



# BEE ALGORITHM INTEGRATED WITH SYSTEM IDENTIFICATION TECHNIQUE FOR MODELLING DYNAMIC SYSTEMS

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## ABSTRACT

System identification has been widely used in modelling dynamic system whereby the input-output data from real system are undergo the model structure selection, parameter estimation and model validation procedure. However, the most complicated part in modelling the dynamic system is selecting the model structure to represent the system. In this project, bee algorithm (BA) is integrated with system identification technique to optimize the model structure selection in modelling the dynamic system. This project describes the procedure and investigates the performance and effectiveness of BA based on a few case studies. The result indicates that the proposed algorithm is able to select the model structure of a system successfully. The validation test carried out demonstrates that BA is capable of producing adequate and parsimonious models effectively.

**Keywords:** bee algorithm, system identification, modelling, optimization.

## INTRODUCTION

System identification is the study on creating equivalent mathematical models based on the input-output data provided (Zadeh, 1962). It consists of four procedure, 1) Collecting input-output data, 2) Model structure selection, 3) Parameter estimation and 4) Model validation. In order to estimate a good and adequate model using the system identification technique, there are some concepts that required to be fulfill: model, true description, model class, complexity, information, estimation, validation and model fit (Ljung, 2010). However, two main areas that given more concern are the selection of the model structure that consists of adequate number of terms in the final model and the estimation of the parameters of these terms (Ahmad *et al.*, 2008; Zakaria *et al.*, 2012 ). The behavior of the dynamic system is investigated and presented in form of a mathematical model. According to Ljung (1999), dynamic system is a system memory where the input value at time  $t$  will influences the output at future time instants. Therefore, the relationship between the input and output data of the dynamic system can be investigate by system identification.

Model structure selection is the most challenging procedure in system identification because by using the data provided, a selection process is done, following the standards and then the characters of the models are compared (Yang *et al.*, 2008). As for the model structure selection, the most popular method is the cross-validation, where the performance of a model is assessed on a validation data set. However, this technique is unreliable as the method is commonly set by an empirical value that needs to be known in order to get the best model structure. Various algorithms are proposed to solve the problem of selecting the optimal model structure to represent the dynamic systems. However for bee algorithm (BA), from the review of applications of BA this is the first attempt in combining the system identification and BA in selecting optimal model structure for modelling dynamic system. The basic of BA is a smart optimization tool that imitates the foraging behavior of honeybees in nature (Pham *et al.*,

2014). This paper presents the effectiveness of BA for identification of dynamic systems that represented by simulated system and real system. To validate the algorithm, this study starts with the identification of simulated single-input-single-output (SISO) systems with known model structures. Later, a real case study is considered where the inputs and outputs for these SISO systems were obtained from real experimental data. The process considered is flexible beam system (Saad *et al.*, 2012). This paper is preliminary study to attempt Bee algorithm as optimizer in finding an optimal model structure for representing behavior of dynamic systems.

## MODEL STRUCTURE REPRESENTATION

Representing a dynamic system from acquired input output data needs to define the type of model representation. Most nonlinear systems are modeled and identified by using mathematical and signal models, block diagram models, and simulation models. In this study, the mathematical and signal model is considered. A very common polynomial linear discrete-time system model representations is ARX (AutoRegressive with eXogenous input) model where the system output can be predicted by the past inputs and outputs of the system (Ljung, 1999) . This model is defined as

$$y(t) = C + a_1 y(t-1) + \dots + a_{n_y} y(t-n_y) + b_1 u(t-1) + \dots + b_{n_u} u(t-n_u) + e(t) \quad (1)$$

where output, input, and noise signal are represented by  $y(t)$ ,  $u(t)$ , and  $e(t)$  respectively, while  $C$ ,  $n_y$ , and  $n_u$  representing a constant, the maximum output and input lags in the model respectively. The coefficients of the model are represented by  $a_1 \dots a_{n_y}$  and  $b_1 \dots b_{n_u}$ . The nonlinear version for ARX model is called NARX model. Chen and Billings (1989) presented a Non-linear AutoRegressive Moving Average with eXogeneous inputs (NARMAX) model which provides a wide class of a



nonlinear model representation with the special case as a NARX model. The NARX model can be defined as

$$y(t) = F^l(C, y(t-1), \dots, y(t-n_y), u(t-1), \dots, u(t-n_u)) + e(t) \quad (2)$$

where  $F^l(\cdot)$  is a polynomial non-linear function with  $l$  degree of nonlinearity. The NARX model can be transformed into a linear regression model represented by

$$y(t) = \sum_{i=1}^M \theta_i \phi_i(t) + e(t), \quad n_y \leq t \leq N \quad (3)$$

where  $\theta_i$  and  $\phi_i(t)$  are unknown coefficients or parameters and regressors respectively,  $M$  is the maximum number of terms of the regressors and  $N$  is the size of data. The maximum number of possible terms,  $Lt$  in the NARX model in Equation (2) can be calculated as Chen and Billings (1989)

$$Lt = M + 1$$

where

$$M = \sum_{i=1}^l n_i$$

and

$$n_i = \frac{n_{i-1}(n_y + n_u + i - 1)}{i}, \quad n_0 = 1 \quad (4)$$

For example, a NARX model with  $n_y = n_u = 3$  and  $l = 2$ , a second order degree of nonlinearity, would contain 28 terms respectively. Thus, the possible models need to be considered can be calculate as  $2^{Lt} - 1$ , which is 268 435 455. Thus, increasing the orders of input and output lags and degree of nonlinearity, will increase the maximum number of terms of NARX model and the possible models that need to be searched. Thus, the user defined parameters such as the orders of input and output lags, and degree of nonlinearity will affect the difficulty of model structure selection. The search space become large and impractical when large user defined parameters are used. Therefore, selecting significant terms to be included in the final model become challenging and needs an automated and suitable tool for this task.

## BEE ALGORITHM

### Introduction to bee algorithm

Bee algorithm (BA) is an algorithm that was motivated from the food-searching activities of the honeybees. In optimization fields, it has effectively been used as the optimizing method. Furthermore, it is simple and straightforward to be applied in optimization fields as it follows the behaviour of nature (Hussein *et al.*, 2013).

The basic of BA is a smart optimization tool that imitates the foraging behaviour of honeybees in nature.

There are two kinds of search method used by the bees in natural environment; global random search and local search. Global random search is when bees around the hive are sent randomly to search for honeys, and they are called scout bees. The scout bees that doing the random search will later recruit more bees to the potential food sources that were discovered by them (Packianather *et al.*, 2009). They will perform a dance; "waggle dance" on the "dance floor" as they found the food sources that exceed the quality limit (measured as sugar content) and the dance is important as it carries three vital information to the colony which are, the direction of the source, distance from the hive and its quality (Pham *et al.*, 2006; Von Frisch, 1950). This information allows the bees to travel to the food sources accurately without needed any lead from the scout bees.

Rationally, as the quality improve and distance of the food sources decrease, the number of recruited bees will also increase. This allows the food-collecting activities done faster and efficiently. These food-searching activities of the recruited bees are called the local search and the remaining bees will continue doing the random search. The iteration continue to be in cycle to ensure that all food sources whether it is above the quality limit or not, are discovered (Pham *et al.*, 2006).

Figure-1 shows the simple of form of the BA. The basic of the BA is presented in the form of a pseudocode (Pham *et al.*, 2006):

1. Initialize population with random solutions
2. Evaluate fitness of the population
3. While (stopping criterion not met) // Forming new population
4. Select sites for neighbourhood search
5. Recruit bees for selected sites (more bees for best  $e$  sites) and evaluate fitness
6. Select the fittest bee from each patch
7. Assign remaining bees to search randomly and evaluate their fitness
8. End While

Figure-1. Pseudocode for the basic BA.

### Bee algorithm for model structure selection

This paper presents the study on modelling dynamic system by integrating system identification technique and BA. The most complicated part in modelling dynamic system is on selecting the optimal model structure to represent the dynamic system. Then, an adequate and parsimonious model is going to be chose from the modelling process.

Bee algorithm (BA) is a population-based metaheuristic algorithm that imitates the foraging behavior of honeybees in nature. Foraging process is done by the scout bees where the scout bees will travel randomly from patch to patch to search for favorable flower patches. For model structure selection using BA, there are some parameters that need to be defined. The parameters are: number of scout bees ( $n$ ), number of sites selected out of  $n$

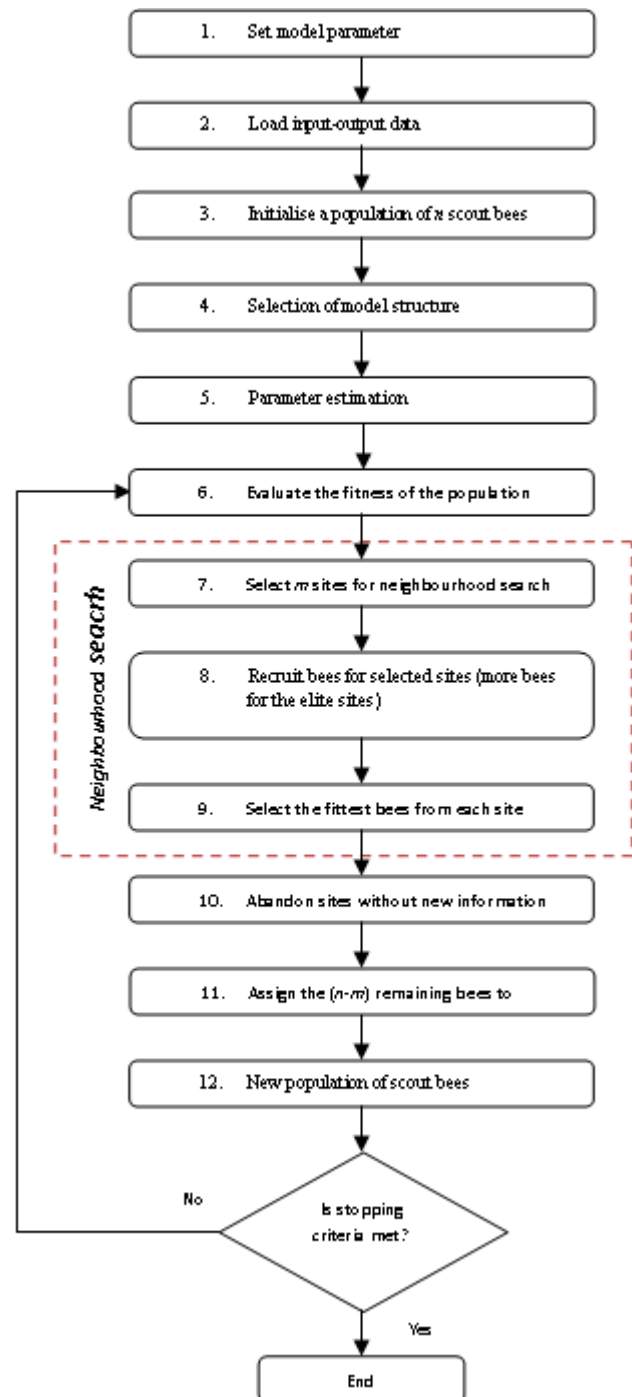


visited sites ( $m$ ), number of best sites out of  $m$  selected sites ( $e$ ), number of bees recruited for the best  $e$  sites ( $nep$ ), number of bees recruited for the other ( $m-e$ ) selected sites ( $nsp$ ), initial size of patches ( $ngh$ ) which include site and its neighborhood and stopping criterion (Pham and Kalyoncu, 2009) while the flowchart for BA combined with system identification technique is shown in Figure-2.

First of all, the parameter of the model is set and the input-output data is loaded into the system identification. Then, the population of  $n$  scout bees which represent the overall number of terms that need to be set for the system is defined. These terms are depends on the input and output lag and nonlinearity degree that is going to represent the system. Then, the fitness of the population is evaluated where in this step, the fitness of the bees are represent by the mean square error (MSE) of the system that can be collected as soon as the data is simulated in the MATLAB.

Then,  $m$  sites are selected for neighborhood search, where each of the sites represents one term in the model. For this project, a term is represented by a bee. This will later brought to neighborhood search where model structure in the system takes place. The fittest bee is selected and it refers to the terms that will represent the model. Next, the terms that are not selected will be ignored and new population is formed to continue the search of terms for best model structure.

The following parameter values of the algorithm were set for this test: population  $n=50$ , number of selected sites  $m=4$ , number of elite sites  $e=1$ , initial patch size  $ngh=3$ , number of bees around elite points  $nep=3$ , number of bees around other selected points  $nsp=1$ . Note that  $ngh$  defines the initial size of the neighborhood in which follower bees are placed.



**Figure-2.** Flowchart of the BA combined with system identification technique.

## RESULTS AND DISCUSSIONS

### Simulation systems

The effectiveness of the proposed algorithm is investigated for two SISO systems before implementing it to an experimental data. Two SISO systems were simulated to produce input-output data for structure selection using the algorithm as follows in Table-1.

**Table-1.** The systems used in the simulations.

Systems		Descriptions
Simulated system	SS1	$y(t) = 1.5y(t-1) - 0.54y(t-2) + 3.2u(t-6) - 0.96u(t-7)$
	SS2	$y(t) = 0.95y(t-1) + 0.7u(t-1) - 0.5u(t-3) - 0.37y(t-3)u(t-3)$
Real system	RS1	Flexible beam vibration (Saad <i>et al.</i> , 2012)

For the simulated system, the input-output data consists of 500 measurements. These known structure models are selected to validate the ability for the algorithm to determine the correct model structure. For Simulated System 1, SS1, the manipulated parameters are:  $(n_u=n_y=8)$ ,  $(n_u=n_y=9)$ , and  $(n_u=n_y=10)$  and degree of nonlinearity,  $l=1$ . While for Simulated System 2, SS2, the manipulated variables are:  $(n_u=n_y=4)$ ,  $(n_u=n_y=5)$ , and  $(n_u=n_y=6)$  and degree of nonlinearity,  $l=2$ .

Then, for further investigation, a real data is considered, labeled as RS1 for data collected from

experimental. As for RS1, the data consists of 1500 measurements and parameters that manipulated are:  $(n_u=n_y=10)$  and one degree of nonlinearity ( $l=1$ ) noted as T10101, and  $(n_u=n_y=5)$  and two degree of nonlinearity ( $l=2$ ) noted as T552.

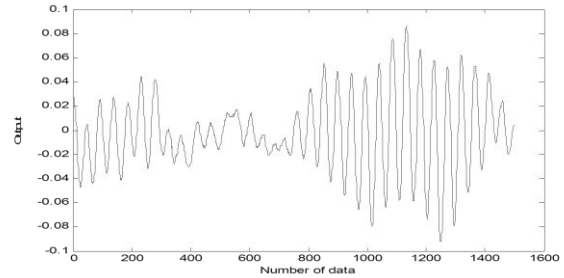
**Figure-3.** The output signal for RS1

Figure-3 shows the output signal for the real system RS1. This output signal is the actual output of the system that will be compared with the predicted output from the optimal model using the system identification and BA.

**Table-2.** Details of model structure RS1 (T10101).

Terms	Run 1	Run 2	Run 3	Run 4	Run 5
y(t-1)	-	1.27804	-	-	-
y(t-3)	-	-0.25670	-	-	-
y(t-4)	-	-	-	0.88576	2.04650
y(t-5)	2.95482	-	-	-	-
y(t-7)	-2.20030	-	0.65829	-	-1.24607
y(t-8)	-	-	-	-	-
y(t-9)	-	-0.06570	-	-	-
u(t-1)	0.00012	0.00003	0.00057	0.00051	0.00010
u(t-10)	-	-	0.00069	-	-
Number of term	3	4	3	2	3
MSE	$1.785932 \times 10^{-5}$	$1.495026 \times 10^{-6}$	$2.766435 \times 10^{-4}$	$1.104563 \times 10^{-4}$	$9.795745 \times 10^{-6}$

**Table-3.** Details of model structure RS1 (T552).

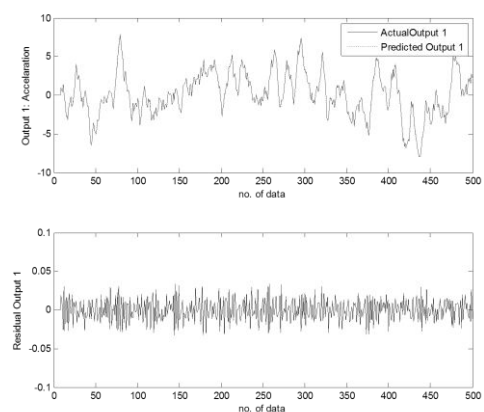
Terms	Run 1	Run 2	Run 3	Run 4	Run 5
$y(t-1)$	-	-	<b>1.44826</b>	-	<b>1.84606</b>
$y(t-2)$	-	1.58016	-	1.88347	-0.86212
$y(t-3)$	0.93061	-	-0.47186	-	-
$y(t-4)$	-	-	-	-0.94463	-
$y(t-5)$	-	-0.65682	-	-	-
$u(t-5)$	-	-	-	-0.00004	-
$y(t-1)y(t-3)$	-	-	-	-0.11909	-
$y(t-2)y(t-2)$	-	-	-	-	-0.00381
$y(t-2)y(t-3)$	-	0.02789	-	-	-
$y(t-3)y(t-5)$	0.2456044	-	-	-	-
$y(t-3)u(t-4)$	-	-	-0.00017	-	-
$y(t-3)u(t-5)$	0.02155	-	-	-	-
$y(t-4)y(t-4)$	-	0.04558	-	-	-
$y(t-4)u(t-4)$	-	-	-	-0.00015	-
$y(t-4)u(t-5)$	-	-	-	-	-0.00020
$y(t-5)u(t-4)$	-0.02419	-	-	-	-
$u(t-1)u(t-1)$	-	-	-0.00006	-	-
$u(t-2)u(t-3)$	-	-	-	-	-0.00003
$u(t-3)u(t-3)$	-	-0.00002	-	-	-
Number of terms	4	5	4	5	5
MSE	$6.585636 \times 10^{-5}$	$2.512107 \times 10^{-6}$	$1.612194 \times 10^{-6}$	$3.213187 \times 10^{-6}$	$2.221808 \times 10^{-6}$

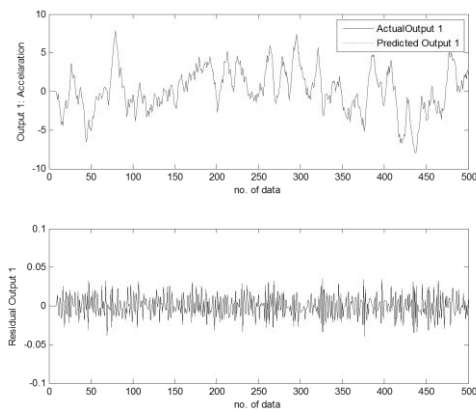
Tables-2 and 3 list the details of predicted model structure of RS1 T10101 and T552 for five time runs (Run 1 to Run 5). Table-2 lists the predicted model structure for simulation study T10101. From five time runs, only one term that exist for every each run which is  $u(t-1)$ . The smallest MSE is come from Run 2 which is  $1.495 \times 10^{-6}$  where considered as fitness value in BA. Meanwhile in Table-3 lists the predicted model structure for simulation study T552. From observation of five time runs in Table-3, the existed terms is scattered and not consistent for every each run. However, the smallest MSE is come from Run 3 which is  $1.61 \times 10^{-6}$  compared the other models. In system identification, the final procedure is model validation, next sub-section describe this final procedure.

### Model validation

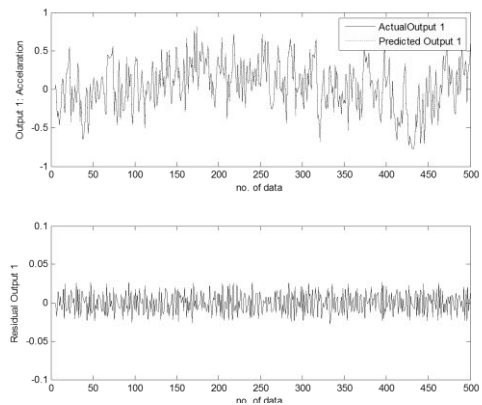
Once the models have been identified and the parameters have been estimated, it is important to know whether the models have successfully captured the system dynamics. Therefore, it is important to evaluate the adequacy and validity of the identified models. Finally, some validation tests are conducted on the identified models that are the OSA test as shown in Figures-4 to 9 for SS1 and SS2. While for RS1, the validation tests that

are conducted are OSA test as shown in Figures-10 to 11 and regression analysis as shown in Figures-12 to 13.

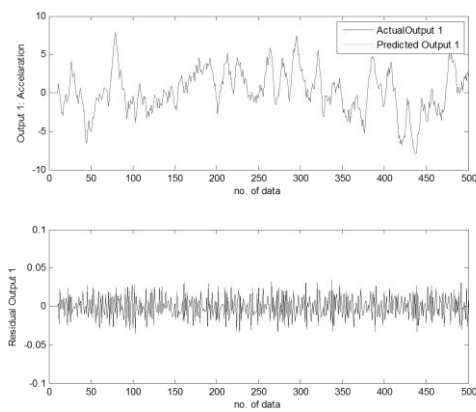
**Figure-4.** OSA test for SS1 ( $n_u=n_y=8$ ,  $l=1$ ).



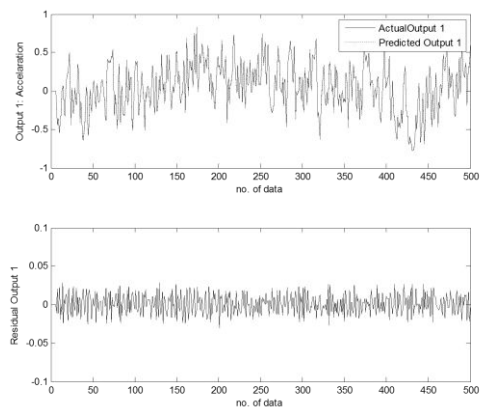
**Figure-5.** OSA test for SS1 ( $n_u=n_y=9$ ,  $l=1$ ).



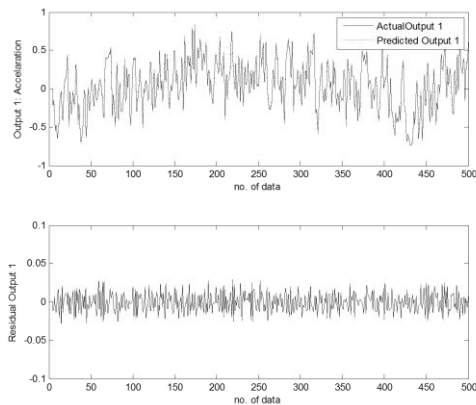
**Figure-8.** OSA test for SS2 ( $n_u=n_y=5$ ,  $l=2$ ).



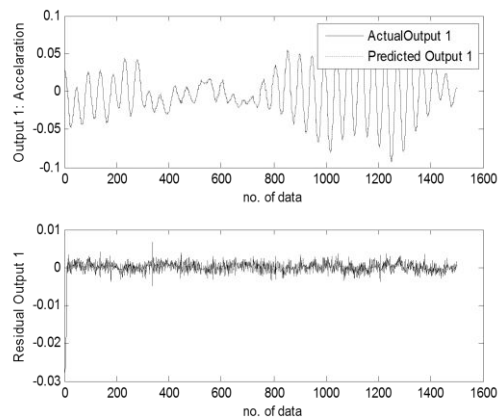
**Figure-6.** OSA test for SS1 ( $n_u=n_y=10$ ,  $l=1$ ).



**Figure-9.** OSA test for SS2 ( $n_u=n_y=6$ ,  $l=2$ ).

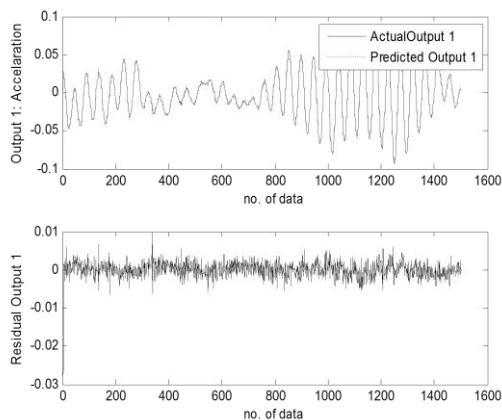


**Figure-7.** OSA test for SS2 ( $n_u=n_y=4$ ,  $l=2$ ).



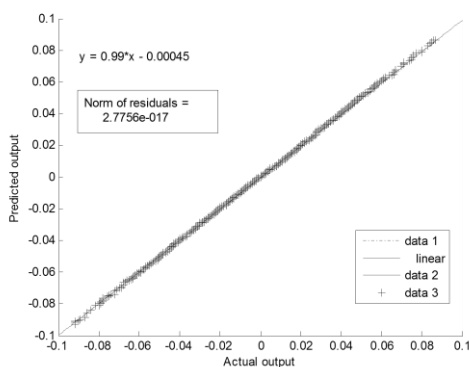
**Figure-10.** OSA test for RS1 (T10101).



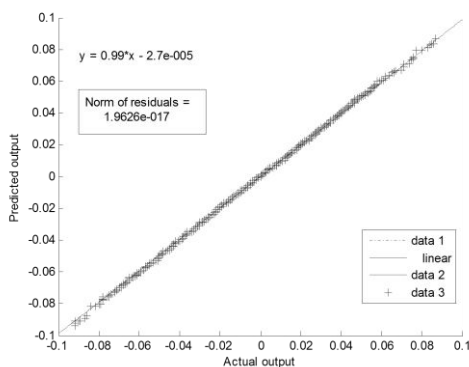


**Figure-11.** OSA test for RS1 (T552).

The OSA tests carried out for SS1 as shown in Figures-4 to 6 and SS2 as shown in Figures-7 to 9, respectively are good because the predicted output follows the actual outputs very well. The values of the residual output for the models are relatively small between the range of  $[0.05, -0.05]$ . From the simulations, the predicted output are consistent and almost the same as the actual model of the system. However, in some simulations, it shows that the value are a bit scattered due to the increase in defining input and output lags and nonlinearity degree of the system.



**Figure-12.** Regression analysis for RS1 (T10101).



**Figure-13.** Regression analysis for RS1 (T552).

For T10101, model structure from Run 2 as listed in Table-2 is selected to go through the model validity test. Meanwhile for T552, Run 3 is chosen as model structure as shown in Table-3 for testing the validity. From the OSA test that was carried out to the real systems RS1, it shows that the results are good because the predicted outputs follow the system outputs very well. Then, the regression analysis that was carried out as shown in Figures-12 to 13 confirm that the models selected are adequate because the predicted output maintain plotted on the linear regression graph against the actual output and at the same time the norm of residuals in every run are relatively small. These validation results also confirm that the algorithm provide good approximation for modelling the dynamic systems of flexible beam.

## CONCLUSIONS

In this paper, BA is proposed to solve the model structure selection in system identification technique for dynamic system modelling. The main problem encountered in this study is on how to select an adequate and parsimonious model from several models developed during the study. The results indicate that the effectiveness of the BA to model the dynamic system has been performed. As in the future, other models than ARX and NARX model could be used as possible continuation to the study in order to deal with complex real process systems. Furthermore other state-of-art methods beside MSE can be considered as future research in comparing the performance of the selected terms. Other types of algorithm can also be used to solve the model structure selection. Furthermore, the application of BA for multivariable system can be considered to continue the research. Hence, future research would be attempting to perform basic research on system identification with focus on a real-world problem.

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