following parameters are defined:

Ratio of connecting rod length to crank radius,

$$R = L_3 / L_2$$

Ratio of length d to crank radius, $C = d / L_2$

Therefore, the relationship between the two parameters is given as follows:

$$C = R - 2 \quad (23)$$

In this analysis, only one of the crank rocker mechanism dimensions from part 1 was selected for analysis, i.e. for $k = 1.2$. The main objective was to determine the best dimension, d which can produce the optimum results. Therefore, the dimensionless parameter C was analyzed for general solutions. Different values of C from 0 to 7 with an incremental value of 0.5 were used to compute the torque outputs and the angular acceleration. Similar to part 1, the crank speed was fixed at 2000 rpm. The maximum value of C was chosen due to the requirements for the design of a connecting rod for crank slider engine. For internal combustion engines, the ratio of connecting rod length to crank radius (R) is between 3 to 4 for small and medium size engines, while for large engines it is between 5 to 9 [18].

Verification of results using ADAMS software

In order to verify the results, 3D models of crank-rocker mechanisms were developed using ADAMS software. The models of the crank-rocker mechanisms were simulated using a constant force of magnitude 100N on the rocker tip and perpendicular to the linkage at all



times while their directions change when extreme positions are reached. The **dimensions** as well as the moment of inertias were set to be the same as the theoretical analysis.

RESULTS AND DISCUSSION

The results of the kinematic and dynamic analysis for different rocker ratio calculated using MATLAB programs are plotted in Figures-4 and 5. The results from the ADAMS software are also included in these figures for comparison purposes.

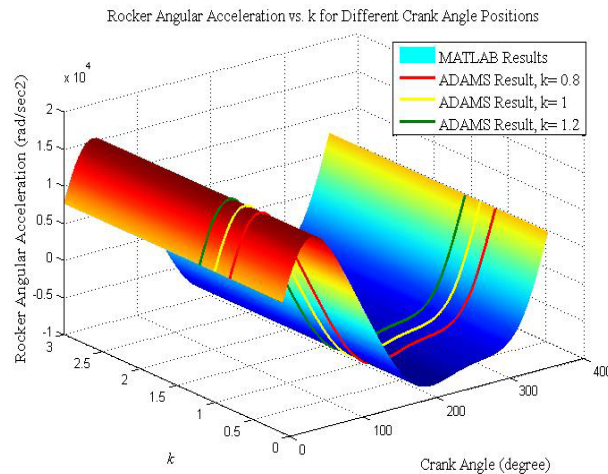


Figure-4. The rocker angular acceleration vs rocker ratios (k) at different crank angle position.

From Figure-4, it can be noted that the rocker angular acceleration remain constant for all the mechanisms for any value of k . For the amplitude of oscillation at 30° , the peak positive value of the rocker angular acceleration is found to be $15,133 \text{ rad/s}^2$ which occurs at 40° crank angle (i.e. 7° before the left extreme position of the rocker).

It is concluded that the value of the rocker ratio does not have any effect on the rocker angular acceleration. The explanation to this is because all the links, except the extended rocker link, are reduced in proportion. Therefore, all the angles are kept constant and this means that the kinematic for the mechanisms remain the same. Based on the selected axis, the extreme positions occur at 47° and 227° . It can be observed from figure 4 that at a crank angle of about 125° (i.e. 78° after the left extreme position of the rocker), the rocker angular acceleration begins to change from positive to negative values and reverses direction again from negative to positive at about 350° (i.e. 57° before the left extreme position of the rocker).

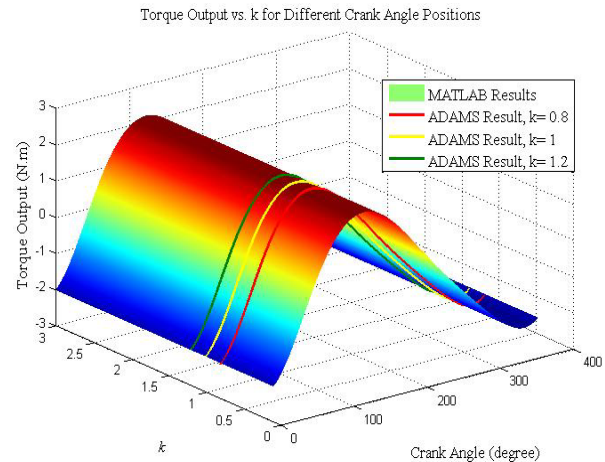


Figure-5. Output torque vs rocker ratios (k) at different crank angle position.

The simulation result in figure 5 shows the torque tracking performance of the proposed crank-rocker mechanisms at a constant speed ($N = 2000 \text{ rpm}$) and the same mechanism stroke. Despite the variation of the rocker ratio k , the mechanism still produces the same output torque. This means that the torque output is independent of the rocker ratio. The explanation is mainly due to the fact that the moment of inertia and angular acceleration of the rocker links remain the same.

It can be noticed that the maximum negative value of the torque output is -2.21 N.m which occurs at 345° (i.e. 62° after the left extreme position of the rocker), while the peak positive value is 2.12 N.m which occurs at 125° (i.e. 78° after the left extreme position of the rocker). Based on the kinematic and dynamic analyses, it can be validly concluded that the variation of the rocker ratio k while keeping other parameters constant does not have any effect on the proposed crank-rocker performance.

For validation purposes, the output results for rocker angular acceleration and torque computed using ADAMS software are also plotted in figures 4 and 5 respectively. Three rocker ratios were selected with the values of k of 0.8, 1 and 1.2 for this purpose. It was found that the curves match almost perfectly with an error less than 2%, indicating agreement of the results from MATLAB and ADAMS software.

The results for the angular acceleration and torque output for different ratio of C using the MATLAB code are plotted in Figures-6 and 7 respectively. For validation purposes, the results from ADAMS software are also plotted on the same figures.

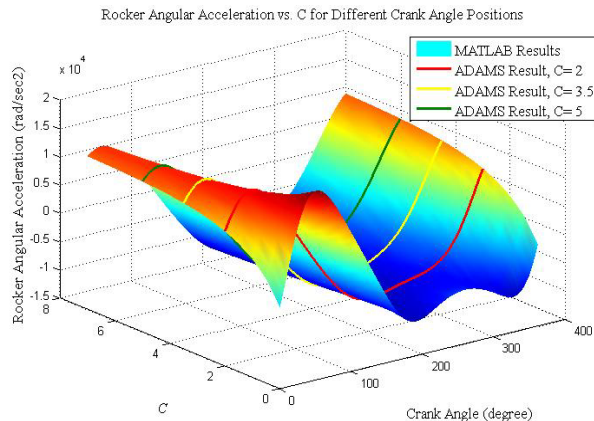


Figure-6. The rocker angular acceleration vs ratios (C) at different crank angle position.

From Figure-6, it can be observed that for different ratio of C and at any fixed crank angle, the values of angular acceleration and their peaks vary as well. It was found that the maximum acceleration was about $18,330 \text{ rad/s}^2$ which occurred at an angle about 50° (i.e. 12° before the left extreme position of the rocker) when the ratio C is zero, while the minimum peak value was about $13,092 \text{ rad/s}^2$ and it occurred at an angle about 20° (i.e. 3° before the left extreme position of the rocker) when the ratio C was 7. In general, the smaller values of C resulted in higher values of acceleration.

The explanation to this is because each mechanism has different extreme position angles. The mechanism with smaller ratio reaches the extreme position late in terms of the crank angle and this leads to the higher peak points of angular acceleration. In other words, variations in the lengths of connecting rods result in the difference of extreme positions.

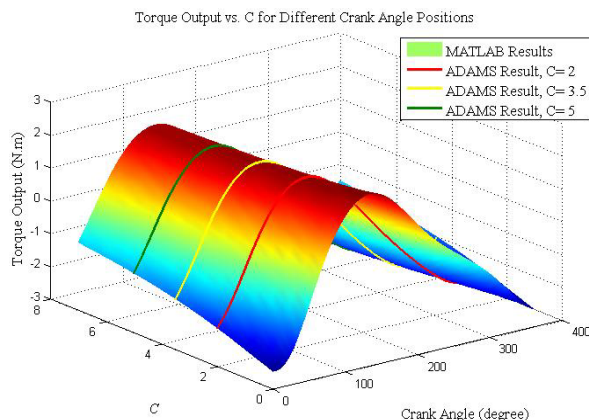


Figure-7. Torque output vs. ratio (C) at different crank angle position.

Figure-7 shows the torque output against crank angle for different ratios of C . As can be seen from Figure-7, when C is varied at any fixed crank angle, the output torque is found to vary. This trend is similar to the characteristics of angular accelerations.

It can be concluded that at any fixed crank angle, the smaller ratio of C resulted in higher torque. However, for all the cases, there is only a small variation in the peak torque values. This is because the peak values occurred at the late crank angles for small values of C as compared to larger values. The magnitudes of the peak torque when C equals to 0, 3.5 and 7 are 2.23, 2.1, and 2.08 N.m and occurred at 135° , 115° , and 105° (i.e. 73° , 82° , and 81° after the left extreme position of the rocker) respectively.

For validation purposes, three different ratios were selected with the values of C to be 2, 3.5 and 5. The results from figures 8 and 9 shows that they tend to agree to the simulation results obtained from ADAMS software. As can be seen, slight difference or error between MATLAB and ADAMS software results is less than 1.5%.

SUMMARY AND CONCLUSIONS

To summarize, this work presents the analysis of a general four-bar mechanism with extended rocker arm link in order to determine the optimized dimensions of the rocker ratio and connecting rod lengths. The effect of the rocker ratios, k and the distance between the crank and rocker extreme positions, d in obtaining the optimal results which meet the design requirement was analyzed.

It is concluded that varying the rocker ratio k while keeping other parameters constant does not have any effect on the crank-rocker performance. There is a significant effect of varying the distance d on the angular acceleration and torque output. It was found that the shorter connecting rod is better than the longer connecting rod in terms of the peak values of angular acceleration and torque.

For better power outputs and packaging advantage, the mechanism can be designed to have smaller crank radius and shorter connecting rod. The presented results are useful and reliable since they can be used to design any similar mechanisms with different dimensions and parameters. Moreover, the accuracy of results was further validated using ADAMS software.

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