ANALYSIS OF VARIANTS OF DIFFERENTIAL TORQUE CONTROL APPLIED TO INDUCTION MOTOR WITH SHORT-CIRCUITED ROTOR

Andrey Edwardovich Evstratov¹, Valery Mihailovich Zavyalov¹, Alexander Vasilyevich Grigoryev² and Irina Yuryevna Semykina¹
¹National Research Tomsk Polytechnic University, Lenin Avenue, Tomsk, Russia
²T. F. Gorbachev Kuzbass State Technical University, Vesennaya Str., Kemerovo, Russia
E-Mail: semykina@tpu.ru

ABSTRACT
The article discusses the existing control methods of an induction motor electromagnetic torque and offers the new control method, called differential torque control. The authors present a few options of the differential control algorithm and carry out their analysis in various induction motor operation modes. Analysis provides a physical interpretation of the results and contributes to the formation of proposals to improve the algorithms. The research substantiates the adjustment parameters of the algorithms and estimates the torque control quality. The computer simulation confirms the high quality of the proposed method of differential torque control as compared to the direct torque control.

Keywords: electric drive, induction motor, differential torque control, electromagnetic torque.

1. INTRODUCTION
Electric drives in industrial facilities based on induction motors are most common nowadays. Reasons for this are ease of maintenance, low cost, high overload capability and reliability. However, induction motor is difficult to control because of non-linearity of processes taking place in it.

Problems of induction motor position control have been studied for almost eighty years. The first way of control, which had been developed by academician M.V. Kostenko [1], became widespread for feedforward control systems, where high precision and high speed of control are not necessary.

Further development of IM control theory can be divided in two ways: scalar and vector. The first one is frequently used in closed loop systems, where high demands to speed and precision of control are not made. The second one had been intensively developed by foreign researchers like, for example, F. Blaschke and M. Depenbrock and was finally named as field-oriented (vector) control [2] and direct torque control [3, 4]. These control systems became widespread in industry. The key parameter in production intensification is operating speed of electric drive, thereby below in this article criterion of operating speed is considered as determinant in evaluation of different control systems.

Among said IM control systems direct torque control (DTC) has the highest operating speed. Its basic algorithms have discontinuous nature, which leads to significant electromagnetic torque ripples, thereby many researchers offer use of the combination of discontinuous control in transitions and PWM control in static modes [6] or use of the constant switching frequency of transformer’s power switches [7, 8].

Vector control (VC) has lower operating speed than DTC. This method has been improved by such scientists as Pankratov V.V. [9], Koziaruk A.E. [10] and many others [11, 12] to have higher operating speed, however such modifications of algorithms lead to complication of control system.

Besides traditional methods of control there are other methods like sliding mode control [13 17], intelligent control [18] and fuzzy logic based control [19, 20].

Each of these methods lets you reach goals of adjustment theoretically; however there are difficulties in technical implementation of these methods. For example, creation of fuzzy logic based systems requires maintenance specialists, but, according to sliding mode theory [21], one of the conditions for making the system is power switch switching frequency tending to infinity.

Thereby the problem of finding new solutions for creation of high-speed IM control systems is still of current interest. Labor [23] describes the algorithm of electro-mechanic energy transformation process control based on generalized electrical machine, with the perspective of extending the result to basic kinds of electrical machines as private cases of method implementation including IM.

Considering obtaining this control method from adjustable values derivatives forming conditions, further on in this text the method will be called as differential control.
2. THE MODEL

Mathematical model of the motor in labor [23] is represented as generalized electrical machine for arbitrary frame of reference. In vector notation it will be:

\[
\begin{align*}
\Psi_1 &= U_1 - \mathbf{I}_1 R_1 - \omega_0 \mathbf{D} \Psi_1; \\
\Psi_2 &= U_2 - \mathbf{I}_2 R_2 - \left( \omega_0 - \omega \right) \mathbf{D} \Psi_2,
\end{align*}
\]

where \( \Psi_1 = [\Psi_{1\alpha} \Psi_{1\beta}]^T \) — stator flux linkage vector; \( \Psi_2 = [\Psi_{2\alpha} \Psi_{2\beta}]^T \) — rotor flux linkage vector; \( U_1 = [u_{1\alpha} \ u_{1\beta}]^T \) — stator voltage vector; \( U_2 = [u_{2\alpha} \ u_{2\beta}]^T \) — rotor voltage vector; \( R_1, R_2 \) — stator and rotor windings active resistances respectively; \( \omega \) — motor rotor angular velocity; \( \omega_0 \) — frame of reference rotation speed; \( \mathbf{I}_1 = [i_{1\alpha} \ i_{1\beta}]^T \) — stator current vector; \( \mathbf{I}_2 = [i_{2\alpha} \ i_{2\beta}]^T \) — rotor current vector; \( \mathbf{D} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \) — vector rotation matrix.

Generalized control algorithm for this fully controlled motor, according to [23], is:

\[
\begin{align*}
U_1 &= \frac{1}{2} \left( h_1 \Psi_{1\alpha} - h_2 \Psi_{1\beta} \right) M_{\text{ref}} + \left( h_3 \Psi_{1\alpha} - h_4 \Psi_{1\beta} \right) \Psi_{1\alpha}; \\
U_2 &= \frac{1}{2} \left( -h_1 \Psi_{2\alpha} + h_2 \Psi_{2\beta} \right) M_{\text{ref}} + \left( -h_3 \Psi_{2\alpha} + h_4 \Psi_{2\beta} \right) \Psi_{2\alpha},
\end{align*}
\]

where \( h_1, h_2, h_3, h_4 \) — positive functions of constant signs; \( M \) — motor rotor electro-magnetic torque; \( \Psi_{1\alpha}, \Psi_{1\beta}, \Psi_{2\alpha}, \Psi_{2\beta} \) — required values of stator and rotor flux linkage modulus respectively; \( M_{\text{ref}} \) — required value of electromagnetic torque derivative.

Since (1) is formed in [23] on the assumption of motor predefined derived quantities, this approach allows to provide high operating speed with defined intensity of transitions. So it is advisable to analyze application of the algorithm in view of characteristics of motor with short-circuited rotor.

3. IM DIFFERENTIAL CONTROL ALGORITHM

The distinctive feature of mathematical model of IM with short-circuited rotor in comparison with [23] is that rotor winding voltage always equals zero. It is because of IM design features. In accordance with that let’s consider control method using only first equation of the system (1). This equation, being expanded to vector components in fixed reference frame \( \alpha-\beta \), becomes:

\[
\begin{align*}
u_{i_{1a}} &= \frac{1}{2} \left( -h_2 \Psi_{1\beta} M_{\text{ref}} - M \Psi_{1\beta} \right) + i_{1a} R_1; \\
u_{i_{1\beta}} &= \frac{1}{2} \left( h_2 \Psi_{1\alpha} M_{\text{ref}} - M \Psi_{1\alpha} \right) + i_{1\beta} R_1; \\
u_{i_{2a}} &= \frac{1}{2} \left( -h_1 \Psi_{2\beta} M_{\text{ref}} - M \Psi_{2\beta} \right) + i_{2a} R_2; \\
u_{i_{2\beta}} &= \frac{1}{2} \left( h_1 \Psi_{2\alpha} M_{\text{ref}} - M \Psi_{2\alpha} \right) + i_{2\beta} R_2.
\end{align*}
\]

Over implementation of this control algorithm (2) functions of fixed signs \( h \) will be formed proportionally to corresponding control error:

\[
\begin{align*}
h_1 &= \frac{M_{\text{ref}} - M}{k_1}; \\
h_2 &= \frac{\Psi_{1\beta} M_{\text{ref}} - \Psi_{1\beta}}{k_3},
\end{align*}
\]

providing continuous change of voltage vector modulus and position. Herewith constant coefficients \( k \) are formed by their normalization relatively to nominal values of controlled variables:

\[
\begin{align*}
k_1 &= \frac{U_{\text{max}}}{M_{\text{H}}}; \\
k_3 &= \frac{U_{\text{max}}}{M_{\text{H}}},
\end{align*}
\]

where \( U_{\text{max}} \) — maximal motor voltage with restrictions; \( M_{\text{H}} \) — nominal motor torque; \( M_{\text{H}} \) — nominal motor flux linkage.

In addition, let’s consider one more way of forming the voltage vector on maximal value level restrictions. To make this, after equations (2) we should normalize voltage vector modulus in relation with:

\[
\begin{align*}
u_{i_{1a}} &= \frac{u_{i_{1a}} U_{\text{max}}}{\sqrt{u_{i_{1a}}^2 + u_{i_{1\beta}}^2}}; \\
u_{i_{1\beta}} &= \frac{u_{i_{1\beta}} U_{\text{max}}}{\sqrt{u_{i_{1a}}^2 + u_{i_{1\beta}}^2}}.
\end{align*}
\]

4. ALGORITHMS ANALYSIS

For examination of the suggested IM control algorithm computer model of electric drive system was used with following motor parameters: \( R_1 = 7.5 \) Ohm, \( R_2 = 5 \) Ohm, stator winding inductance \( L_1 = 0.285 \) H, rotor winding inductance \( L_2 = 0.283 \) H, stator and rotor windings mutual inductance \( L_{\text{m}} = 0.275 \) H, number of poles pairs \( P = 2 \), rotor inertia moment \( J = 0.1 \) kg∙m².

The results of simulation with step change of predefined electro-magnetic torque are shown in Figure-1.
As it can be seen from Figure-1, electro-magnetic torque starts to decrease without reaching the predefined value, i.e. the goal of control is not achieved contrary to the prognosis from [23]. Let’s analyze (1) in the context of IM to explain our results.

According to [1], to achieve the goal of control the next condition should be provided:

$$\text{sign}(E_1D_2\Psi_1 + \text{DE}_2\Psi_1) = \text{sign}M_{ref}$$

(5)

where $E_1$ - stator winding EMF vector; $E_2$ - rotor winding EMF vector; $M_{ref}$ - required value of electro-magnetic torque derivative.

Let’s consider these conditions in the context of physics. Electro-magnetic torque is formed by interaction of stator and rotor magnetic poles. With stator and rotor flux linkage vector amplitudes being constant, the value of electro-magnetic torque will be maximal when stator poles are disposed to rotor poles at an angle of 90 electrical degrees. If angular disposal is different, the torque is determined as the product of the amplitude of one of the vectors and the projection of the second vector on the axis normal to the first vector.

Let’s consider the hypothesis about the excess of scalar product of $E_1D_2\Psi_1$ over scalar product of $\text{DE}_2\Psi_1$, as an explanation of observed phenomenon. In simulation we’ll consider torque reaction on step change of reference value. The results of simulation are shown in Figure-2.

The analysis of obtained transitions shows that condition (5) is provided only at the moment of start, and then as rotor angular velocity increases electro-magnetic torque deviates to zero. It is caused by the excess of scalar product of $\text{DE}_2\Psi_1$ over scalar product of $E_1D_2\Psi_1$, while at positive error $\text{DE}_2$ and $\Psi_1$ vectors should be codirectional just like $E_1$ and $\text{D}\Psi_2$ vectors.
The next phase of analysis is to test the hypothesis about the effect of weight coefficients $h_1$ and $h_3$ on torque control speed. To test that we’ll make the coefficients halve and then doubled in comparison to the coefficients shown in Figure-1.

The results of simulation are shown in Figure-4.

With having Figure-4 analyzed it is possible to make the conclusion that weight coefficients value determines the speed of the system, but forming of maximal coefficients doesn’t allow to achieve control goals, therefore it is necessary to find another ways to affect on speed.

As one of such ways let’s consider the voltage value effect on speed of the system, because, according to [24], IM voltage should be maximal with current frequency independently from speed criterion to provide maximal speed of the system. To test this effect we made the simulation, the results are shown in Figure-5.

Analysis of the received transients shows that the system performance significantly increased in compare with the Figure-1. Static error of regulation also decreased. But, for technical realization of voltage, which showed on the figure 5b with help of autonomic invertor, it is required high frequency of modulation.

From a physical point of view, in time of formation of the voltage vector at the level of maximum value considering limitations, vector $E_1$ shifting in position, which is according with nominal mode of induction motor, how it is showed at the Figure-6. This fact is linked with such situation as forming component of equation (4) in form of $1/\omega$, which formed accordingly with (2) is negligible in compare with $U_1$ and not make an effect on spatial position of $E_1$. In result we can see stabilization of electromagnetic torque at nominal level.
In view of this, we analyze another case, when control system, depending on the error control, forms such condition of inverter switch as we need. We shall assume that generated voltage vector will create the necessary signs of the derivatives of the moment and flux.

![Figure-6. Vector diagram of induction motor in the voltage formation at the level of maximum value considering limitations.](image6)

For this we use function (2). From these components of the voltage vector we will find its angular position by the formula:

$$\gamma = \arctan \frac{u_{s\beta}}{u_{s\alpha}}.$$

After we had identified the position of voltage vector in number of sectors, which shown at the picture 7, it is forms condition of switches, providing realization of nearest of the six possible vectors.

![Figure-7. Location of vectors on the coordinate space.](image7)

In the simulation, as in previous cases, it is considering the influence of electromagnetic torque reaction at the step excitation. From the results of simulation shown on the Figure-8, it is easily to seen that by the increasing of angular velocity of rotor rotation we can see that pulsations of electromagnetic torque near with given value are also increasing.

Based on the above, it can be concluded that failing of opportunity to forming rotor EMF (electromotive force) it can’t be possible to reach the objective of regulation with using of algorithm (2), to decide the problem of electromagnetic torque and flux regulation it is necessary to modify generalized algorithm, which described in [23].

5. ALGORITHM MODIFICATION

Analyzing control algorithm, which offered in [23], applied to induction motor it was revealed that working on high angular velocities, in time when EMF of rotation is near with maximum voltage, there is deviation of voltage vector from the required direction (1) that is prevents the achieve of the objectives of regulation. To decide this problem there is offering to introduce in equation the vector equal in value to the EMF rotation, but rotated by 90 electrical degrees, resulting in it forms modified control algorithm:

$$U_i = \frac{1}{2} \left( k_1 \Delta \Psi \text{Sign} (M_{\text{ref}}) + k_1 \Delta \Psi \text{Sign} (\Psi_{\text{ref}}) \right) + I_0 + \omega_{m} \Delta \Psi_1.$$  \hspace{1cm} (7)
where \( \omega_{\text{el}} = p \omega \) – electrical angular rotor velocity.

6. COMPARATIVE ANALYSIS

We will carry out comparative analysis of existing control ways of induction motor condition with differential control which offered in this article. We will consider field oriented control and direct torque control, and for simulation we will used the same computer model of electric drive system.

Comparative analysis of simulation results, presented on Figure-9 show us that is unlike of algorithm (2), formative derivatives proportional to error, static error using of (7) stays constant, however its value is reduced when weighting coefficient is increasing. Also by reason of shifting of voltage vector clockwise in the direction of the vector \( \Psi_1 \), it’s visibly decreased the system performance.

*Figure-9. Transitions of electromagnetic torque: a) the voltage vector is not limited in amplitude and it formed as continuous value; b) the voltage vector formed by the pulse-width modulation.*

Comparative analysis of simulation results at the maximum switching frequency of inverter switches \( f_m = 10kHz \) has shown that developed control algorithms has speed which comparable with speed of direct moment control and superior the speed of field oriented control approximately 1000.

*Figure-10. Transitions of electromagnetic torque: a) field oriented control; b) direct moment control.*
Table-1.

<table>
<thead>
<tr>
<th>Control Type</th>
<th>Time of transition in reverse</th>
<th>Static error</th>
<th>The maximum amplitude of moment pulsations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential control</td>
<td>0.005 s</td>
<td>-</td>
<td>±5 % ($f_m=10$ kHz)</td>
</tr>
<tr>
<td>Differential control with override voltage</td>
<td>0.005 s</td>
<td>2.86 %</td>
<td>±90.33 % ($f_m=10$ kHz)</td>
</tr>
<tr>
<td>Modified differential control</td>
<td>0.01 s</td>
<td>2.14 %</td>
<td>±5.18 % ($f_m=10$ kHz)</td>
</tr>
<tr>
<td>Vector control</td>
<td>0.1 s</td>
<td>0.1 %</td>
<td>±7.18 % ($f_m=10$ kHz)</td>
</tr>
<tr>
<td>Direct moment control</td>
<td>0.0001 s</td>
<td>3.35 %</td>
<td>±33.78 % ($f_m=10$ kHz)</td>
</tr>
</tbody>
</table>

The same time in point of view of moment pulsations, differential equation at frequency $f_m = 10$ kHz has the lesser level of high frequency interferences than in field oriented control, and, on this basis, the most lesser pulsations in compare with direct moment control. The algorithm of differential control, forming the maxim value of voltage vector, has the pulsations level, which significantly exceed the pulsations level in direct moment control.

7. CONCLUSIONS

Comparing the value of static error of moment regulation there is can be to make a conclusion that algorithms of differential control take a place an intermediate position between the field oriented control and direct moment control.

So, analyzing the full totality of performance criterion, we can see that proposed algorithms make a worthy rival to applicable at present time ways for such plants in what it is required to make a high control accuracy in high speed.

ACKNOWLEDGEMENTS

This article was prepared by the state assignments «Science» of the project «Intelligent Mechatronic Systems», No. 3852.

REFERENCES


