



APPLICATION OF NATURAL AND MODEL EXPERIMENT METHODOLOGY IN TWO-DIMENSIONAL ELECTRICAL IMPEDANCE TOMOGRAPHY

Grayr Karenovich Aleksanyan¹, Nikolay Ivanovich Gorbatenko¹, Valeriy Viktorovich Grechikhin¹,
Tran Nam Phong² and Tran Dai Lam³

¹Platov South-Russian State Polytechnic University (NPI), Prosvescheniyaul., Novocherkassk, Russian Federation

²Le Quy Don Technical University, Hoang Quoc Viet, Ha Noi, Viet Nam

³Vietnam Academy of Science and Technology, Graduate University of Science and Technology, Ha Noi, Viet Nam

E-Mail: graer@yandex.ru

ABSTRACT

An analytical overview of the methods for solving problems in the field of electrical impedance tomography was performed. Realization of natural and model experiment methodology in electrical impedance tomography was reviewed. The structural installation diagram for natural and model experiment was shown. The use of conditionally well-posed problems was suggested. The algorithm of natural and model experiment was introduced and functional test was performed. Test results show relatively high accuracy and operation speed of the algorithm. It was traced the possibility of image reconstruction and biological objects anomaly detection based on the natural and model experiment methodology.

Keywords: electrical impedance tomography, biological object, natural and model approach, direct problem, inverse problem, iterative process, reconstruction, model, finite element mesh.

1. INTRODUCTION

Currently for diagnostics medicine and industry use electrical impedance tomography (EIT) which is the way to obtain tomography image based on the natural and model experiment methodology.

Low-voltage currents (1-5 mA) of high-frequency are carried to the study subject by a system of electrodes attached to its surface. Electric potentials are measured on the electrodes. Values of potentials and currents converted to digits are input into computer. Its software has a computer model that includes a mathematical model that describes the potential distribution in the object, techniques of modeling and functionals minimizing. By solving the inverse problem an unknown distribution of electrical conductance and shape of abnormal inclusions inside the object are found, which can be used in medical tomography, non-destructive inspection of industrial products and processes. The problem relates to inverse problems of mathematical physics. They are characterized by several features: they are, in most cases, non-linear; they may have more than one solution or no solutions at all, they are not stable with respect to small changes in the input data. For the inverse problems measurement error can have a significant impact on the error in the determination of any properties of the object. Such problems are usually called ill-posed problems in the sense of Hadamard.

2. RESULTS

A considerable amount of research is devoted to problems of EIT. [1] describes the technology of the finite element model of biological objects construction. As an example the construction of a model of the human head. It is suggested a criterion of optimizing the finite elements and estimated the maximum amount of information obtained by electrical impedance measurements. [2]

describes the experimental installation for EIT and the method of inverse problem solution based on the use of artificial neural network models.

In [3, 4] the electrical impedance tomography uses famous iterative method for solving the system of equations, the Newton-Raphson method which is as follows. For the various configurations of the current running to the object functionals like the following are constructed.

$$J_i = \|\bar{\varphi}_i - \bar{\varphi}_i^*\|^2; \quad i = 1, 2, \dots, n, \quad (1)$$

where $\|\cdot\|$ is the norm in L_2 , $\|\cdot\| = \sum_{j=1}^n (\varphi_{ij} - \varphi_{ij}^*)^2$; $\bar{\varphi}_i$ is the vector of potential values, which are determined by solving the direct problem of EIT; $\bar{\varphi}_i^*$ is the vector of the measured values of potentials; i is the current configuration index, n is the number of configurations equal to the number of variables; j is the electrode index.

It is estimated that conductances σ have piecewise constant distribution. The functional (1) is expanded in Taylor's series without sacrificing linear components

$$J_i^{k+1}(\bar{\sigma}^k + \Delta \bar{\sigma}^k) = J_i^k(\bar{\sigma}^k) + \frac{\partial J_i^k}{\partial \sigma_1} \Delta \sigma_1^k + \frac{\partial J_i^k}{\partial \sigma_2} \Delta \sigma_2^k + \dots + \frac{\partial J_i^k}{\partial \sigma_n} \Delta \sigma_n^k = 0;$$

$i = 1, 2, \dots, n$, where k is the index of the iteration step.

As a result we have a system with matrix of Jacobi of the form

$$\begin{bmatrix} \frac{\partial J_1^k}{\partial \sigma_1} & \dots & \frac{\partial J_1^k}{\partial \sigma_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial J_n^k}{\partial \sigma_1} & \dots & \frac{\partial J_n^k}{\partial \sigma_n} \end{bmatrix} \begin{bmatrix} \Delta \sigma_1^k \\ \vdots \\ \Delta \sigma_n^k \end{bmatrix} = \begin{bmatrix} -\|\bar{\varphi}_1 - \bar{\varphi}_1^*\|^2 \\ \vdots \\ -\|\bar{\varphi}_n - \bar{\varphi}_n^*\|^2 \end{bmatrix} \quad (2)$$



After the system (2) is solved we find the following initial guess

$$\bar{\sigma}^{(k+1)} = \bar{\sigma}^{(k)} + \Delta \bar{\sigma}^{(k)}$$

Then we check if it is met the condition

$$J_i^{k+1} \leq \varepsilon$$

where ε is the functional minimum determination accuracy.

We shall note that a vector $\bar{\varphi}_i^*$ can be obtained on a physical model or by computer modeling. The variables can be the conductance, currents, geometrical parameters of internal abnormalities.

The approach described above has a significant drawback: with increase of k the value of the Jacobi matrix elements tends to zero. This increases the effect of rounding errors. We have to apply the regularization.

A genetic algorithm is applied in [4] to solve the problem of EIT. It is stated that this algorithm can overcome the incorrectness of the inverse problem. The drawback of this work is that the algorithm is verified on a two-dimensional model that includes only one nonhomogeneity.

The problem of the electrical impedance tomography is solved using the circuit analysis in [5]. It is supposed to use matrix inversion which complicates the calculations. This approach may be useful for determining the initial values of the desired conductance. It is proposed in [6] to use method of conductivity band in circuit analysis instead of the Newton-Raphson method, which reduces the order of systems of equations. However, disadvantages of this approach using the derivatives are retained.

[7] Uses the final element analysis (FEA) to solve the direct problem of EIT, the inverse problem is solved by the Newton-Raphson method.

It is suggested in [8] to solve the problems of EIT by modifying the finite difference method. Method schemes are determined by the method of finite volumes. The article compares the suggested method and FEA and shows that the suggested method does not have any advantages. The drawback of the method is the necessity to build a new program to solve the direct problem of EIT. The analysis of the sources [1 - 8] leads to the following conclusions:

a) EIT has a number of advantages over other types of tomography: compact equipment, devices simplicity, low cost, relatively little time to gather information, security for biological tissues. Disadvantage of EIT is poor quality of the images. EIT could be a promising direction provided hardware improvements and computer models based on the use of natural and model experiment method.

b) It is reasonable to solve the direct problems of EIT by using the finite element method, which is implemented in many commercial and non-commercial software systems and provides acceptable accuracy. However, the methods which significantly reduce the direct problems order of EIT are not described. These

include boundary element methods, fundamental system of solutions, and mixed methods.

c) Application of the Taylor series method and the Newton-Raphson method necessitates to solve the system of « n » equations with ill-conditioned matrix (n is the number of unknown quantities: conductance and geometrical parameters of abnormal inclusions). In our view, in order to minimize the functional (1) it is reasonable to use gradient methods that would reduce the number of operations in n^2 times. This is due to the fact that it is required $\sim n^3$ operations to solve the system of « n » equations with « n » unknowns. When using gradient methods one uses « n » relations and follows \sim « n » steps.

d) The direct problem of EIT resolves into a boundary value problem for the elliptic equation of the second order.

e) It is noted that the inverse problem is ill-posed. We further propose to use conditionally well-posed problems. At the same time the existence and uniqueness of solutions of the boundary value problems for elliptic equations is proved. Stability of the problem is achieved by finding solutions within the restricted class of functions - the class of constrained functions. These functions are used in the finite element method and other numerical methods.

f) The natural and model experiment allows the use of the physical models of the study subject instead of the subject itself when checking-out measuring equipment and computer model of the electrical impedance tomography. It is convenient to use computer simulations of the subject when testing a computer model of EIT.

Image reconstruction in EIT depends on the correct choice of the mathematical model of the study subject, initial approximation of the unknown parameters, measuring equipment error, the choice of minimizing the target functional method. We suggest using the method of the natural and model experiment to solve the problem. The effectiveness of such an approach for determining the magnetic parameters of electrical products is shown in [22, 23]. Its special feature is that the results of the experiment are used as input data for the simulation of electromagnetic field of the system the product is tested in, and as a criterion for evaluating the accuracy of the problem numerical solution.

Consider the implementation of the natural and model experiment method in the EIT. In [24 - 28] a hardware-software complex for electrical impedance tomography of biological objects is developed. Generalized structural installation diagram for the experiment is shown in Figure-1.

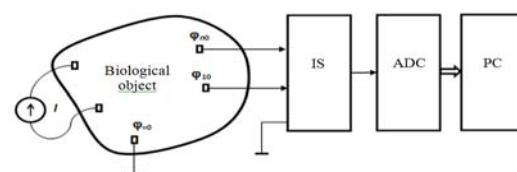


Figure-1. Structural installation diagram for the natural and model experiment in EIT.



The experimental information about the currents I and potentials on electrodes φ attached to the surface of the study subject is put in the personal computer (PC), after it has been amplified in the input section (IS) and the measuring signals have been digitized by analog-to-digital converter (ADC). The software required for the solution of the following inverse problem is installed. It is necessary to reconstruct the structure of the subject, the distribution of the specific electrical conductance and the geometry of the anomalies based on the solution of an elliptic boundary value problem for the differential equation of the second order

$$\frac{\partial}{\partial x} \left[\sigma(x, y) \frac{\partial \varphi}{\partial x} \right] + \frac{\partial}{\partial y} \left[\sigma(x, y) \frac{\partial \varphi}{\partial y} \right] = 0 \quad (3)$$

with the boundary conditions:

$$\frac{\partial \varphi}{\partial n} = 0 \quad \text{out of the current electrodes;} \quad (4)$$

$$\frac{\partial \varphi}{\partial n} = \frac{I}{\sigma S} \quad \text{on the current electrodes of S in area.} \quad (5)$$

Here φ is electric potential, $\sigma(x, y)$ is specific electric conductance.

Electric induction currents are neglected.

The described problem belongs to the class of Neumann inner models which solutions are defined up to a constant. To obtain a unique solution set a potential of one electrode is $\varphi = 0$. As mentioned above, additional information for the solution of the inverse problem are the measured values of current I_i^* and potential φ_i^* of the electrodes.

We propose the following algorithm of the natural and model experiment in the EIT to solve the problem:

1. Set the initial values of the specific electrical conductance of the subdomains of the object σ_i^0 and geometric parameters g_i^0
 $\vec{x}^0 = (\sigma_1^0, \sigma_2^0, \dots, \sigma_{m1}^0, g_1^0, g_2^0, \dots, g_{m2}^0)$.

2. Solve the direct problem (3 - 5) by one of the numerical methods (see clause 1.3). The result is a first approximation to the distribution of potential on the electrodes $\varphi_i (i=1, 2, \dots, N)$, where N is a number of measurements.

3. Calculate the square of the difference norm
 $J = \|\vec{\varphi}_i - \vec{\varphi}_i^*\|^2$.

4. Check the condition $J_i \leq \varepsilon$, where ε is determining accuracy of the functional J_i minimum, $\varepsilon = 5 \cdot 10^{-11}$.

5. If the condition (4) is met, then the solution is found and it is \vec{x}^0 . Otherwise, by applying a gradient

method for minimizing the functional J (see clause 1.4), we find the following approximation \vec{x}^1 . The following relation is used

$$\vec{x}^{k+1} = \vec{x}^k - [\alpha] \text{grad} J^k,$$

where $[\alpha]$ is a diagonal matrix of steps.

6. Return to step 2, using the new values of \vec{x} .

Computer simulations of the study subject was used when testing the proposed computer model of EIT.

Set two-dimensional region (Figure-2) and the values of current I (amplitude 1 mA, frequency 50 kHz), electrical conductance of the subdomain 1 ($\sigma_1^* = 200 \text{ sm/m}$) and 2 ($\sigma_2^* = 0.5 \text{ sm/m}$).

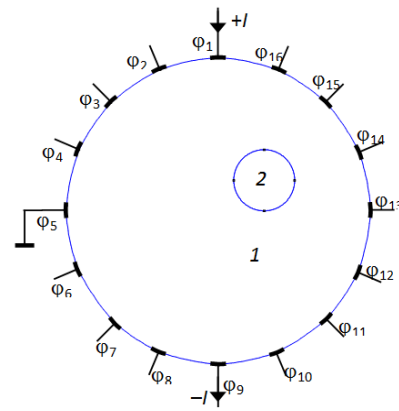


Figure-2. Calculation area (number of electrodes 16).

Solve the direct problem (1.34-1.36) by the finite element method. Figure-3 shows the finite element mesh covering the calculation area and containing 8375 components.

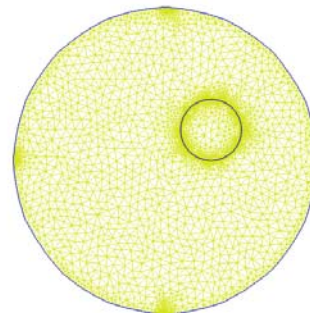


Figure-3. Calculation area with the finite element mesh.

The electric field distribution in the subject is shown in Figure-4.

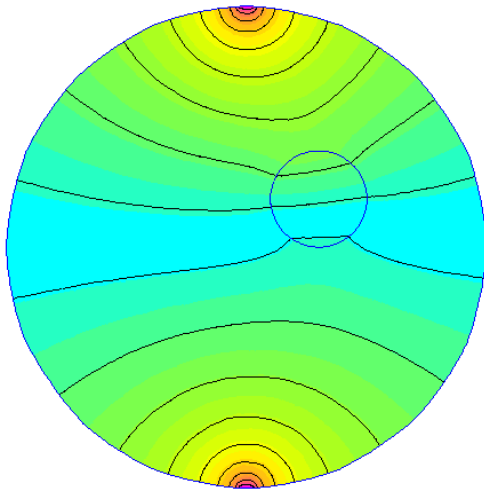


Figure-4. The electric field distribution in the subject.

We now turn to the problem of solving the inverse problem, setting $\sigma_1^0 = 180 \text{ sm/m}$ and $\sigma_2^* = 0.4 \text{ sm/m}$ and using the proposed algorithm. The results of iterative process for solving the inverse problem of EIT are shown in Table-1.

Table-1. The results of iterative process for solving the inverse problem of EIT.

Measured value	1	2	3	4	5	6	7
σ_1	0.045552	0.050615	0.047950	0.046600	0.045576	0.045546	0.045550
σ_2	0.015313	0.017015	0.016119	0.015665	0.015321	0.015311	0.015312
σ_3	0.008399	0.009332	0.008841	0.008592	0.008403	0.008398	0.008398
σ_4	0.004018	0.004465	0.004230	0.004110	0.004020	0.004017	0.004018
σ_5	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
σ_6	-0.003761	-0.004179	-0.003959	-0.003847	-0.003762	-0.003760	-0.003761
σ_7	-0.008733	-0.009704	-0.009193	-0.008934	-0.008738	-0.008732	-0.008733
σ_8	-0.014463	-0.016071	-0.015225	-0.014796	-0.014471	-0.014462	-0.014463
σ_9	-0.044874	-0.049861	-0.047236	-0.045906	-0.044898	-0.044869	-0.044879
σ_{10}	-0.015304	-0.017006	-0.016110	-0.015657	-0.015312	-0.015303	-0.015306
σ_{11}	-0.008953	-0.009949	-0.009425	-0.009159	-0.008958	-0.008952	-0.008954
σ_{12}	-0.004877	-0.005420	-0.005134	-0.004990	-0.004880	-0.004877	-0.004877
σ_{13}	-0.000676	-0.000751	-0.000712	-0.000691	-0.000676	-0.000676	-0.000676
σ_{14}	0.003882	0.004313	0.004086	0.003971	0.003884	0.003881	0.003882
σ_{15}	0.009205	0.010229	0.009690	0.009417	0.009210	0.009205	0.009205
σ_{16}	0.015615	0.017352	0.016438	0.015975	0.015624	0.015614	0.015617
J	6.662 $\cdot 10^{-8}$	5.168 $\cdot 10^{-8}$	2.854 $\cdot 10^{-8}$	1.534 $\cdot 10^{-8}$	7.400 $\cdot 10^{-11}$	6.700 $\cdot 10^{-11}$	1.110 $\cdot 10^{-11}$
σ_1	200.000	180.000	190.000	195.504	199.896	200.026	199.98
σ_2	0.5000	0.4000	0.4500	0.4781	0.4963	0.4850	0.4987

For the case $J = 1.11 \cdot 10^{-11}$ we received $\sigma_1 = 200.010 \text{ sm/m}$, $\sigma_2 = 0.4996 \text{ sm/m}$.

Errors in the reconstruction of the conductance are:

$$\delta(\sigma_1) = \left| \frac{\sigma_1 - \sigma_1^*}{\sigma_1^*} \right| \cdot 100\% = 5 \cdot 10^{-3} \%,$$

$$\delta(\sigma_2) = \left| \frac{\sigma_2 - \sigma_2^*}{\sigma_2^*} \right| \cdot 100\% = 0.08 \, \%$$

Test results show relatively high accuracy and operation speed of the algorithm.

3. CONCLUSIONS

Image reconstruction and biological objects anomaly detection based on the natural and model experiment methodology can be proved possible by the following circumstances:

- an iterative process to determine the desired image converges fair quickly, as experimentally

determined parameters (capacity, conductance), when properly organized experiment, have a high degree of correlation with the true object parameters because they are, in fact, a display (projection) of the desired parameters.

- mathematical modeling drawbacks, including errors, appear significantly less as the calculation results are verified at each step of the iterative process by comparison with experimental results.

ACKNOWLEDGEMENTS

Research is being conducted with the financial support of the state represented by Ministry of Education and Science of the Russian Federation (agreement number #14.574.21.0029). The unique identifier of the project RFMEFI57414X0029.

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