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AN IMPROVED LOAD-DISPLACEMENT PREDICTION FOR A CONED DISC SPRING USING THE ENERGY METHOD

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ABSTRACT

An improved computation of the load-displacement prediction for a coned disc spring is proposed. This work is an extension work of the previous proposed energy-based computation for coned disc springs. To show the validity of the improved method, the load-displacement results using the improved computation, the previous proposed method, and the finite element analyses are compared. In this work, the improved computation and the finite element analyses have accounted for the radial deflection of the disc spring in the form of energy method. This is different in the previous work where the radial deflection of the disc spring was ignored and the comparison to the finite element analyses without accounting the radial deflection is not practical. The present work makes the comparison to the finite element analyses to be more reasonable and more practical.

Keywords: coned disc spring, energy method, load-displacement, radial stresses, tangential stresse.

INTRODUCTION

A coned disc spring is commonly designed to obtain large loads under small deflections [1-3]. The nonlinear load-displacement curve is often exploited by designer in order to preserve the span of null slope zone to delay further increasing loads. This type of spring is used in many engineering applications where a large amount of energy at high loads is absorbed to avoid unnecessary damage [1-10].

By referring to the previous research work, the Almen-analytical load-displacement prediction for a coned disc spring has been widely used for many years [1-10]. This analytical expression derived in the Ref. [2] depends on the tangential stresses and the deflections of the disc spring [2-6]. This work is referred to as a standard method for stress and displacement calculations.

However, the referred analytical research work cannot exactly match with the 3-dimensional finite element analyses. The developed analytical model also cannot evaluate the actual load-displacement features such as the deflection in the radial direction [7-10]. The work by Almen is limited to a number of assumptions and approximations [1, 2]. To improve the accuracy of the work by Almen, Curti proposed an improved calculation by not only considering the tangential stresses and deflections, but also the radial stresses [4-10]. Even though it was found that the influence of the radial stresses were very low, the computation results using the improved equations agreed more closely with the finite element analyses results in comparison to those by the theory of Almen [4].

In this work, another formed of calculation method is proposed in the form of energy method which is different in comparison to the Curti computation. In comparison to the analytical Almen computation, the radial stress is considered.

IMPROVED LOAD-DISPLACEMENT FOR CONED DISC SPRING IN THE FORM OF ENERGY METHOD

The work by Almen [2-4] assumes that the cross section of the coned segment does not distort at all and only rigidly rotates. The tangential stress and the displacement s of the coned disc segment are expressed as [2, 4];

$$\sigma_t = \frac{4Es}{d_e^2}\left[A\left(h_0 - \frac{s}{2}\right) + Bt\right] \tag{1}$$

$$F = \frac{4Es}{d_e^2}\left[(h_0 - s)\left(h_0 - \frac{s}{2}\right)Ct + Dt\right] \tag{2}$$

where A, B, C, and D are defined based on the ratio $\delta = d_e/d_i$ and the Poisson ratio μ which can be calculated as follows [2,4]:

$$A = \frac{1}{1-\mu^2}\left(\frac{\delta-1}{\ln\delta} - 1\right)\left(\frac{\delta}{\delta-1}\right)^2 \tag{3}$$

$$B = \frac{1}{1-\mu^2}\left(\frac{\delta-1}{2}\right)\left(\frac{\delta}{\delta-1}\right)^2 \tag{4}$$

$$C = \frac{1}{1-\mu^2}\left(\frac{\delta+1}{\delta-1} - \frac{2}{\ln\delta}\right)\pi\left(\frac{\delta}{\delta-1}\right)^2 \tag{5}$$

$$D = \frac{1}{1-\mu^2}\left(\frac{\pi}{6}\ln\delta\right)\left(\frac{\delta}{\delta-1}\right)^2. \tag{6}$$

The The proposed equations by Curti [4] not only account for the tangential stress and the displacement s of the coned disc spring, but also the effect of the radial stress. The radial stress σ_r is neglected in the work of

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Almen where the ring is assumed to be constrained in the radial direction at both outer and inner edges [2, 4]. By accounting the radial stress σ_r effect, the proposed theory by Almen can be reasonably assimilated to a circular flat thin plate [4]. By including the tangential stress σ_t, the radial stress σ_r and the displacement s, previous equations (3) to (6) are calculated as follows [4]:

$$\bar{A}=\left(\frac{\delta}{\delta-1}\right)^2\left(\frac{d}{d_i}-1\right) \quad (7)$$

$$\bar{B}=\frac{1}{2}\left(\frac{\delta^2}{\delta-1}\right) \quad (8)$$

$$\bar{C}=\frac{2\pi}{1-\mu}\cdot\frac{\delta^2}{(\delta-1)^3}\left[\frac{1+\delta}{2}+\frac{\mu}{1+\mu}\cdot\frac{\delta^{(\mu+1)}-1}{1-\delta^{\mu}}\right] \quad (9)$$

$$\bar{D}=\frac{\pi}{6}\cdot\frac{d_e}{d}\cdot\frac{\delta}{\delta-1} \quad (10)$$

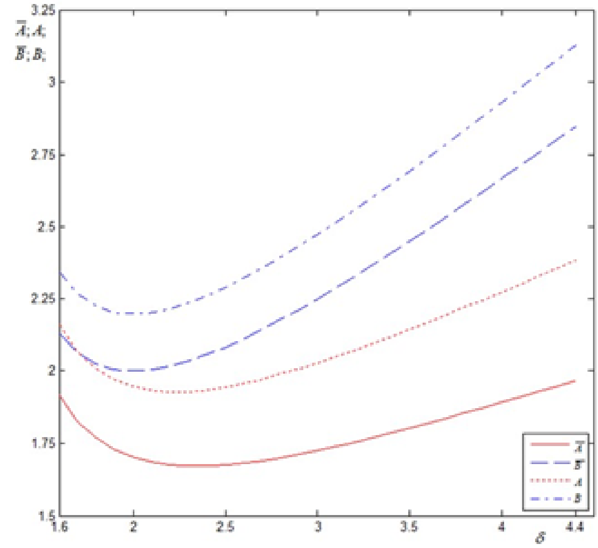
In order to examine the effect of the radial stress σ_r, the function A, B, $\bar{A}$, and $\bar{B}$ in equations (7), (8), (9), and (10) are shown with respect to the variation of $\delta = d_e/d_i$ in Figure-1(a). The dimensions of the disc spring and the plotted curves are similar to the Ref. [4]. The changes of the plotted curves are significant between the equations proposed by Curti and the theory of Almen. This shows even though by using the same input parameters of free heigh h_0, poisson ratio μ=0.3 and δ, the maximum tangential stresses which can be calculated using equation (1) are always smaller in the Curti's proposed calculations in comparison to the theory of Almen. Thus, significant different load-displacement curves are to be obtained once the radial stress is considered.

Similarly, significant change can also be observed for functions C, D, $\bar{C}$ and $\bar{D}$ as shown in Figure-1(b). The functions of the ratio between the theory of Almen $R=C/D$ and the work by Curti $R=\bar{C}/\bar{D}$ are shown in Figure-1(c) with the changes of Poisson ratio μ. In the theory of Almen, the $R=C/D$ ratio depends only on the ratio $\delta = d_e/d_i$ and differs very little from unity [2, 4]. However, the functions of the ratio proposed by Curti $R=\bar{C}/\bar{D}$ not only depend on the ratio $\delta = d_e/d_i$, but also Poisson ratio μ. Moreover, these can differ from unity more than as shown in Figure- 3 as the functions have significant change with the variation of μ = 0.3, 0.4 and 0.5.

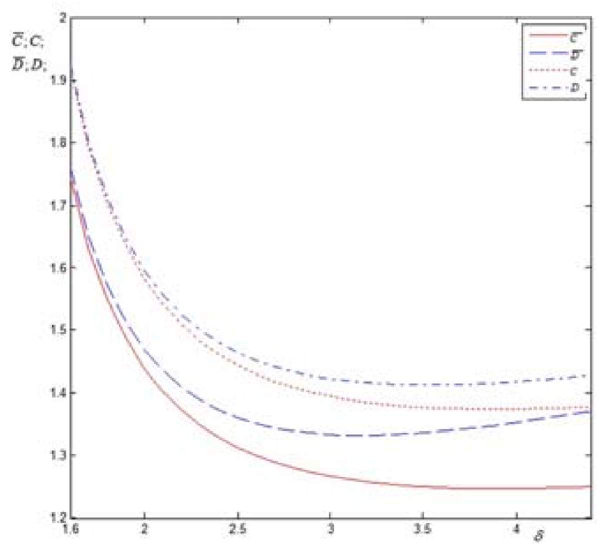
By accounting the radial deflection, the load-displacement for coned disc springs in Ref. [1] is shown in Figure-2 which can be modified as follows;

$$F_1=F_3(\bar{C},\bar{D})\cdot\frac{d\delta_3}{d\delta_1} \quad (11)$$

$$F_1=\frac{4Es}{d_e^2}\left[(h_0-s)\left(h_0-\frac{s}{2}\right)\bar{C}t+\bar{D}t\right]\cdot\frac{l_3}{l_1}\left[\frac{1}{\cos\gamma-[\tan(\alpha-\theta)\sin\gamma]}\right] \quad (12)$$



(a)



(b)

VOL. 11, NO. 9, MAY 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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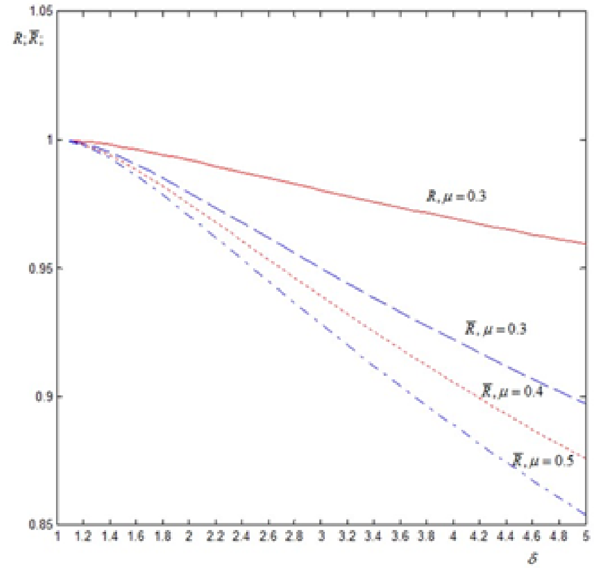

(c)

Figure-1. (a) Coefficients A, B, $\overline{A}$ and $\overline{B}$ depending on the ratio (b) Coefficients C, D, $\overline{C}$ and $\overline{D}$ depending on the ratio $\delta = d_e/d_i$ for steel spring (μ=0.3) (c) Ratio $R=C/D$ and $\overline{R} = \overline{C} / \overline{D}$ depending on the ratio $\delta = d_e/d_i$.

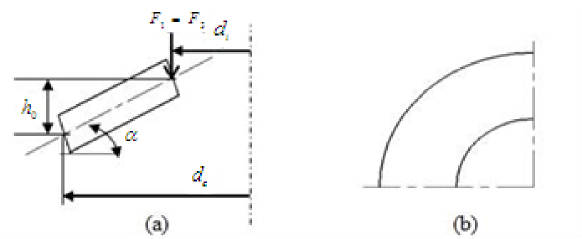

Figure-2. (a) Coned Disc (b) quarter top view.

COMPUTATION RESULTS

The comparison results between the improved energy method and the previous work [1] are shown in Figure-3 to Figure-5. The improved computation results approximately match the load-displacements using the finite element analyses. The present work clearly improves the previous work in order to obtain good results. At the same time, the radial deflection was not considered in the previous proposed energy method1. In contrast, the finite element analyses freely allowed the deflection in the radial direction. By considering the radial deflection in the improved energy method, the comparison is more practical and reasonable since both computations account for radial deflection.

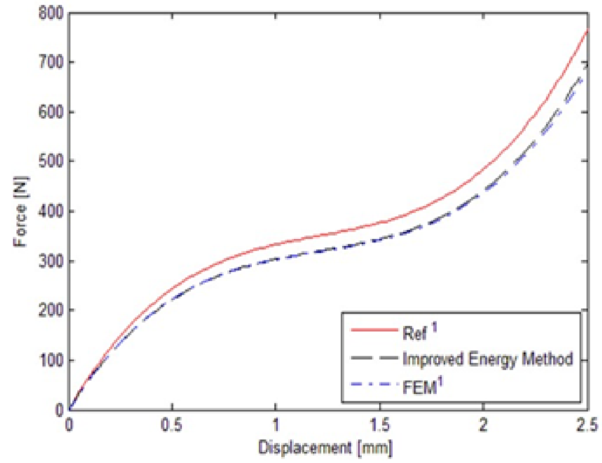

Figure-3. Case A.

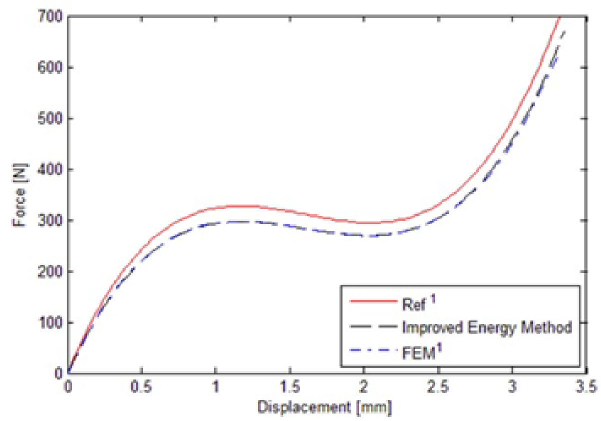

Figure-4. Case B.

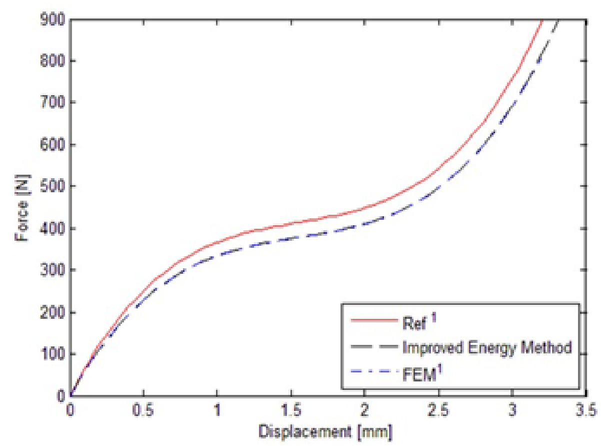

Figure-5. Case C.

CONCLUSIONS

An improved analytical load-displacement for coned disc springs is presented by considering the radial deflection. This extended analytical prediction is proposed to match with the three-dimensional finite element analyses results besides accounting for the radial

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deflection which is similar to the boundary conditions of the finite element analyses.

ACKNOWLEDGEMENT

This work was supported by Malaysian Government under Fundamental Research Grant Scheme (FRGS/2/2014/TK01/UTM/02/7).

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