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DEVELOPMENT OF SHEAR STRESS EQUATION CONTRIBUTED BY STEEL FIBRE IN REINFORCED CONCRETE

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ABSTRACT

The rise of extensive research and development on the application of steel fibre inside concrete is a tremendous topic in the last few decades. The obvious benefits of adding steel fibre inside concrete can be seen from the improvement of the mechanical properties of steel fibre reinforced concrete (SFRC). The improvement of the mechanical properties can be seen easily on the higher value of tensile and flexural strengths for SFRC compared to plain concrete. However, a question arises on to what extent does the steel fibre exerts an additional stress to increase the strength. Currently, this additional stress is called “shear stress supplement”. In this study, shear supplement model developed by Rilem was quantified with some statistical modification to predict the additional shear exerted by the steel fibre when it is placed inside the concrete. To achieve the objective, 51 prism specimens (150 mm × 150 mm × 750 mm) were prepared and differ in terms of the type of steel fibre and the fibre volume fraction, V_f. Two types of hooked-end steel fibre labeled as SF60 and SF50 were used in this research. Meanwhile, each type of steel fibre was mixed into a plain concrete with respect to the V_f of 0.25%, 0.50%, 0.75%, 1.00%, 1.25%, 1.50%, 1.75%, and 2.00%. Plain concrete prisms were prepared as the control specimens. After 28 days of curing period, the prism was then transferred to the test frame to undergo flexural strength test. The results from the flexural strength test were used to develop the shear stress equation to predict the contribution of steel fibre to increase the shear stress. The modified equation was then used to predict the value of shear stress supplement. The results showed that, the modified model well predicted the shear stress supplied by the steel fibres well which was shown by higher coefficient of correlation.

Keywords: shear stress supplements, shear stress, steel fibre, steel fibre reinforced concrete.

INTRODUCTION

Concrete is a composite material, where the matrix is made from the combination of fine aggregate, coarse aggregate, cement and water (Hibbeler, 2011), (Sarbini, 2014). However, the combination of those materials in the concrete matrix is not enough to sustain the tensile load. This is due to the strength of the concrete matrix which is good against compressive load. Therefore, other materials are added to the concrete matrix to sustain the tensile load. This includes using steel reinforcement, which is a common material used in building construction. Even so, nowadays people are looking for something that is more cost effective. Hence, it opposes the idea of using fibre instead of steel. The randomly distributed discreet fibres to reinforce plain concrete provide three-dimensional resistances against the applied load as compared to the plain concrete (Paine, 1997), (Altun *et al*., 2007), (Holschemacher *et al*., 2010), (Ibrahim *et al*., 2011).

The extensive use of steel fibre reinforced concrete (SFRC) is due to its performance in restraining shear stresses induced by the concrete element. Therefore, randomly discreetly distributed steel fibres are added into plain concrete expecting that the shear stress is effectively transferred within the developed cracks. There were a lot of works concentrated on the tensile and flexural resistance of SFRC. Previously, Pierre *et al.* (1999) have studied the differences in the behavior of steel fibres when it is placed inside concrete and mortar. They found out that the mechanical properties for both types of composites improved due to the addition of steel fibres. The improvement is seen by the higher strength of steel fibre concrete and steel fibre mortar, compared to the plain ones. On the other hand, Gao *et al*. (1997) reported the differences in the behaviour of the steel fibres reinforced high strength and lightweight concrete. Though, both types of concrete show improvement due to the addition of steel fibres, the improvement by lightweight concrete is less. This is due to the lightweight aggregate which act as a weak component and lessen the strength improvement compared to other type. Other studies have shown that SFRC improved the properties of toughness and modulus of elasticity even with small dosage of fibre volume fraction (example 0.25%) and even at a fibre aspect ratio less than 40 (Song and Hwang , 2004), (Teng *et al*., 2004), (Altun *et al*., 2007), (Thomas and Ramaswamy, 2007), (Yazici *et al*., 2007), (Xu and Shi, 2009), (Ramli and Dawood, 2010), (Pawade *et al*., 2011), (Soulioti *et al*., 2011).

Unfortunately, their works mostly reported on the strength values and the specimen behaviour. To the concern of the study on shear stress restrained by steel fibres inside the concrete element, it is a great step to focus on the additional stress contributed by steel fibres. This special attention on shear supplement by steel fibres is somehow a study that is rarely focused by any researcher, while other usually studies on the tensile and flexural performances.

It is well-known that the failure of SFRC due to the applied flexural load is associated with the de-bonding

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process of steel fibres from the surrounding concrete. During the early stage of the de-bonding process, the cracks initiated are slow due to the ability of bridging fibres. At some stage where the element is no longer capable of sustaining the flexural load, the de-bonding process occurred rapidly. Due to the unstable cracks propagation at the point of where the steel fibres are pulled out from the specimen, failure will occur. This is also associated with exceeding the maximum limit of ultimate bond strength between the steel fibres and surrounding concrete (Gao *et al*., 1997). This shows why the prediction of additional shear stress contributed by steel fibres is important; it is to determine the capacity of such fibres when restraining concrete.

RESEARCH METHODOLOGY

Material preparation

Two types of hooked-end steel fibre were used in this study labeled as SF60 and SF50. The details of the steel fibres are given in Table-1, while Figure-1 shows the illustration of the hooked-end steel fibre.

Table-1. Properties of steel fibres.

Properties	SF60	SF50
Length, mm	60	50
Diameter, mm	0.75	0.75
Aspect ratio, *L/D*	80	67
Tensile strength, MPa	1100	1200

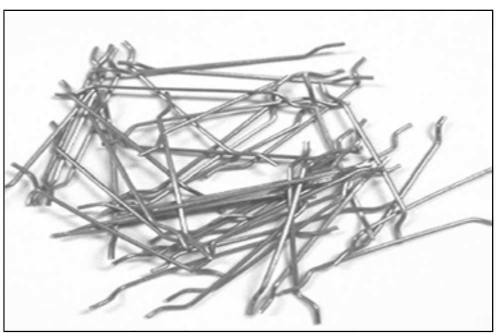

Figure-1. Schematic diagram of flexural strength test.

Eight (8) different fibre volume fractions, V_f are used for each type of steel fibre. The V_f were 0.25%, 0.50%, 0.75%, 1.00%, 1.25%, 1.50%, 1.75%, and 2.00%. At the same time, one plain concrete (without any fibre addition) was prepared as the control batch. The specimen was a prism specimen with a cross-section of 150 mm × 150 mm, and length of 750 mm. The concrete batch was prepared for grade 40 N/mm^2 and the mix proportion is given in Table-2. For each type of steel fibre and fibre volume fraction, three (3) prism specimens were prepared (Figure-2).

Table-2. Mix composition.

Sand (kg/m^3)	Gravel (kg/m^3)	Water, (kg/m^3)	Cement, (kg/m^3)	Admixture (kg/m^3)
762	1011	230	400	0.8

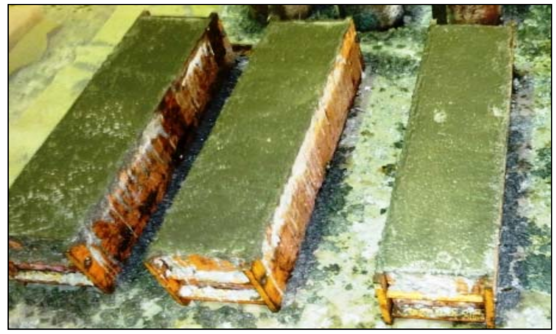

Figure-2. Prism specimens (illustration before being de-mould).

Testing procedures

The schematic diagram of the flexural strength test is shown in Figure-3. Three units of linear variable differential transformer (LVDT) were positioned at the mid-span and at both supports. LVDT was used to measure the deflection and deformation throughout the test. The load was applied incrementally at every 2 kN until the first crack line appeared. After the first crack had been observed, the additional load was controlled by the deflection at every 0.01 mm until the specimen fails. During the test, data were recorded for the first peak, peak and residual strengths at specified deflections.

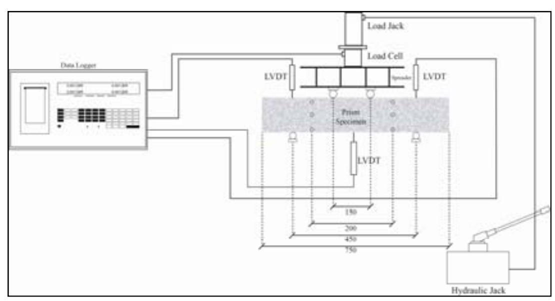

Figure-3. Schematic diagram of flexural strength test.

Analytical works

The additional shear from steel fibres, v_{fd} is the parameter introduced by Rilem to determine the shear strength enhancement due to the addition of steel fibres in plain concrete (Rilem, 2002). The general equation for the additional shear from steel fibres is given as:

$$v_{fd} = Ak_f k\tau_{fd} \quad (1)$$

Where:
A = 0.7 (bond and orientation factors)
k_f = 1.0 (for rectangular section)
$k = 1+\sqrt{(200/d)} \leq 2$, with d in mm

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τ_{fd} = Design value of the increase in shear strength due to steel fibres, N/mm^2

To determine the design value for the increase in shear strength due to steel fibres, τ_{fd}, Rilem proposed two calculation methods (Rilem, 2002). Both calculation methods differed in terms of the test procedure. In the first method, the test procedure followed was as recommended in BS EN 14651:2005, whereby, the second method procedure was as recommended by JCI-SF (BS EN 14651, 2005), (JCI-SF, 1984). In both test procedures, the flexural test was carried out for prism specimen. However, during the test they differed in terms of the measured variables. The first procedure measured the *CMOD* (crack mouth opening displacement) when the prism specimen was applied under flexural load, whereas in the second procedure, the mid-span deflection of the prism specimen was measured.

In the first procedure, the residual strength, f_{Rk4} was calculated based on the measured *CMOD* until a crack width up to 3.5 mm. Meanwhile, for the second procedure, the equivalent flexural ratio was measured from the mid-span deflection up to 3.0 mm. After the residual strength from the first procedure and the equivalent flexural ratio from the second procedure were determined, the design value of the increase in shear strength due to steel fibres can be determined. These are accomplished using either equation (2) for the first procedure or equation (3) for the second procedure.

$$\tau_{fd} = 0.12 f_{Rk4} \tag{2}$$

$$\tau_{fd} = 0.12 R_{e,3} f_p \tag{3}$$

Where:

f_{Rk4} = Residual flexural strength measured in BS EN 14651: 2005

$R_{e,3}$ = Equivalent flexural ratio in JCI-SF: 1984

f_p = Characteristic peak strength of plain concrete

RESULTS AND DISCUSSIONS

The flexural strength test carried out in this research work is related closely to the one that is assigned in JCI-SF test procedure. Therefore, equation (3) was used to determine the design value of the increase of shear strength due to steel fibres. Theoretically, the equation indicates that the shear strength due to steel fibre is the enhancement on flexural/peak strength of plain concrete, when steel fibre is added into the mix. Rilem suggested that the enhancement can be determined using the multiplication from the equivalent flexural ratio at a maximum deflection of 3 mm with the characteristic peak strength of plain concrete (Rilem, 2002).

The equivalent flexural ratio was determined as the ratio between the residual strength at net deflection of $L/150$, f_{150}^D, to the peak strength of SFRC, $f_{p,SFRC}$. The equivalent flexural ratio is given as, $\left[\dfrac{f_{150}^{\ D}}{f_{p,SFRC}}\right]$. Analytical works using SPSS software was used to determine the peak strength of SFRC (Carver and Nash, 2009). The proposed fundamental relationship is given in equation (4) and it is also complied with ACI 318-08, ACI-363R-92 and Eurocode 2 for the prediction of flexural strengths of plain concrete (ACI-318, 2008), (ACI-363R, 1992), (BS EN 1992-1-1, 2004).

$$f_x = A\sqrt{f_{cu}} \tag{4}$$

Where:

A = Material coefficient

f_x = Flexural/peak strength of plain concrete, N/mm^2

f_{cu} = Compressive strength of plain concrete, N/mm^2

In the work by Thomas and Ramaswamy (2007), they suggested that the flexural strength can be predicted using an additive method as given in equation (5). The first term in the equation is the concrete compression contribution, whereas the second term is the contribution from steel fibres to the matrix strength.

$$f_x = \underbrace{A\sqrt{f_{cu}}}_{\text{First term}} + \underbrace{\left\{V_f \, L/D \, x\sqrt{f_{cu}} + V_f \, L/D\right\}}_{\text{Second term}} \tag{5}$$

In this research, the second term was modified. The modification was related to the effects of the peak strength and fibre volume fraction in a quadratic relationship of the second degree. Therefore, the new analytical equation is given in equation (6).

$$f_x = A\sqrt{f_{cu}} + \left\{\sqrt{f_{cu}}\left(BV_f^{\,2} + CV_f\right)\right\} \tag{6}$$

Where:

A, B, C = Regression coefficients

The peak strength from the experimental results was assigned as the dependent variable, whereas the independent variables were $\sqrt{f_{cu}}$, $V_f\sqrt{f_{cu}}$ and $V_f^2\sqrt{f_{cu}}$. Next, the proposed models are given as in equation (7) and equation (8). The coefficient of determination, R^2 were 0.84 and 0.87 for SF60 and SF50, respectively. A higher value of R^2 (greater than 0.80) for both proposed models indicated the suitability of the test results. The correlation between the test results and the proposed models with 1:1 line is shown in Figure-4 and Figure-5.

$$f_p = 0.527\sqrt{f_{cu}} + \left(0.047V_f\sqrt{f_{cu}} + 0.203V_f^{\,2}\sqrt{f_{cu}}\right) \tag{7}$$

$$f_p = 0.551\sqrt{f_{cu}} + \left(0.291V_f\sqrt{f_{cu}} - 0.017V_f^{\,2}\sqrt{f_{cu}}\right) \tag{8}$$

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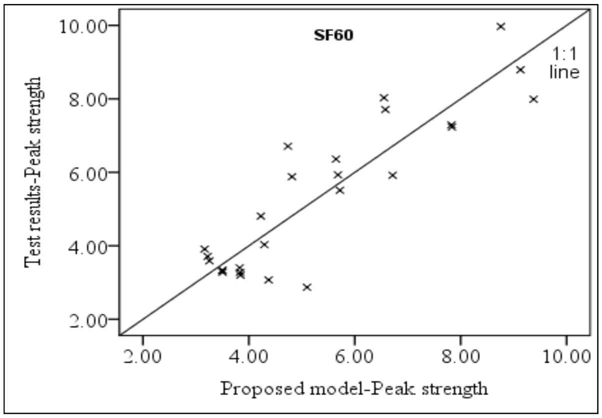

Figure-4. Relationship of peak strength between test results and proposed model for SF60 steel fibre.

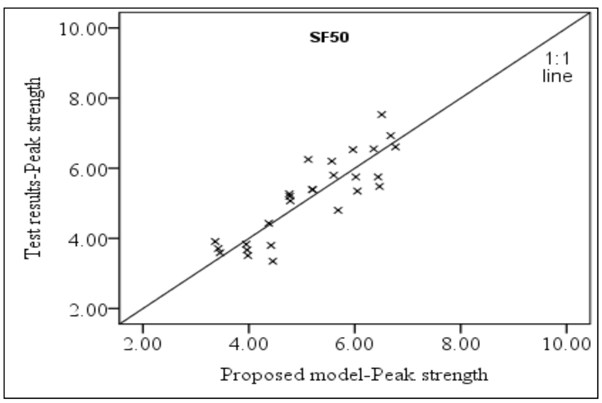

Figure-5. Relationship of peak strength between test results and proposed model for SF50 steel fibre.

Then, the residual strength at net deflection of $L/150$ was derived using regression analysis. The proposed models to derive the residual strength at net deflection of $L/150$ using the SPSS software is shown in Figure-6. From the figure, the relationship between residual strength at net deflection of $L/150$ and fibre volume fraction for SF60 is linear, while for SF50 the relationship is quadratical relationship. These relationships were taken into account during the regression analysis. Equation 9 and equation 10 are the proposed models for SF60 and SF50, respectively. From the results, it was found that higher coefficient of determination, R^2 of 0.89 and 0.86 were obtained for SF60 and SF50, respectively. These showed that the proposed models are suitable to represent the test results of the residual strength at net deflection of $L/150$.

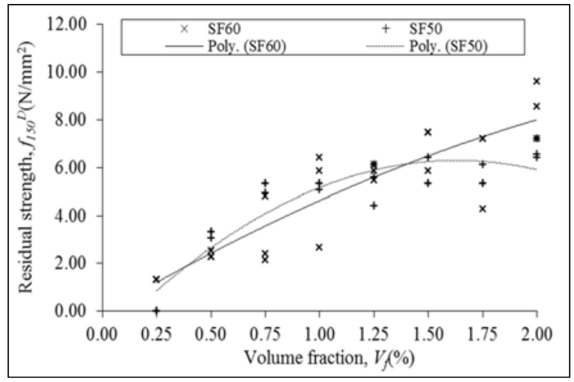

Figure-6. Relationship between the residual strength at net deflection of $L/150$ and volume fraction.

$$f_{150}^D = 0.419 V_f f_{p,SFRC} + 1.875 \quad (9)$$

$$f_{150}^D = -0.511 V_f^2 f_{p,SFRC} + 1.472 V_f f_{p,SFRC} - 0.067 \quad (10)$$

On the other hand, Figure-7 and Figure-8 presents the correlation between the test results and the proposed models with 1:1 line.

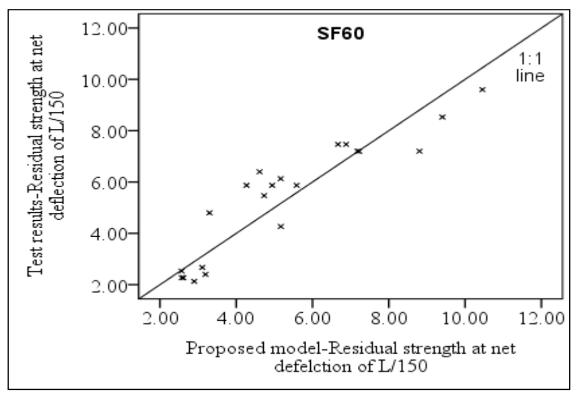

Figure-7. Relationship of residual strength between test results and proposed model for SF60 type of steel fibre.

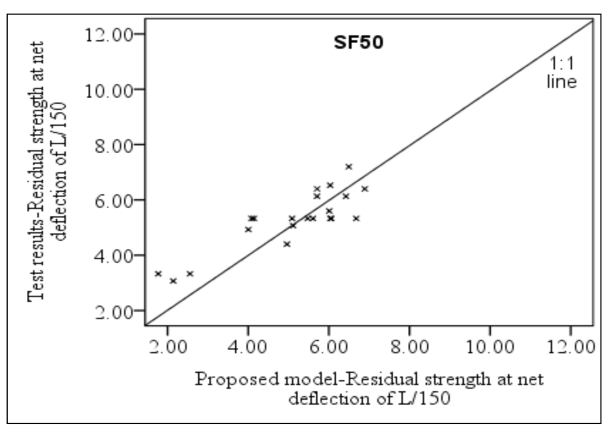

Figure-8. Relationship of residual strength between test results and proposed model for SF50 type of steel fibre.

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The newly proposed models on the residual strength at net deflection of $L/150$ were then applied to equation (3). The new equations on the prediction of the design value of the increase in shear strength due to the steel fibres are given in equation (11) and equation (12) for SF60 and SF50, respectively.

$$\tau_{fd} = 0.12\left(0.419V_f + \frac{1.875}{f_{p,SFRC}}\right)f_p \qquad (11)$$

$$\tau_{fc} = 0.12\left(-0.511{V_f}^2 + 1.472V_f - \frac{0.067}{f_{p,SFRC}}\right)f_p \qquad (12)$$

In order to calculate the additional shear from steel fibre, equation (11) and equation (12) are applied to equation 1. The calculated values are plotted in Figure-9, while the values are tabulated in Table-3. The additional shear supplement was plotted with respect to the fibre reinforcing index, *RI*. The fibre reinforcing index is the multiplication between fibre volume fraction, V_f and fibre aspect ratio, *L/D*. The purpose of plotting the values using the fibre reinforcing index is to illustrate the overall view clearly for both types of steel fibre. Through the Figure-9, it can be seen that all calculated values are closely mapped to the trendline. Therefore, the additional shear calculated from the proposed equation is suitable for predicting the contribution from steel fibres inside concrete.

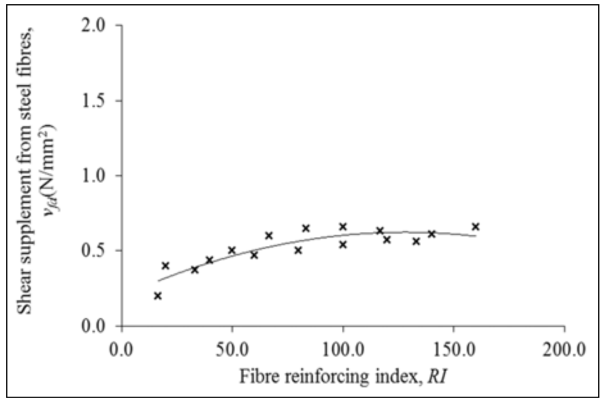

Figure-9. Relationship between additional shear from steel fibres and fibre reinforcing index.

Table-3. Shear supplement values.

Prism	V_f (%)	SF60			SF50		
		$f_{p,SFRC}$	τ_{fd}	v_{fd}	$f_{p,SFRC}$	τ_{fd}	v_{fd}
1	0.25	3.49	0.29	0.40	3.94	0.14	0.20
2		3.49	0.29	0.40	3.94	0.14	0.20
3		3.49	0.29	0.40	3.94	0.14	0.20
1	0.50	3.80	0.32	0.44	4.38	0.27	0.37
2		3.80	0.32	0.44	4.38	0.27	0.37
3		3.80	0.32	0.44	4.38	0.27	0.37
1	0.75	4.28	0.34	0.47	4.80	0.36	0.50
2		4.28	0.34	0.47	4.80	0.36	0.50
3		4.28	0.34	0.47	4.80	0.36	0.50
1	1.00	4.91	0.36	0.50	5.21	0.43	0.60
2		4.91	0.36	0.50	5.21	0.43	0.60
3		4.91	0.36	0.50	5.21	0.43	0.60
1	1.25	5.71	0.38	0.54	5.61	0.46	0.65
2		5.71	0.38	0.54	5.61	0.46	0.65
3		5.71	0.38	0.54	5.61	0.46	0.65
1	1.50	6.66	0.41	0.57	6.00	0.47	0.66
2		6.66	0.41	0.57	6.00	0.47	0.66
3		6.66	0.41	0.57	6.00	0.47	0.66
1	1.75	7.78	0.44	0.61	6.37	0.45	0.63
2		7.78	0.44	0.61	6.37	0.45	0.63
3		7.78	0.44	0.61	6.37	0.45	0.63
1	2.00	9.06	0.47	0.66	6.73	0.40	0.56
2		9.06	0.47	0.66	6.73	0.40	0.56
3		9.06	0.47	0.66	6.73	0.40	0.56

CONCLUSIONS

In this study, an analytical model was developed to propose an equation in predicting the shear stress supplement due to the existence of steel fibre in reinforced concrete. Rilem guideline has proposed an equation (as given in equation (1) to predict the shear supplement. However, there is a restriction on the model as its consideration is very general and applies to all types of steel fibre. In consideration of the current type of hooked-end steel fibre, some modifications were made so that it can be applied to this specific type of fibre. Besides, the modification also considered a wide range of fibre reinforcing index, *RI* from 67 and up to 80. This is believed to be able to represent the actual condition of shear supplement of hooked-end steel fibre when it is inside concrete. This gave many benefits as the hooked-end type of steel fibre is the most used fibre in research and construction industry. The proposed equations were found suitable to be used to predict the shear supplement due to steel fibres as it gives gave close estimation to the actual test results.

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