



A REPORT ON PERFORMANCE OF ANNULAR FINS HAVING VARYING THICKNESS

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ABSTRACT

Studying the characteristics of annular fin is a key research problem in many thermal applications. A comparison of performance of the exponential and parabolic annular fins of varying geometry parameter is reported in the present work. The governing differential equation for the fins has been derived to study the temperature distribution of the fins with insulated tip. A parametric study is then carried out by varying the geometry parameters in the governing equation to investigate the effect of on fin performance and the results are presented in graphical form.

Keywords: annular fins, exponential profile, parabolic profile, aspect ratio.

INTRODUCTION

The main purpose of an extended surface or a fin is to increase the rate of heat transfer from a heated surface to a cold fluid. Of these, annular fins find numerous applications in compact heat exchangers, specialized installations of single and double pipe heat exchangers, electrical apparatus with efficient heat dissipation, cylinders of air cooled internal-combustion engines and fuel cans in nuclear reactors to name a few. The design of a fin is considered to be optimum when the fins require minimum cost of manufacturing, offer the minimum resistance to the fluid flow, are light in weight and are easy to manufacture. A detailed review of literature on optimum design of fins has been carried out starting with Gardner (1945) where upon using a set of idealizing assumptions, the efficiency of various straight fins and spines have been reported. Duffin and McLain (1959, 1968) solved the optimization problem of straight based fins assuming that the minimum weight fin had a linear temperature distribution along its length. Murry (1938) presented an equation for the temperature gradient and the effectiveness of annular fins of constant thickness with a symmetrical temperature distribution around the base of the fin. Brown (1965) reported the optimum dimensions of uniform annular fin by relating fin dimensions to the heat transfer and thermal properties of the fin and heat transfer coefficient between the fin and its surroundings.

The optimized dimensions of the fin can be found in either one of the two ways: the maximum amount of heat dissipation for a given quantity of weight or the minimum weight for dissipating a given quantity of heat. Ullmann and Kalman (1989) adopted the first way and determined the efficiency and optimum dimensions of annular fins with triangular, parabolic, and hyperbolic profiles using numerical techniques. Dhar and Arora (1976) described the methods of carrying out the minimum weight design of finned surfaces of specific type by first obtaining the optimum surface profile of a fin required to dissipate a certain amount of heat from the given surface, with no restriction on the fin height and then extended their study for the case when fin height is

given. (Duffin1959) gave a method for carrying out the minimum weight design of a fin using a rigorous mathematical method based on Variational calculus and assumed constant thermal conductivity of a fin material and a constant heat transfer coefficient along the fin surface. In a recent work, (Arauzo *et al* 2005) reported a ten-term power series method for predicting the temperature distributions and the heat transfer rates of annular fins of hyperbolic profiles. Assuming fixed fin volume, (Arslanturk2005) reported simple correlation equations for optimum design of annular fins with uniform cross sections to obtain the dimensionless geometrical parameters of the fin with maximum heat transfer rates. These simple correlation equations can help the thermal design engineers for carrying out the study on optimum design of annular fins of uniform thickness. In their recent work, (Kundu and Das 2001) reported the performance analysis and optimization of concentric annular fins with a step change in thickness using Lagrange multiplier.

In a recent work, (Iborra and Campo 2009) reported that approximate analytic temperature profiles and heat transfer rates are easily obtainable without resorting to the exact analytic temperature distribution and heat transfer rate based on modified Bessel functions. Kang (2009) reported the optimum performance and fin length of a rectangular profile annular fin using variations separation method. Aziz and Fang (2010) presented alternative solutions for different tip conditions of longitudinal fins having rectangular, trapezoidal and concave parabolic profiles and reported relationship between dimensionless heat flux, fin parameter and dimensionless tip temperature for all the geometries. Aziz and Khani (2010) presented an analytical solution for thermal performance of annular fins of rectangular and different convex parabolic profiles mounted on a rotating shaft, losing heat by convection to its surroundings. In their work, convection heat transfer coefficient was assumed to be a function of radial coordinate and shaft speed.

In an experimental study, heat transfer rate and efficiency for circular and elliptical annular fins were



analyzed for different environmental conditions by (Nagarani2010) and high efficiency was reported for elliptical fins as compared to circular ones. In another recent work, (Aziz and Fang 2011) derived analytical expressions for the temperature distribution, tip heat flow and biot number at the tip and reported thermal performance of the annular fin under both cooling and heating conditions.

The present work reports the performance of exponential and parabolic annular fin subject to heat flow in radial direction with insulated tip boundary condition. The effect of geometry parameters governing the thickness variation is studied on the fin performance and a comparison is drawn amongst the two profiles.

MATHEMATICAL FORMULATION

Assuming the effect of external environment on the surface convection to be negligible, the second order differential equation for the heat transfer through the fins is developed to determine the temperature profile.

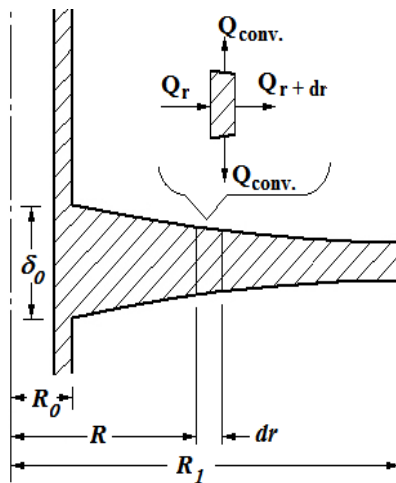


Figure-1. Sectional view of fin geometry.

For calculating the heat balance, the details for a control volume of length 'dr' of a fin is shown in Figure-1. The resulting second order differential equation has been solved using a computational algorithm. Applying the law of conservation of energy or thermal energy balance:

$$Q_r = Q_{r+dr} + Q_{conv.} \quad (1)$$

Using above equation the following equation can be arrived at

$$\frac{d}{dr} \left(k 2\pi r \delta \frac{d\theta}{dr} \right) dr = 4\pi r h \theta dr \sqrt{1 + \left(\frac{d\delta}{2dr} \right)^2} \quad (2)$$

The fin geometry and physical parameters are normalized using the following expressions,

$$\bar{\delta} = \frac{\delta}{\delta_a}, \quad \Phi = \frac{\theta}{\theta_0}, \quad R_f = \frac{R_1}{R_0}, \quad x = \frac{r}{R_0}$$

and the fin parameter

$$m_f = L \sqrt{\frac{2h}{k\delta_a}}$$

Upon introducing the normalized variables, the governing equation becomes,

$$\frac{d^2\Phi}{dx^2} + A_1 \frac{d\Phi}{dx} + A_2 \Phi = 0 \quad (3)$$

Where

$$A_1 = \frac{1}{x} + \frac{1}{\bar{\delta}} \frac{d\bar{\delta}}{dx}$$

and

$$A_2 = \frac{-m_f^2}{(R_f - 1)^2 \bar{\delta}} \sqrt{1 + \frac{1}{4} \left(\frac{d\bar{\delta}}{dx} \right)^2 \left(\frac{\delta_a}{R_0} \right)^2}$$

Equation (3) can be generalized for varying thickness of the fin using parabolic and exponential variation along the length of the fin as defined in the following equations shown in Table-1.

Table-1. Geometry profiles considered in the study.

	Profile nature	Thickness variation
1	Exponential	$\bar{\delta} = e^{-nr^m}$
2	Parabolic	$\bar{\delta} = 1 - nr^m$

In the profile equations, it is to be noted that parameter m , being an index to radius, changes the profile at any radius and is responsible of linear or non-linear variation of the profile while parameter n , being a scalar multiplier to the radius affects the thickness of the profile obtained due to m at any radius. Using Equation (3) and relations from Table 1, for exponential profile, the governing equation becomes

$$\frac{d^2\Phi}{dx^2} + \left(\frac{1}{x} - k_3 x^{2m-1} \right) \frac{d\Phi}{dx} + k_1 \left\{ e^{2nR_0^m} x^m + k_2 \cdot x^{4m-4} \right\}^{0.5} \Phi = 0 \quad (4)$$

where

$$k_1 = -\frac{m_f^2}{(R_f - 1)^2}; \quad k_2 = \left(\frac{nmR_0}{2} \right)^2 \left(\frac{\delta_0}{R_0} \right)^2 R_0^{(4m-2)}; \quad k_3 = nmR_0^{2m}$$

while, for parabolic profile, the governing equation becomes



$$\frac{d^2\Phi}{dx^2} + \left(\frac{1}{x} - \frac{k_4 x^{m-1}}{1 - k_5 x^m} \right) \frac{d\Phi}{dx} - \frac{k_6}{1 - k_5 x^m} \{1 + k_7 \cdot x^{2m-2}\}^{0.5} \Phi = 0 \quad (5)$$

where

$$k_4 = nm R_0^{-m}; k_5 = n R_0^{-m}; k_6 = \frac{m_f^2}{(R_f - 1)^2}; k_7 = \left(\frac{k_4 \delta_0}{2 R_0} \right)^2$$

Equation (4) and (5) are solved using the following boundary conditions:

- (i) $\Phi = 1$ at $x = 1$
- (ii) $\frac{d\Phi}{dx} = 0$ at $x = R_f$ (insulated tip)

Efficiency of the fin is obtained from the general equation as follows:

$$\eta_{fin} = \frac{-\Phi'_{x=1}}{\frac{m_f^2}{R_0} \int_1^{R_f} \left\{ (Lx + R_0) \sqrt{1 + x^{2m-2} k_1} \right\} dx} \quad (6)$$

Using Equation (6), the fin efficiency for exponential and parabolic profile respectively are derived as follows (Equation.7-8):

$$\eta_{exp} = \frac{-\Phi'_{x=1}}{\frac{m_f^2}{(R_f - 1)^2} \int_1^{R_f} \{A_E\} dx} \quad (7)$$

Here,

$$A_E = \sqrt{\left(\frac{1}{4} \cdot \frac{\delta_a^2}{R_0^2} \cdot n^2 \cdot R_0^{2m} \cdot e^{-2n \cdot x^m} \cdot R_0^m \cdot m^2 x^m + 1 \right) x^2}$$

$$\eta_{par} = \frac{-\Phi'_{x=1}}{\frac{m_f^2}{(R_f - 1)^2} \int_1^{R_f} A_p dx} \quad (8)$$

Here,

$$A_p = \sqrt{\frac{1}{4} \cdot \frac{\delta_a^2}{R_0^2} \cdot n^2 \cdot m^2 R_0^{2m} x^{2m} + x^2}$$

RESULTS AND DISCUSSIONS

The dimensionless temperature Φ being a function of normalized variables depends on n and m due to chosen fin shape, m_f , R_f and x due to fin geometry. Considering x as the only independent variable, the Equation.(4-5), are solved for a range of n and m . The system properties are taken as $h=25$ W/m², $k=200$ W/m-K, $\delta_a=0.01$ m, and $R_0=0.1$ m.

To calculate the fin efficiency the first derivative of temperature at the fin base i.e. $\Phi'_{x=1}$ is evaluated by solving second order differential equation in MATLAB

using *bvp4c* routine. A proven technique to solve boundary value problems is to choose a continuous piecewise polynomial or a spline that adjusts the boundary conditions. The remaining unknown coefficients are determined by collocating the algebraic equations at several points. Code *bvp4c* (Shampine et. al. 2010) is an adaptive finite difference code that employs three-stage Lobatto III-a collocation formula by computing a cubic spline on each subinterval $[x_i, x_{i+1}]$ of a mesh. The important requirements are that the piecewise polynomial must satisfy the boundary conditions and collocate the ODE at endpoints and midpoint of the subinterval. The approximate continuous solution is obtained by controlling the residual over each subinterval $[x_i, x_{i+1}]$ approximated using a five-point Lobatto quadrature formula.

The solution is validated with benchmark results available for rectangular profile (Kreith and Bohn 2001). The results are found to be in good agreement with benchmark results as shown in Figure-2. The effect of geometry parameters on fin performance for exponential and parabolic profiles are studied and reported next.

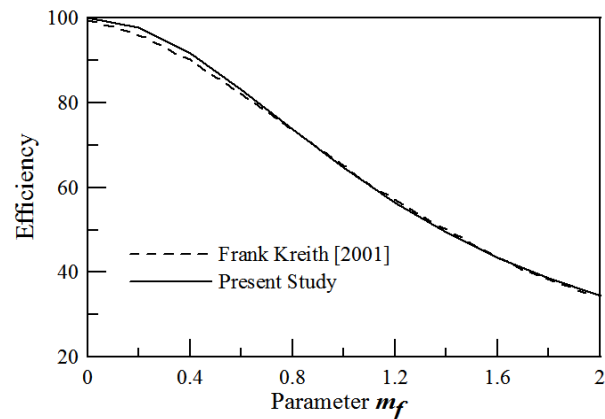


Figure 2: Validation of the present study with Kreith and Bohn (2001)

Exponential profile

For exponential profile, the effect of geometry parameters m and n on the efficiency is shown in Figure-3 and 4 respectively. It is observed from Figure-3 that efficiency decreases with increase in m and approaches the efficiency for rectangular profile i.e. $n=0$. However, from Figure-4, it is observed that variation of n monotonically increases efficiency. This is obvious due to the fact that increasing n increases the thickness of any profile obtained at a given value of m . A contour plot of efficiency with n and m is presented in Figure-5 and can be used as a design monogram to determine the efficiencies of exponential annular fin for any combination of n and m .

Parabolic profile

Similar studies are carried out for parabolic profile and results are plotted in Figure-6 and 7 for the identical value of aspect ratio. The effect of geometry



parameter m on efficiency in case of parabolic profile is more pronounced as compared to exponential profile. In Figure-8, a contour plot of efficiency with n and m is plotted and can also be used as a design monogram as the expected efficiency at any combination of parameter n and m can be predicted from the plot i.e. Figure-8. The above results indicate that although parameter n has a monotonic effect on efficiency of both exponential and parabolic annular fins at a given value of m , the same does not apply the other way round.

In fact a comparison of the variation of m and its effect on the efficiencies of both profiles gives an interesting insight. As a result a comparative plot of efficiency versus geometry parameter m ($n=0.5$) for both the profiles is shown in Figure-9. It is observed from the plot that the selection of profile is governed by a limiting value of geometry parameter m (given by m_L). For the value of m less than the m_L parabolic profile yields better efficiency as compared to exponential profile while for values of m greater than m_L the exponential profile yields better performance. This implies that for the manufacturer m_L is an important design parameter that decides the shape of annular fin. It is, thus, evident from Figure-9 that a limiting value of m always exists, denoted by m_L , which decides the criteria for selection of fin profile amongst exponential and parabolic geometries for a given value of n and R_f .

To study the variation of m_L with R_f a plot between m_L and n is shown in Figure. 10 for different R_f . It is observed that initially as the n increases, m_L increases. However for larger values of n , m_L tend to reach an optimum value. An interesting insight obtained from the plot is that as n directly affects the value of m_L at any given aspect ratio and hence the suitability of parabolic profile increases over the exponential one, as fabricating an exponential profile is more typical. However, increasing aspect ratio has an inverse effect on m_L . Thus for larger aspect ratio, exponential profiles possess broader limits to have an advantage over the parabolic counterparts.

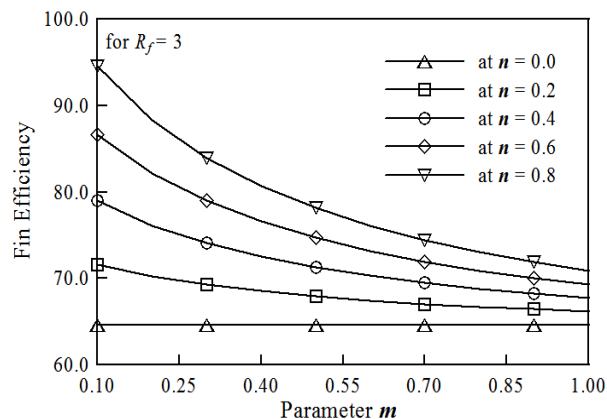


Figure-3. Variation of efficiency with parameter m at different n values for exponential geometry ($R_f=3$).

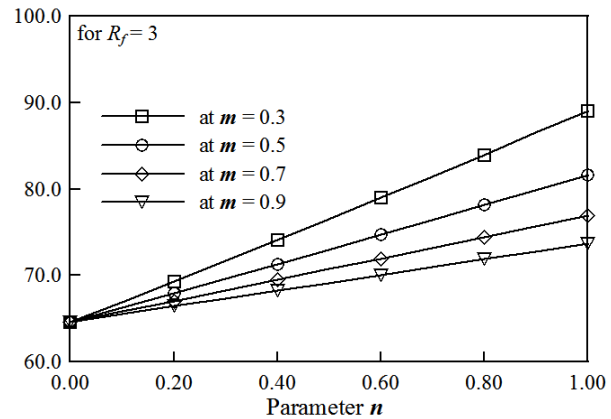


Figure-4. Variation of efficiency with parameter n at different m values for exponential geometry ($R_f=3$).

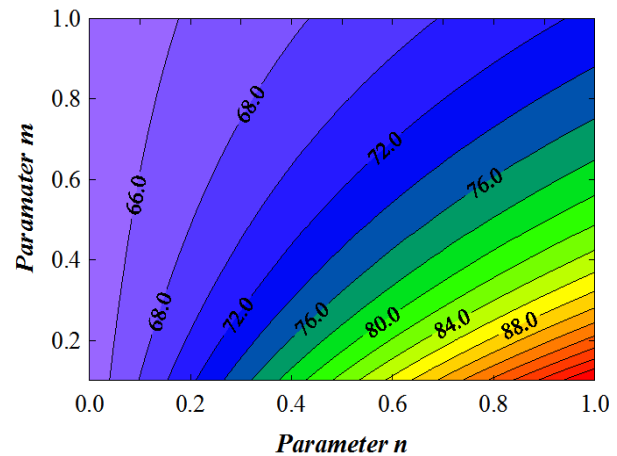


Figure-5. Contour plot of variation of efficiency with geometry parameters for exponential profile ($R_f=3$).

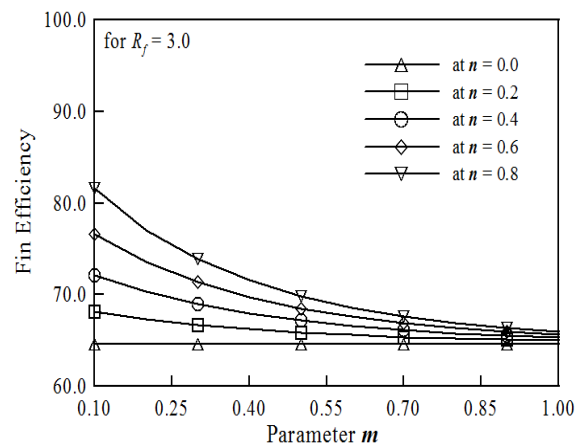


Figure-6. Variation of efficiency with parameter m at different n values for parabolic geometry ($R_f=3$).

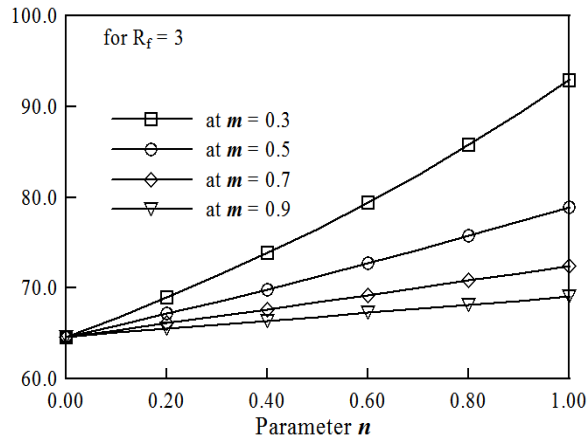


Figure-7. Variation of efficiency with parameter n at different m values for parabolic geometry ($R_f=3$).

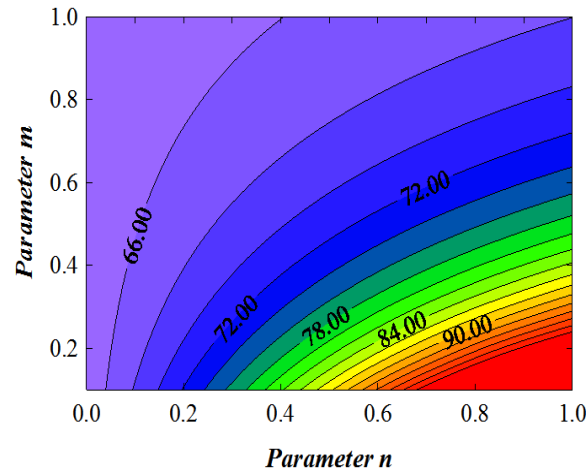


Figure-8. Contour plot of variation of efficiency with geometry parameters for parabolic profile ($R_f=3$).

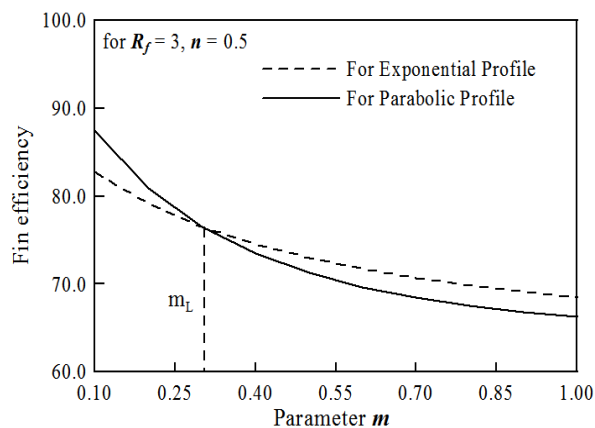


Figure-9. A comparison of variation of fin efficiency for both profiles with parameter m at $n = 0.50$, $R_f = 3$.

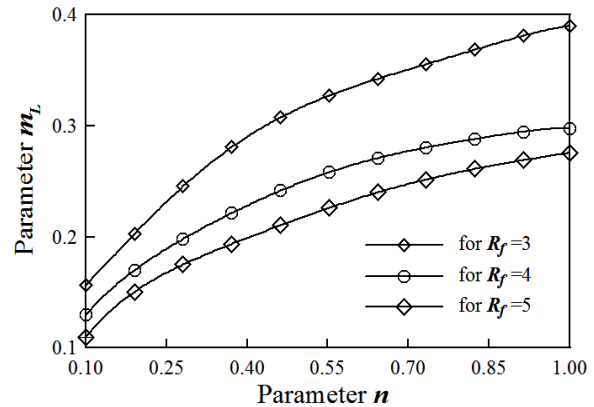


Figure-10. Plot of m_L , with parameter n , for different aspect ratio.

CONCLUSIONS

The performance of annular fins having exponential and parabolic thickness variation is reported. The study is carried out for different values of geometry parameters n and m and aspect ratio. It is observed that for both exponential and parabolic, as the parameter m increases, the efficiency of the fin decreases and approaches to minimum. The effect of variation of n on efficiency is direct in nature. As n increases, due to more convection area available at the base of fin, the efficiency also increases. A comparative study reveals that parabolic profile is having higher efficiency for the lower values of m as compared to exponential profile but as the value of m increases and exceeds a certain value the efficiency of exponential profile becomes better.

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