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GENERALIZED SCATTERING MATRIX METHOD FOR ANALYSIS OF CASCADED UNI-AXIAL DISCONTINUITIES

Chaabane Faten[1], Benzina Hafedh[2], ElmirMabrouk Lassaad[1] and Jun Wu Tao[3]
[1]Laboratory of Physics of Materials and Nanomaterials applied at Environment (LaphyMNE), Faculty of Sciences of Gabes, ErriadhManaraZrig, Gabes, Tunisia
[2]Research Unit HatemBettaher Computer, Networks, Communication Systems and Mathematics (IReSCoMath) National School of Engineering of Gabes, Omar IbnElkhattab, Zrig, Gabes, Tunisia
[3]Laboratory of Plasma and Energy (LAPLACE) National School of Electrical Engineering, Electronics,Computer Science, Hydraulicsand Telecommunications, Charles Camichel, Toulouse, cedex France
E-Mail: Faten85@live.fr

ABSTRACT

In this paper we present a Generalized Scattering Matrix (GSM) approach using Mode Matching Method (MMM) for characterizations of cascaded uniaxial discontinuities in rectangular waveguides. An analysis of single, double and multiple step discontinuities for rectangular waveguides loaded of ferrite magnetized longitudinally is obtained. To validate the result of (MMM), another analysis is carried out by using commercial software, namely HFSS. There is a good agreement between the calculated scattering (S) parameters and these obtained with HFSS.

Keywords: discontinuities, ferrites, GSM, rectangular waveguide.

1. INTRODUCTION

Waveguide discontinuities have been the subject of intense research interest in the past several years.

For assessment of the effects of such discontinuities on the transmission and reflection properties of rectangular waveguide, many analytical techniques can be used.

An equivalent circuit approach based on an electrostatic approximation and variational principle has been used to analyze these discontinuities. However, these approximate techniques may not be accurate for electrically large discontinuities. Furthermore, only single mode interactions are accounted in these simple representations.

For these different reasons, many researchers have been applied a numerical methods in the analysis of discontinuities between waveguides because they were more efficient.

We can cite: the mode matching method (MMM) [1],[2],[3], the finite element method (FEM) [4], [5], the finite-difference time-domain method (FDTD) [6], [7], the method of lines (MoL) [8], [9] and the multimodal variational formulations (MVM) [10], [11] etc.

In this paper we have applied the mode matching method (MMM) for designing the rectangular waveguides and analyzing the discontinuities. In the MMM, the fields in each region across the junction are expressed in terms of infinite number of waveguide modal functions. Application of continuity of tangential components of electric and magnetic fields across the junction in conjunction with the Galerkin's method yields a matrix equation with tangential fields over the junction as an unknown variable. From the solution of the matrix equation, the reflection and the transmission properties of the waveguide junction are determined. There are different versions of the MMM reported in the literature such as Generalized Scattering Matrix (GSM) techniques, multimodal network representation methods using admittance, or impedance matrix representation.

In this article, we employ the GSM technique to analyze the discontinuity problem between ferrite loaded waveguides.

Many investigations concerning ferrite-loaded waveguides have been reported, including analysis of discontinuity problems by a coupled-line model [12], modal expansion methods [13]-[14], transverse operator method and generalized scattering method [15].

The ferrite can be magnetized transversely or longitudinally. The studies of waveguides with transverse magnetization are more abundant than those with longitudinal magnetization. However, the research works about the rectangular waveguide loaded with ferrite with longitudinally magnetization present a considerable lack. At hyperfrequencies, the ferrites are characterized by scalar permittivity (diagonal permittivity tensor) and tonsorial permeability [16] which represents their inducted anisotropy under a magnetic field.

$$\overline{\overline{\mu}}_f = \mu_0 \cdot \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} = \mu_0 . \overline{\overline{\mu}}_{rf}$$

With elements μ, k and μ_{rz} given in [17]- [18], [19]-[20].

Modal analysis of single step discontinuities, two cascaded junctions discontinuities and multiple step discontinuities have been reported. All simulations are developed in Matlab and will be compared with HFSS.

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2. METHOD OF ANALYSIS

In this section, Generalized Scattering Matrix (GSM) is used to characterize the uni-axial discontinuities between rectangular waveguides and determine the S parameters as a function of frequency. The discontinuities are considered without losses. This method based on the modal development of the transverse fields.

A. Analysis of scattering matrix method

We consider the junction between two rectangular waveguides represented on the Figure-1, where a^i and b^i are respectively the incident and reflected waves in the guides forming the discontinuity. The transverse sections of the guides are S_I and S_{II} respectively of guide 1 and 2.

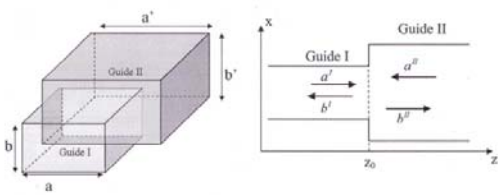

Figure-1. The incident and reflected waves at the discontinuity.

The transverse electric and magnetic fields (E_T, H_T) in the waveguides can be written in the modal bases as follows [10], [11].

$$E_T = \sum_{m=1}^{\infty} A_m^i \left(a_m^i + b_m^i\right) e_m^i \tag{1}$$

$$H_T = \sum_{m=1}^{\infty} B_m^i \left(a_m^i - b_m^i\right) h_m^i \tag{2}$$

Where E_T and H_T are the transverse electric and magnetic fields (the index T refers to the components in the transverse plane i.e in the plan Oxy). A and B are complex coefficients which are determined from the normalization of the power (i=I, II; m is the index of the mode). e_m^i (respectively h_m^i) represents the mth real orthonormal basis vector mode of electric fields (magnetic respectively) in the guide i.

At the level of discontinuity, i.e. on the surface S_I in the plan z = z_0, the continuity of the fields allows writing the following equations:

$$E_t^{II} = \begin{cases} E_t^I & sur\ S_I \\ 0 & sur\ S_I - S_{II} \end{cases} \tag{3}$$

$$H_t^I = H_t^{II} \tag{4}$$

For the reason of numerical calculation, we are going to take N_1 modes in guide 1 and N_2 modes in guide 2.

By using equations (1-2) in (3-4), we obtain

$$\sum_{m=1}^{N_1} A_m^I \left(a_m^I + b_m^I\right) e_m^I = \sum_{p=1}^{N_2} A_p^{II} \left(a_p^{II} + b_p^{II}\right) e_p^{II} \tag{5}$$

$$\sum_{m=1}^{N_1} B_m^I \left(a_m^I - b_m^I\right) h_m^I = \sum_{p=1}^{N_2} B_p^{II} \left(-a_p^{II} + b_p^{II}\right) h_p^{II} \tag{6}$$

The use of the Galerkin's method, which consists in taking the inner product by the basis vector mode both members of each of the equations (5-6), leads to the following systems

$$\sum_{m=1}^{N_1} A_m^I \left(a_m^I + b_m^I\right) \left\langle e_m^I \middle| e_p^{II} \right\rangle = A_p^{II} \left(a_p^{II} + b_p^{II}\right) \tag{7}$$

$$B_m^I \left(a_m^I - b_m^I\right) = \sum_{p=1}^{N_2} B_p^{II} \left(-a_p^{II} + b_p^{II}\right) \left\langle h_p^{II} \middle| h_m^I \right\rangle \tag{8}$$

The scalar product is defined on the surface of the junction as:

$$\left\langle e_m \middle| e_p \right\rangle = \int_{S_I} e_m^* e_p \, dS \tag{9}$$

Where e_m^* is the complex conjugate of e_m.

The equations (7-8) give

$$-a_p^{II} + \sum_{m=1}^{N_1} \frac{A_m^I}{A_p^{II}} a_m^I \left\langle e_m^I \middle| e_p^{II} \right\rangle = b_p^{II} - \sum_{m=1}^{N_1} \frac{A_m^I}{A_p^{II}} b_m^I \left\langle e_m^I \middle| e_p^{II} \right\rangle \tag{10}$$

$$a_m^I + \sum_{p=1}^{N_2} \frac{B_p^{II}}{B_m^I} a_p^{II} \left\langle h_p^{II} \middle| h_m^I \right\rangle = b_m^I + \sum_{p=1}^{N_2} \frac{B_p^{II}}{B_m^I} b_p^{II} \left\langle h_p^{II} \middle| h_m^I \right\rangle \tag{11}$$

Which can be written as the matrix form

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$$\begin{bmatrix} U & M_1 \\ M_2 & -U \end{bmatrix} \begin{bmatrix} a_1^I \\ . \\ a_{N_1}^I \\ a_1^{II} \\ . \\ a_{N_2}^{II} \end{bmatrix} = \begin{bmatrix} U & M_1 \\ -M_2 & U \end{bmatrix} \begin{bmatrix} b_1^I \\ . \\ b_{N_1}^I \\ b_1^{II} \\ . \\ b_{N_2}^{II} \end{bmatrix} \quad (12)$$

Where U is the identity matrix M_1 and M_2 (of dimension respectively $(N_1 \text{x } N_2)$ and $(N_2 \text{x } N_1)$) are matrixes of general term

$$M_{1ij} = \frac{B_j^{II}}{B_i^I} \left\langle h_j^{II} \middle| h_i^I \right\rangle \quad (13)$$

$$M_{2ij} = \frac{A_i^I}{A_j^{II}} \left\langle e_i^I \middle| e_j^{II} \right\rangle \quad (14)$$

So we obtain the diffraction matrix of the discontinuity (or generalized S matrix)

$$S = \begin{bmatrix} U & M_1 \\ -M_2 & U \end{bmatrix}^{-1} \begin{bmatrix} U & M_1 \\ M_2 & -U \end{bmatrix} \quad (15)$$

of the dimensions ((N1 + N2) x (N2 + N1)).

B. Analysis of double discontinuity by scattering matrix method

Figure-2 represents a double uni-axial discontinuity, the total matrix will be obtained by making the S transfer matrices of discontinuities separated by waveguide of equal lengths to the distances between the discontinuities.

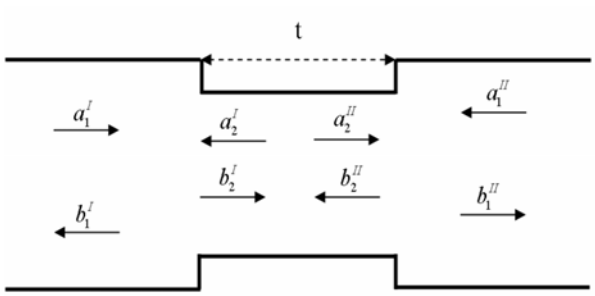

Figure-2. Double discontinuity.

From the incident and reflected waves, we can write the following equations

$$\begin{pmatrix} \begin{bmatrix} b_1^I \end{bmatrix} \\ \begin{bmatrix} b_2^I \end{bmatrix} \end{pmatrix} = S^I \begin{pmatrix} \begin{bmatrix} a_1^I \end{bmatrix} \\ \begin{bmatrix} a_2^I \end{bmatrix} \end{pmatrix} \quad (16)$$

$$\begin{pmatrix} \begin{bmatrix} b_1^{II} \end{bmatrix} \\ \begin{bmatrix} b_2^{II} \end{bmatrix} \end{pmatrix} = S^{II} \begin{pmatrix} \begin{bmatrix} a_1^{II} \end{bmatrix} \\ \begin{bmatrix} a_2^{II} \end{bmatrix} \end{pmatrix} \quad (17)$$

$$\begin{bmatrix} b_1^I \end{bmatrix} = S_{11}^I \begin{bmatrix} a_1^I \end{bmatrix} + S_{12}^I \begin{bmatrix} a_2^I \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} b_2^I \end{bmatrix} = S_{21}^I \begin{bmatrix} a_1^I \end{bmatrix} + S_{22}^I \begin{bmatrix} a_2^I \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} b_1^{II} \end{bmatrix} = S_{11}^{II} \begin{bmatrix} a_1^{II} \end{bmatrix} + S_{12}^{II} \begin{bmatrix} a_2^{II} \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} b_2^{II} \end{bmatrix} = S_{21}^{II} \begin{bmatrix} a_1^{II} \end{bmatrix} + S_{22}^{II} \begin{bmatrix} a_2^{II} \end{bmatrix} \quad (21)$$

There are

$$\begin{bmatrix} a_2^I \end{bmatrix} = D \begin{bmatrix} b_2^{II} \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} a_2^{II} \end{bmatrix} = D \begin{bmatrix} b_2^I \end{bmatrix} \quad (23)$$

With

$$D = \begin{bmatrix} e^{-\gamma_1 . t} & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & e^{-\gamma_N . t} \end{bmatrix} \quad (24)$$

γ_i is the propagation constant of the ith mode of the central guide and N is the number of mode in the same guide. t is the length of the middle waveguide.

Using the equations (22) and (23), we get

$$\begin{bmatrix} b_1^I \end{bmatrix} = S_{11}^I \begin{bmatrix} a_1^I \end{bmatrix} + S_{12}^I D \begin{bmatrix} b_2^{II} \end{bmatrix} \quad (25)$$

$$\begin{bmatrix} b_1^{II} \end{bmatrix} = S_{11}^{II} \begin{bmatrix} a_1^{II} \end{bmatrix} + S_{12}^{II} D \begin{bmatrix} b_2^I \end{bmatrix} \quad (26)$$

$$\left[b_2^I\right] = S_{21}^I\left[a_1^I\right] + S_{22}^I D\left[b_2^{II}\right] \quad (27)$$

$$\left[b_2^{II}\right] = S_{21}^{II}\left[a_1^{II}\right] + S_{22}^{II} D\left[b_2^I\right] \quad (28)$$

Equation (28) becomes in using (27)

$$\left[b_2^{II}\right] = S_{21}^{II}\left[a_1^{II}\right] + S_{22}^{II} D S_{21}^I\left[a_1^I\right] + S_{22}^{II} D S_{22}^I D\left[b_2^{II}\right] \quad (29)$$

We put

$$E = \left[U - S_{22}^{II} D S_{22}^I D\right]^{-1} \quad (30)$$

U is the identity matrix. As a result

$$\left[b_2^{II}\right] = E S_{22}^{II} D S_{21}^I\left[a_1^I\right] + E S_{21}^{II}\left[a_1^{II}\right] \quad (31)$$

$$\left[b_1^I\right] = \left[S_{11}^I + S_{12}^I D E S_{22}^{II} D S_{21}^I\right]\left[a_1^I\right] + \left[S_{12}^I D E S_{21}^{II}\right]\left[a_1^{II}\right] \quad (32)$$

$$\left[b_1^{II}\right] = S_{12}^I D\left[U + S_{22}^I D E S_{22}^{II} D\right] S_{21}^I\left[a_1^I\right] + \left[S_{11}^{II} + S_{12}^{II} D S_{22}^I D E S_{21}^{II}\right]\left[a_1^{II}\right] \quad (33)$$

The matrix S of the double discontinuity is given by

$$S = \begin{bmatrix} S_{11}^I + S_{12}^I D E S_{22}^{II} D S_{21}^I & S_{12}^I D E S_{21}^{II} \\ S_{12}^I D\left[U + S_{22}^I D E S_{22}^{II} D\right] S_{21}^I & S_{11}^{II} + S_{12}^{II} D S_{22}^I D E S_{21}^{II} \end{bmatrix} \quad (34)$$

Thus, the matrix S of several discontinuities in cascade may be determined from the equation (34) by transfer two matrices S_i to two.

3. NUMERICAL RESULTS

A. Simple discontinuity

We consider a simple discontinuity between two rectangular waveguides (WR62) of the same dimensions (a=15.9 mm, b=7.9 mm) loaded with isotropic materials;

(i) empty or hollows rectangular waveguide (ε_{r1}=1, μ_{r1}=1)

(ii) rectangular waveguide fully loaded with ferrite (TT1-414) such as (ε_{r2}=11.3, 4πMs=750 Gauss) as shown in Figure-3.

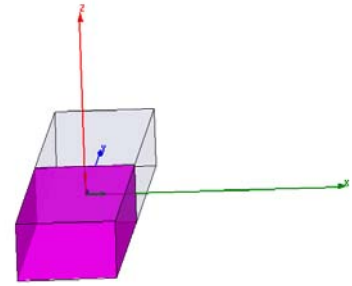

Figure-3. Simple discontinuity.

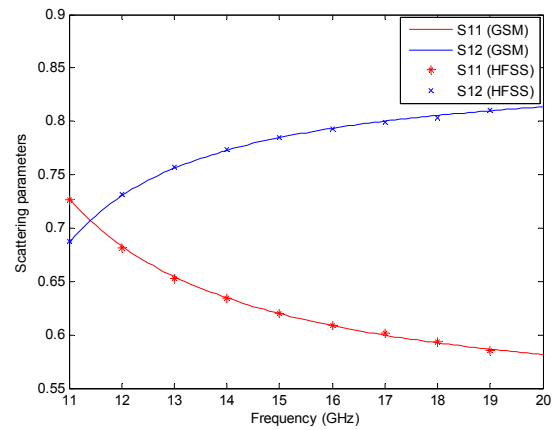

Figure-4. Reflection and transmission coefficients of the simple discontinuity.

Figure-4 represents the coefficients of reflection and transmission of discontinuity as a function of the frequency using the GSM. Eight modes were used in the two waveguides.

To validate the numerical results obtained from (GSM) method, the simulation results using commercial software, namely HFSS from Ansoft are depicted together for comparison in Figure-4.

It is seen from these results, an exellent agreement between the simulated and calculated S_{ij} parameters.

However, GSM is much faster than HFSS.

B. Double discontinuity

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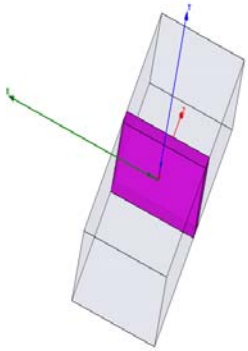

Figure-5. Double discontinuity.

Figure-5. represents a double discontinuity between three rectangular waveguides of the same dimensions (a=15.9 mm, b=7.9mm). The guides of the input and output are empty. The resonator of width d=5mm is filled with the ferrite (TT86-6000) such as (ε_{r2}=12.5, 4πMs=5000 Gauss).

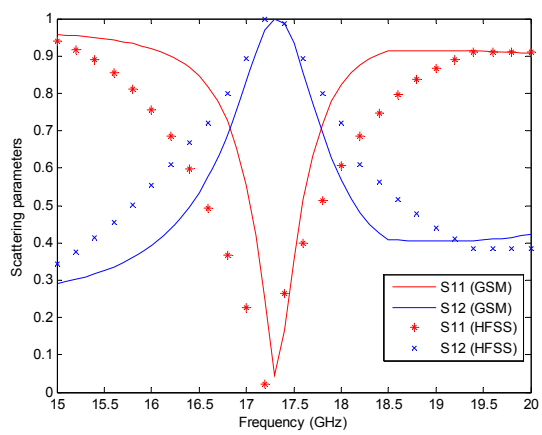

Figure-6. Reflection and transmission coefficients of the double discontinuity as functions of frequency

The magnitude of the calculated scattering coefficients of the resonator using HFSS and the chaining of matrixes S obtained by the GSM is shown in Figure-6. For the GSM, we used 8 modes in the three guides.

From the result depicted in Figure-6, it shows that the GSM simulation result is comparable to the result of commercial software with some disparities in the amplitude of reflection coefficient in some frequency range.

The resonate frequency is equal to 17.3 GHz.

C. Multiple-step discontinuities

Finally, in order to demonstrate the cascade of Generalized Scattering Matrix (GSM), we considere twelve discontinuities constituted by thirteen rectangular waveguides (see Figure-7) of the same dimensions (a=15.9 mm, b=7.9mm).

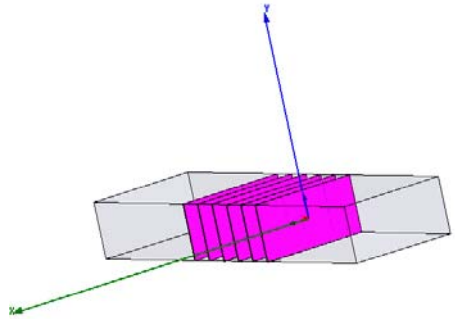

Figure-7. Multiple-step discontinuities.

The circuit is formed in alternation by empty guide (ε_{r1}=1, μ_{r1}=1) of width l=8mm and guide filled by the ferrite (TT1-414) such as (ε_{r2}=11.3, 4πMs=750 Gauss) of width d=5 mm.

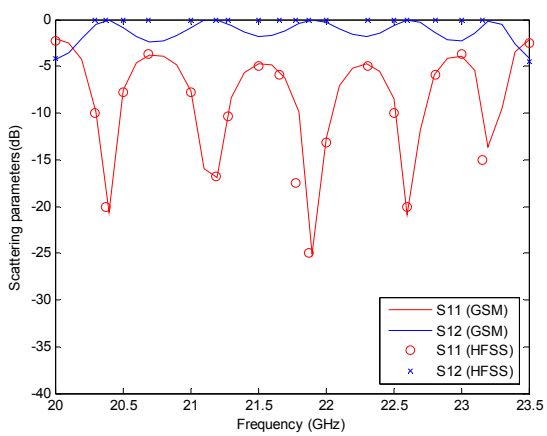

Figure-8. Reflection and transmission coefficients of multiple-step discontinuities as functions of frequency.

Figure-8 shows typical results of the amplitude of the reflection and transmission coefficients from a cascaded step discontinuities. The results are found to agree very well with the simulation of HFSS.

However, the GSM is significantly faster than the HFSS especially if the number of discontinuities increases. We obtain a band-pass filter with a band width of 4 GHz.

4. CONCLUSIONS

In this paper, the Generalized Scattering Matrix technique (GSM) has been applied to the problems of a cascaded step discontinuity in rectangular waveguides filled with anisotropic materials (ferrite).

The numerical results of the approach are in good agreement with HFSS.

The proposed method reduces the global computing time of convergence with about 95% in all studied cases compared to HFSS.

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