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ANALYTICAL APPROXIMATION TO DIFFERENCIATE BETWEEN THE EFFECT OF IONOSPHERIC HORIZONTAL GRADIENT AND ELEVATION ANGLE IN DGPS

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ABSTRACT
In differential GPS or DGPS, it is generally considered that the ionospheric correction is the same for both reference and unknown target (or mobile or user) receivers. However, it is clear that the ray paths from the satellite to the two receivers will generally traverse the ionosphere at slightly different elevations. Any horizontal ionospheric gradient present would also introduce errors into the DGPS correction. These two effects have been investigated in the present work in order to obtain a more accurate ionospheric correction for DGPS and have been found to be roughly comparable showing that they are both important. To accomplish this, the differential delay between the two paths for different elevations and azimuths has been done using an analytical ray tracing program. By performing ray-tracing calculations with and without a linear horizontal ionosphere gradient, the effects of elevation angle and horizontal gradient have been separated and a ratio of these effects has been determined for different elevations and horizontal ionosphere gradients. Empirical models have been introduced to model these variations based on the ray-tracing results.

Keywords: DGPS, ionospheric gradient, ray tracing, empirical model.

INTRODUCTION
In differential GPS or very well known as DGPS, it is generally considered that the ionospheric correction is the same for both reference and user stations because of the short baseline (up to about 10 km) that is normally employed. However, it is clear that the ray paths from the satellite to the two receivers will generally traverse the ionosphere at slightly different elevations due to the presence of ionospheric horizontal gradient.

If a user station is exposed to drastically large ionosphere gradient, the large difference of ionosphere delays between the reference and user stations can result in significant position error for the user. Several examples of extremely large ionosphere gradients that could cause the significant user errors have been observed (Datta-Barua *et al.*, 2002 and Walter *et al.*, 2004). In the paper (Datta-Barua *et al.*, 2002) had found an ionosphere gradient as large as 316mm/km which is more than 60 times the one-sigma nominal gradient (one-sigma nominal gradient is at 3~5mm/km) (McGraw, 2000 and Pullen, 2000).

In this paper, by performing ray-tracing calculations with and without a linear horizontal ionosphere gradient, the effects of difference in elevation angle (between the paths to the reference and mobile receivers) and horizontal gradient have been separated and a ratio of these effects has been determined for different elevations and horizontal ionosphere gradients.

A relationship to estimate the gradient of electron density, *Ne*, at the IPP (Ionospheric Pierce Point) using the approximately known slant Total Electron Content (TEC_{slant}) at the reference and mobile stations is also described. In most cases, the IPP has been approximated to 350km as this has been presumed as the average altitude where the content of the *Ne* is maximum.

Differential GPS (DGPS)
Some of the errors in GPS, especially ionospheric error, are presumed to be highly correlated for closely spaced paths from the satellite to two receivers separated by a short baseline length on the Earth. Thus, these errors can be mitigated by using relative positioning or DGPS rather than point positioning, which utilize at least two Earth stations the minimum (one reference and one user) and a number of GPS satellites (minimum four satellites for an accurate positioning). Currently, these are the most widely used ionosphere mitigation techniques in GPS positioning. However, due to presence of ionospheric horizontal gradients, the effect of the ionosphere for two closely spaced paths may not be the same.

In DGPS positioning, the pseudorange error (the difference between the geometric range and measured pseudorange) will be calculated for each satellite at a surveyed GPS reference station. Then, this error will be broadcasted to the user on a radio link as differential corrections, especially for real-time users (navigation). If the distance between those two receivers is not 'too large' (spatial) and the corrections from the reference station is 'not old' (temporal), it is assumed that the ionospheric, tropospheric, satellite clock and the ephemeris errors are the same at both the reference station and the user.

Nevertheless, the effect of the ionosphere will generally be different for both the reference station and the user due to the baseline distance between these two locations. As short as 10km of baseline distance could result in an appreciable difference in the TEC along the

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trajectory of the paths from a satellite to these two locations. This would be particularly true for equatorial or polar regions (especially at early morning, noon and after sunset) as at times of strong geomagnetic storms. The problem occurs due to the horizontal variation of the ionosphere between the reference station and the user.

Due to this, DGPS is not able to correct accurately for the actual ionospheric error which can then leads to inaccuracy in obtaining the position of the user.

Ionospheric horizontal gradient

The ionospheric horizontal gradient is the variation of *Ne* with latitude and longitude which can cause the azimuthal deviation of the GPS ray path. It can either increase or decrease the propagation time of the GPS signal depending on the path or trajectory of the signal with respect to the gradient direction. During periods of high solar activity with strong geomagnetic disturbance, large TEC gradients can exist, making ionospheric modeling difficult, especially in the equatorial and mid-auroral or polar region (Komjathy, 1997 and Fedrizzi *et al.*, 2001).

The ionospheric horizontal gradients (north-south or noon to midnight and east-west or dawn to dusk) vary diurnally (Strangeways, 2000), seasonally and altitudinally (Soicher and Gorman, 1985). They also depend on geographical location, the level of the Sun's activity and can vary with geomagnetic field variations. Sudden variations such as traveling ionospheric disturbances (TIDs) in the F_2 layer and small-scale irregularities (SSIs) in the E layer can also cause changes of the electron density horizontally in the ionosphere. Realizing the severity of this effect, much research has been done and is still continuing to determine and mitigate the ionospheric horizontal gradient by using a variety of methods (Lejeune *et al.*, 2003, Bhuyan and Borah, 2004, Luo *et al.*, 2005, Pryse *et al.*, 2006, Konno *et al.*, 2006, Yizenghaw *et al.*, 2006 and Niranjan *et al.*, 2007) in order to improve the final GPS user positioning.

3D ray tracing program

The Versatile Three-Dimensional Ray Tracing Computer Program for Radio Waves in The Ionosphere, well known as the Jones 3-D Ray Tracing program (Jones and Stephenson, 1975), is a program that will be used in this project with the provision of a new *Ne* subroutine for precise determination of transionospheric paths in an ionosphere with horizontal gradients. This important and much used program in the FORTRAN 77 computer language was produced by R. Michael Jones, Judith J. Stephenson and their colleagues. This program can trace radio waves through an ionospheric medium whose refractive index varies continuously in three dimensions.

For each ray path, the program can calculate group path length, phase path length, Doppler shift due to a time-varying ionosphere and geometrical path length to determine the characteristics of the ray path through the ionosphere. The most general ionospheric ray tracing method used in the Jones 3-D program was described by authors (Haselgrove, 1954) and (Haselgrove, 1957).

METHODOLOGY

By performing ray-tracing calculations with and without a linear horizontal ionosphere gradient, the effects of elevation angle and horizontal gradient have been separated and a ratio of these effects has been determined for different elevations and horizontal ionosphere gradients. An analytical method has been used to separate the effect of elevation angle and a linear ionospheric horizontal gradient, as being illustrated in Figure-1 below.

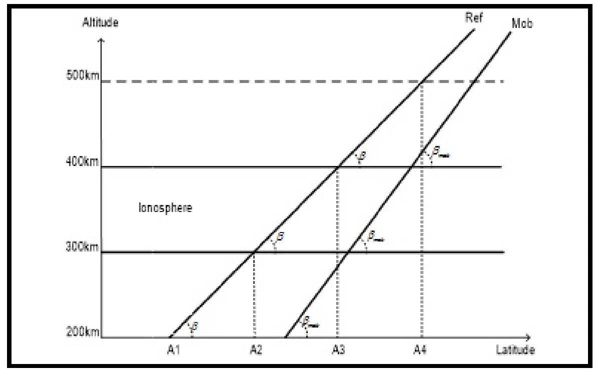

Figure-1. Analytical approach showing the propagation of ray from the satellite to mobile and reference through the ionosphere.

From the figure, it is assumed that the altitude of a GPS satellite at 20200 km and the ray are propagating through the ionosphere to both reference and user stations. As the explanation above, due to the slight different in the elevation angles of the ray to both reference and user, so, the mathematic also will be different. Then the elevation angle at reference station will be;

$$\tan(\beta) = \frac{20200}{x} \quad (1)$$

x = distance along the ground of the ray path from the satellite

β = elevation angle at reference station

Similarly, since the user station has been fixed to 10 km away from reference station and nearer to the satellite in the same direction, then β_{mob} will be:

$$\tan(\beta_{mob}) = \frac{20200}{x-10} \quad (2)$$

β_{mob} = elevation angle at user station

Then, the slant TEC (TEC_{slant}) of the ray for different elevation angles and for different value of C can be obtained by using:

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$$TEC_{slant} = \int Ne_{max} \bullet \exp\left(-\left(\frac{h-h_{max}}{s_{max}}\right)^{200}\right) \bullet \left(1 + C\left(\theta - \theta_{ref}\right)\right) \sec(\beta)\, dh \quad (3)$$

C = fractional increase or decrease in *Ne* per radian away from the reference latitude, θ_{ref}

In equation (3), C should give the change in *Ne* per km rather than per radian.

$$C(per\,km) = \frac{C(per\,rad)}{6370.0} \quad (4)$$

The same is true for the variation of $\left(\theta - \theta_{ref}\right)$,

$$Variation\ in\ latitude(in\,km) = \frac{h}{\tan(\beta)} \quad (5)$$

So, the actual TEC_{slant} due to the variation of elevation angle is being obtained by;

$$TEC_{slant}(C,\beta) = \frac{\left(\int Ne_{max} \bullet \exp\left(-\left(\frac{h-h_{max}}{s_{max}}\right)^{200}\right) \bullet \left(1 + \frac{C}{6353.143}\left(\frac{h}{\tan(\beta)}\right)\right) dh\right)}{\cos(\beta)} \quad (6)$$

Using the similar method, the TEC_{slant} at user station will be obtained.

After the TEC_{slant} at both the reference (TEC_{ref}) and user (TEC_{user}) stations have been obtained, the analysis is furthered to separate the effect of elevation angle and horizontal gradient for two short baseline (10 km) stations.

The effect of elevation angle is obtained by taking the difference of TEC_{slant} between the paths to the reference station and the user given by:

$$Difference\ in\ slant\,TEC = TEC_{ref} - TEC_{user} \quad (7)$$

Then, by another approximation, the effect of ionospheric horizontal gradient also can be obtained as shown by the equation below.

$$Effect\ of\ the\ gradient = \left(TEC_{ref} - TEC_{user}\right)_{WG} - \left(TEC_{ref} - TEC_{user}\right)_{NG} \quad (8)$$

WG = With Gradient
NG = No Gradient (C=0.0)

To further, the ratio between the effect of the linear gradient and the effect of elevation angle has also been obtained. It was done using equation (9) below.

$$Ratio = \frac{Effect\ of\ Gradient}{\left(TEC_{ref} - TEC_{mob}\right)_{NG(C=0.0)}} \quad (9)$$

TEC_{ref} = TEC_{slant} at reference station

TEC_{mob} = TEC_{slant} at mobile station

Finally, another equation has also been formulated that can estimate the gradient of *Ne* at the IPP utilizing the approximately known TEC_{slant} at the reference station and the user. This ratio can be used to approximate the TEC_{slant} for the path to the unknown user's location in terms of the TEC_{slant} at the reference receiver, as given as equation (10) below.

$$TEC_{slant}\,user = \frac{Ne_{user}}{Ne_{reference}} \bullet TEC_{slant}\,reference \quad (10)$$

RESULTS AND ANALYSIS

Effect of the elevation angle at both stations

By using equation (7), the difference in the TEC_{slant} between the paths to the stations for different values of C is shown in Figure-2. The baseline distance between both stations is 10km.

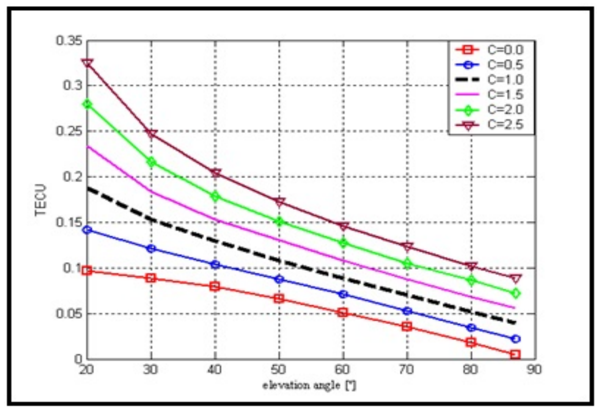

Figure-2. Difference in TEC_{slant} between reference station and the user's receiver due to the difference in elevation angle at both stations for different C values.

Due to the difference in the elevation angle at these stations that view the same satellite, it can be seen that as gradient increases, the difference in TEC_{slant} also increases, which was measured in the unit of TEC unit (TECU). This shows that even both reference and user view the same satellites, the difference in their elevation angle does effect the TEC_{slant}. This phenomena become very obvious with the presence of ionospheric horizontal gradient.

Effect of the Ionospheric horizontal gradient over the ray path

Then in order to determine only the effect of ionospheric gradient, its effect than was extracted by utilizing equation (8). It was done by eliminating the difference in between TEC_{slant} between the reference and user stations, at times of with gradient and without gradient (C=0.0). Figure-3 shows the results for this. Seems, the greater the horizontal gradient, the greater the TECU will be. However, at higher elevation angles, the TECU will be lower compare to the TECU at lower elevation angles.

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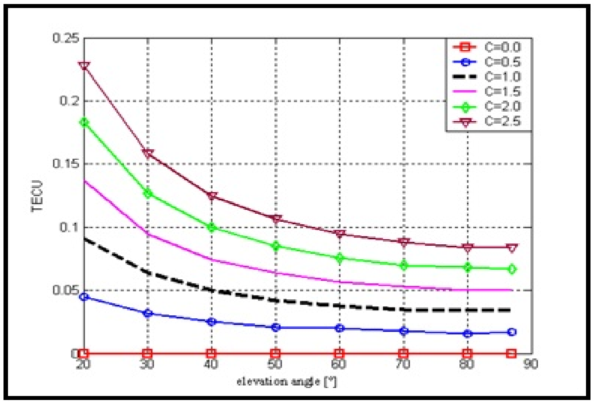

Figure-3. Difference in TEC_{slant} 'with' and 'without' gradient between reference station and the user for different C.

Ratio between Ionospheric linear gradient and elevation angle

By utilizing equation (9), the ratio was obtained just to show the importance of ionospheric horizontal gradient difference than the elevation angle difference. This is to understand that the ionospheric horizontal gradient difference in DGPS more pronounce than the elevation angle effect. The result is shown in Figure-4 below.

It can be seen that the ratio is lower during lower elevation angle and however keep increasing as the increase in the elevation angle.

Horizontal variation of electron density at IPP

From equation (10), is has been formulated to estimate the gradient of *Ne* at the IPP utilising the approximately known TEC_{slant} at the reference and mobile stations. This ratio can be used to obtain the approximate TEC_{slant} for the path to the unknown location receiver in terms of the TEC_{slant} at the reference receiver.

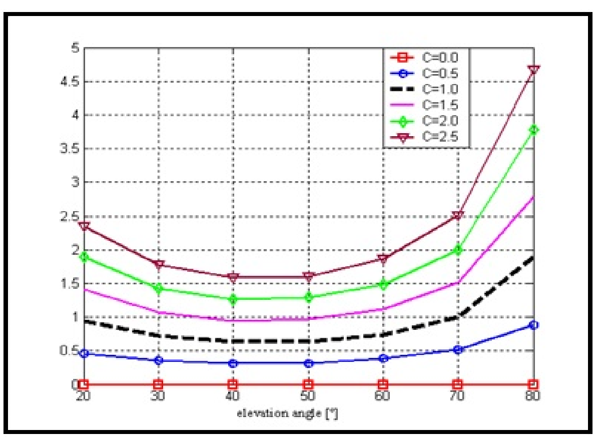

Figure-4. Ratio of the effect of the gradient for different values of C with the difference in sTEC between reference station and the user when C =0.0.

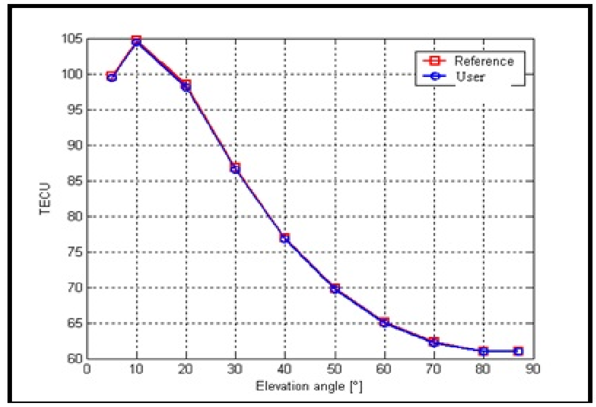

Figure-5. The TEC_{slant} at the reference station (from 3-D Jones ray tracing) and at the user station (obtained from equation (10)).

From Figure-5, it can be seen that the TEC_{slant} at the reference station is a bit greater than the TEC_{slant} at the mobile station because the reference station is further away from the satellite (baseline length between reference station and the user is 10km). Then, equation (10) can be used to obtain the TEC_{slant} at the user. In order to validate the TEC_{slant} that was obtained from the equation above, another value of TEC_{slant} for the same location of the user was obtained by using the 3-D Jones ray tracing program. The TEC_{slant} from both the equation and the 3-D Jones ray tracing program are very similar as shown in Figure-6.

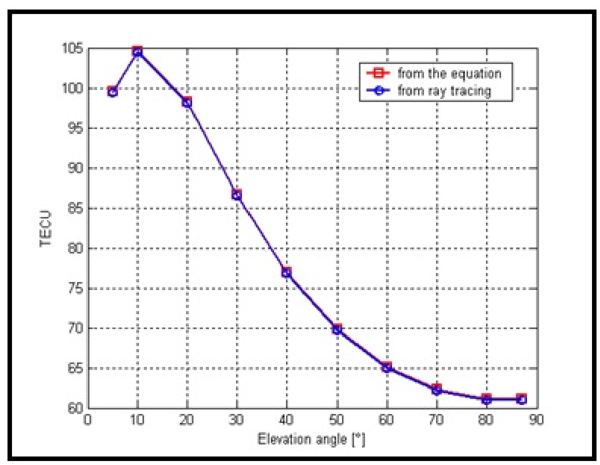

Figure-6. The TEC_{slant} of the user from the 3-D Jones ray tracing and equation (10).

However, this approximation can be used only for certain cases, such as for propagation of GPS ray at mid-latitude as then the IPP can be approximated to 350km, though it is still not very precise. For other geographic regions, such as the equatorial region, due to the presence of ionospheric horizontal gradient, the IPP cannot be approximated to 350km (section 4.8). Thus, this approximation will not be valid at any other region than mid-latitudes.

CONCLUSIONS

In this paper, the ionospheric range error introduced by the spacing and different inclination of the paths from the satellite to the reference and user stations has been determined for both cases; with and without the presence of linear ionospheric horizontal gradients. By separating the two effects, their magnitude and relative importance as a source of range error has been determined over a wide range of elevation.

In order to make an account correction in practice, the horizontal electron density gradients need to be known which could be estimated from the gradient in TEC observed from the received GPS satellites (since it has also been shown that the ratio of TEC_{slant} gives the ratio of *Ne* at the IPPs) or based on an empirical model updated with real-time data, or both.

Eventually, an improved ionospheric correction, which corrects both the elevation angle difference and ionospheric horizontal gradient must be obtained in order to determine the highest precise final DGPS positioning.

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