



## LINGO BASED REVENUE MAXIMIZATION USING AGGREGATE PLANNING

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### ABSTRACT

Predictable variability is change in demand that can be forecasted. Products that undergo this type of change in demand create numerous problems in the supply chain, ranging from high levels of stock outs during peak demand periods to high levels of excess inventory during periods of low demand. Faced with predictable variability, a company's goal is to respond in a manner that balances supply with demand to maximize profitability. The goal of sales and operations planning is to appropriately combine two broad options to handle predictable variability. In this article we use aggregate planning to maximize revenue. LINGO is used as the optimization tool.

**Keywords:** predicted variability, aggregate planning, LINGO.

### 1. INTRODUCTION

Predictable variability is change in demand that can be forecasted. Products that undergo this type of change in demand create numerous problems in the supply chain, ranging from high levels of stock outs during peak demand periods to high levels of excess inventory during periods of low demand. These problems increase the costs and decrease the responsiveness of the supply chain. Supply and demand management through sales and operations planning can significantly improve performance when applied to predictably variable products.

Faced with predictable variability, a company's goal is to respond in a manner that balances supply with demand to maximize profitability. The goal of sales and operations planning is to appropriately combine two broad options to handle predictable variability:

1. Manage supply using capacity, inventory, subcontracting and backlogs.
2. Manage demand using short-term price discounts and promotions.

### 2. LITERATURE REVIEW

Kizito Paul Mubiru (1), in this paper, an optimization method for allocating hotel rooms to occupants is proposed. The objective of the model is to optimally admit customers in order to maximize revenue. Adopting a Markov decision process approach, the states of a Markov chain represent possible states of demand for hotel room occupancy. The decision of whether or not to admit additional occupants is made using dynamic programming over a finite period planning horizon. The approach demonstrates the existence of an optimal state dependent admission policy for occupants as well as the corresponding hotel room revenue.

Michael Collins, H.G. Parsa (2), the current study presents that there is a directional relationship between room rates and price-ending strategies. It demonstrates that as average room rates decrease, the price-ending

strategies change from whole dollar practice to dollar and cents practice. Results from the qualitative investigation were compared with the room rates from the Internet for 10 US cities. Based on this study, an innovative pricing strategy is presented with a potential gain of \$251 million dollars by conservative estimations (nearly \$555 million if estimated liberally) annually for the hotel industry in the USA. These potential sales are about 0.54% of revenues and 3.9% of industry wide pre-tax profits. Further studies in consumer acceptance of the recommended pricing strategy are suggested.

Maher Said (3), a seller has an uncertain number of perishable goods to sell in each period. Privately informed buyers arrive stochastically to the market. Buyers are risk neutral, patient, and have persistent private values for consuming a single unit. We show that the seller can implement the efficient allocation using a sequence of ascending auctions. The buyers use memory less strategies to reveal all private information in every period, inducing symmetric behaviour across different cohorts. We extend our results to revenue maximization, showing that a sequence of ascending auctions with asynchronous price clocks is an optimal mechanism.

Richard J. Cebula (4), The objective of this study is identify key factors that determine the ticket-sales revenue of minor league baseball teams and, in doing so, to verify that marketing strategies can lead to greater success for free enterprise in this milieu. To ensure greater comparability of data between teams and hence greater relevance of the results, this study focuses upon a single grouping of minor league teams, the Carolina League, and a single minor league baseball season, 2006. The minor league teams studied are treated as constrained ticket-sales revenue maximisers. Although certain forms of marketing prove ineffective (2-for 1 beer specials, bands or concerts) in promoting ticket-sales revenue, several marketing tools at the disposal of these minor league teams are found to be highly statistically and financially significant in elevating ticket-revenue sales, including fireworks displays, low-value merchandising, high-value merchandising, and food/drink specials, as well as occasional family discounts



and group discounts, suggesting that even in such a constrained environment, the free market works and marketing strategies can promote private enterprise.

J. Andrew Petersen *et al* (5), in this article, we assess the marketing literature with regard to marketing metrics. Subsequently, we develop a framework that identifies key metrics that firms should focus on that can give a firm a better picture of how they got to where they are now and insights towards how they can continue to grow into the future. We then identify several organizational challenges that need to be addressed in order for firms to build the capabilities of collecting the right data, measuring the right metrics, and linking those metrics to customer value and firm performance. Finally, we offer guidelines for future research with regard to marketing metrics to help firms establish successful marketing strategies, measure marketing effectiveness, and justify marketing expenditures to top management.

Changhyun Kwon *et al* (6), This paper recognizes that in many decision environments in which revenue optimization is attempted, an actual demand curve and its parameters are generally unobservable. Herein, we describe the dynamics of demand as a continuous time differential equation based on an evolutionary game theory perspective. We then observe realized sales data to obtain estimates of parameters that govern the evolution of demand; these are refined on a discrete time scale. The resulting model takes the form of a differential variational inequality. We present an algorithm based on a gap function for the differential variational inequality and report its numerical performance for an example revenue optimization problem.

### 3. THE PROBLEM

We illustrate linear programming through the discussion of Red Tomato Tools, a small manufacturer of gardening equipment with manufacturing facilities in Mexico. Red Tomato's products are sold through retailers in the United States. Red Tomato's operations consist of the assembly of purchased parts into a multipurpose gardening tool. Because of the limited equipment and space required for its assembly operations, Red Tomato's capacity is determined mainly by the size of its workforce.

For this example we use a six-month time period because this is long enough time horizon to illustrate many of the main points of aggregate planning.

**Table-1.** Demand forecast at red tomato tools.

S. No.	Month	Demand Forecast
1	January	1600
2	February	3000
3	March	3200
4	April	3800
5	May	2200
6	June	2200

### Red tomato tools

The demand for Red Tomato's gardening tools from consumers is highly seasonal, peaking in the spring as people plant their gardens. This seasonal demand ripples up the supply chain from the retailer to Red Tomato, the manufacturer. Red Tomato has decided to use aggregate planning to overcome the obstacle of seasonal demand and maximize profits. The options Red Tomato has for handling the seasonality are adding workers during the peak season, subcontracting out some of the work, building up inventory during the slow months, or building up a backlog of orders that will be delivered late to the customers. To determine how to best use these options through an aggregate plan, Red Tomato's vice president of supply chain starts with the first task – building a demand forecast. Although Red Tomato could attempt to forecast this demand itself, a much more accurate forecast comes from a collaborative process used by both Red Tomato and its retailers to produce the forecast shown in Table-1.

Red Tomato sells each tool to the retailers for \$40. The company has a starting inventory in January of 1000 tools. At the beginning of January the company has a workforce of 80 employees. The plant has a total of 20 working days in each month and each employee earns \$4 per hour regular time. Each employee works eight hours per day on straight time and the rest on overtime. As discussed previously, the capacity of the production operation is determined primarily by the total labour hours worked. Therefore, machine capacity does not limit the capacity of the production operation. Because of labour rules, no employee works more than 10 hours overtime per month. The various costs are shown in Table-2.

**Table-2.** Various costs.

S. No.	Item	Cost
1	Material cost	\$10/unit
2	Inventory holding cost	\$2/unit/month
3	Marginal cost of stock out/ backlog	\$5/unit/month
4	Hiring and training costs	\$300/worker
5	Layoff cost	\$500/worker
6	Labour hours required	4/unit
7	Regular time cost	\$4/hour
8	Overtime cost	\$6/hour
9	Cost of subcontracting	\$30/unit

Currently, Red Tomato has no limits on subcontracting, inventories and stock outs /backlog. All stock outs are backlogged and supplied from the following month's production. Inventory costs are incurred on the ending inventory in the month. The supply chain manager's goal is to obtain the optimal aggregate plan that allows Red Tomato to end June with at least 500 units (i.e., no stock outs at the end of June and at least 500 units in inventory).



The optimal aggregate plan is one that results in the highest profit over the six month planning horizon. For now, given Red Tomato's desire for a very high level of customer service, assume all demand is to be met, although it can be met late. Therefore the revenues earned over the planning horizon are fixed. As a result, minimizing cost over the planning horizon is the same as maximizing profit. In many instances, a company has the option of not meeting certain demand, or price itself may be a variable that a company has to determine based on the aggregate plan. In such a scenario, minimizing cost is not equivalent to maximizing profits.

Red Tomato estimates that discounting a tool from \$40 to \$39 in any period results in the period demand increasing by 10% because of increased consumption or substitution. Further, 20 percent of each of the two following months' demand is moved forward. Management would like to determine whether it is more effective to offer the discount in January or April. We analyse the two options by considering the impact of a promotion on demand and the resulting optimal aggregate plan. Table-3 shows the various discounting in the months.

**Table-3.** Various discounting.

S. No.	Month	Discount %
1	January	0.1
2	February	0.2
3	March	0.2
4	April	0
5	May	0
6	June	0

The team first considers the impact of offering the discount in January. The new forecast for the fact that consumption will increase by 10 percent in January and 20 percent of the demand from February and March is moved forward to January. Thus with a January promotion, the new demand forecast for January is 3000. The new demand for February is 2400 and the new demand for March is 2560.

#### 4. WHAT IS LINGO?

LINGO is a simple tool for utilizing the power of linear and nonlinear optimization to formulate large problems concisely, solve them, and analyze the solution. Optimization helps you find the answer that yields the best result; attains the highest profit, output, or happiness; or achieves the lowest cost, waste, or discomfort. Often these problems involve making the most efficient use of your resources-including money, time, machinery, staff, inventory, and more. Optimization problems are often classified as linear or nonlinear, depending on whether the relationships in the problem are linear with respect to the variables.

LINGO includes a set of built-in solvers to tackle a wide variety of problems. Unlike many modeling

packages, all of the LINGO solvers are directly linked to the modelling environment. This seamless integration allows LINGO to pass the problem to the appropriate solver directly in memory rather than through more sluggish intermediate files. This direct link also minimizes compatibility problems between the modelling language component and the solver components.

Local search solvers are generally designed to search only until they have identified a local optimum. If the model is non-convex, other local optima may exist that yield significantly better solutions. Rather than stopping after the first local optimum is found, the Global solver will search until the global optimum is confirmed. The Global solver converts the original non-convex, nonlinear problem into several convex, linear sub problems. Then, it uses the branch-and-bound technique to exhaustively search over these sub problems for the global solution. The Nonlinear and Global license options are required to utilize the global optimization capabilities.

#### 5. THE MODELING FRAMEWORK

##### Decision variables

The first step in constructing an aggregate planning model is to identify the set of decision variables whose values are to be determined as part of the aggregate plan.

- $W_t$  = workforce size for month  $t$ ,  $t = 1, \dots, 6$
- $H_t$  = number of employees hired at the beginning of Month  $t$ ,  $t = 1, \dots, 6$
- $L_t$  = number of employees laid off at the beginning of Month  $t$ ,  $t = 1, \dots, 6$
- $P_t$  = number of units produced in Month  $t$ ,  $t, \dots, 6$
- $I_t$  = Inventory at the end of Month  $t$ ,  $t, \dots, 6$
- $S_t$  = number of units stocked out/backlogged at the end of Month  $t$ ,  $t, \dots, 6$
- $C_t$  = number of units subcontracted for Month  $t$ ,  $t, \dots, 6$
- $O_t$  = number of overtime hours worked in Month  $t$ ,  $t, \dots, 6$

##### Objective function

Denote the demand in Period  $t$  by  $D_t$ . The values of  $D_t$  are as specified by the demand forecast in Table 1. The objective function is to minimize the total cost (equivalent to maximizing total profit as all demand is to be satisfied) incurred during the planning horizon. The cost incurred has the following components:

- Regular - time labor cost: Recall that workers are paid a regular - time wage of \$640 (\$4/hour x 8 hours/day x 20 days/month) per month. Because  $W_t$  is the number of workers in period  $t$ , the regular - time labour cost over the planning horizon is given by

$$\sum_{t=1}^6 640W_t$$



2. Overtime labor cost: As the labour cost is \$6 per hour and  $O_t$  represents the number of overtime hours worked in period  $t$ , the overtime cost over the planning horizon is

$$\sum_{t=1}^6 6O_t$$

3. Cost of hiring and layoffs: The cost of hiring a worker is \$300 and the cost of laying off a worker is \$500.  $H_t$  and  $L_t$  Represent the number of hired and the number laid off, respectively in Period  $t$ . Thus, the cost of hiring and layoff is given by

$$\sum_{t=1}^6 300H_t + \sum_{t=1}^6 500L_t$$

4. Cost of inventory and stockout: The cost of carrying inventory is \$2 per unit per month and the cost of stocking out is \$5 per unit per month.  $I_t$  and  $S_t$  represent the units in inventory and the units stocked out respectively in period  $t$ . Thus the cost of holding inventory and stocking out is

$$\sum_{t=1}^6 2I_t + \sum_{t=1}^6 5S_t$$

5. Cost of materials and subcontracting: The material cost is \$10 per unit and the subcontracting cost is \$30/unit.  $P_t$  represents the quantity produced and  $C_t$  represents the quantity subcontracted in period  $t$ . Thus the material and subcontracting cost is

$$\sum_{t=1}^6 10P_t + \sum_{t=1}^6 30C_t$$

6. Total Cost: The total cost incurred during the planning horizon is the sum of all the aforementioned costs and is given by

$$\sum_{t=1}^6 640W_t + \sum_{t=1}^6 6O_t + \sum_{t=1}^6 300H_t + \sum_{t=1}^6 500L_t + \sum_{t=1}^6 2I_t + \sum_{t=1}^6 5S_t + \sum_{t=1}^6 10P_t + \sum_{t=1}^6 30C_t$$

7. Revenue: The amount multiplied by sales  
 $39 \cdot D1 + 40 \cdot (D2 + D3 + D4 + D5 + D6)$
8. Objective:  
 Max: Total profit  
 Total profit = Revenue – Total cost

## 6. LINGO PROGRAM

Model:

Max = Total\_profit;

Total\_profit = Revenue - Total\_Cost;

Revenue =  $39 \cdot DD1 + 40 \cdot (DD2 + DD3 + DD4 + DD5 + DD6)$ ;

$U1 = 0.1$ ;

$U2 = 0.2$ ;

$U3 = 0.2$ ;

$U4 = 0$ ;

$U5 = 0$ ;

$U6 = 0$ ;

$DD1 = D1 + D1 \cdot U1 + D2 \cdot U2 + D3 \cdot U3$ ;

$DD2 = D2 - D2 \cdot U2$ ;

$DD3 = D3 - D3 \cdot U3$ ;

$DD4 = D4 - D4 \cdot U4$ ;

$DD5 = D5 - D5 \cdot U5$ ;

$DD6 = D6 - D6 \cdot U6$ ;

Total\_cost = RTLC + OLC + CHL + CIS + CMS;

$RTLC = 640 \cdot (W1 + W2 + W3 + W4 + W5 + W6)$ ;

$OLC = 6 \cdot (O1 + O2 + O3 + O4 + O5 + O6)$ ;

$CHL = 300 \cdot (H1 + H2 + H3 + H4 + H5 + H6) + 500 \cdot (L1 + L2 + L3 + L4 + L5 + L6)$ ;

$CIS = 2 \cdot (I1 + I2 + I3 + I4 + I5 + I6) + 5 \cdot (S1 + S2 + S3 + S4 + S5 + S6)$ ;

$CMS = 10 \cdot (P1 + P2 + P3 + P4 + P5 + P6) + 30 \cdot (C1 + C2 + C3 + C4 + C5 + C6)$ ;

$W0 = 80$ ;

$W1 = W0 + H1 - L1$ ;

$W2 = W1 + H2 - L2$ ;

$W3 = W2 + H3 - L3$ ;

$W4 = W3 + H4 - L4$ ;

$W5 = W4 + H5 - L5$ ;



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W6 = W5 + H6 - L6;
P1 <= 40 * W1 + O1/4;
P2 <= 40 * W2 + O2/4;
P3 <= 40 * W3 + O3/4;
P4 <= 40 * W4 + O4/4;
P5 <= 40 * W5 + O5/4;
P6 <= 40 * W6 + O6/4;
S0 = 0;
I0 = 1000;
I6 = 500;
S6 = 0;
D1 = 1600;
D2 = 3000;
D3 = 3200;
D4 = 3800;
D5 = 2200;
D6 = 2200;
I0 + P1 + C1 = DD1 + S0 + I1 - S1;
I1 + P2 + C2 = DD2 + S1 + I2 - S2;
I2 + P3 + C3 = DD3 + S2 + I3 - S3;
I3 + P4 + C4 = DD4 + S3 + I4 - S4;
I4 + P5 + C5 = DD5 + S4 + I5 - S5;
I5 + P6 + C6 = DD6 + S5 + I6 - S6;
O1 <= 10 * W1;
O2 <= 10 * W2;
O3 <= 10 * W3;
O4 <= 10 * W4;
O5 <= 10 * W5;
O6 <= 10 * W6;
W1 >= 0;W2 >= 0;W3 >= 0;
W4 >= 0;W5 >= 0;W6 >= 0;
O1 >= 0;O2 >= 0;O3 >= 0;
O4 >= 0;O5 >= 0;O6 >= 0;
H1 >= 0;H2 >= 0;H3 >= 0;
H4 >= 0;H5 >= 0;H6 >= 0;
L1 >= 0;L2 >= 0;L3 >= 0;
L4 >= 0;L5 >= 0;L6 >= 0;
I1 >= 0;I2 >= 0;I3 >= 0;
I4 >= 0;I5 >= 0;I6 >= 0;
S1 >= 0;S2 >= 0;S3 >= 0;
S4 >= 0;S5 >= 0;S6 >= 0;
P1 >= 0;P2 >= 0;P3 >= 0;
P4 >= 0;P5 >= 0;P6 >= 0;
C1 >= 0;C2 >= 0;C3 >= 0;
C4 >= 0;C5 >= 0;C6 >= 0;
@GIN(W1);@GIN(W2);@GIN(W3);
@GIN(W4);@GIN(W5);@GIN(W6);
@GIN(H1);@GIN(H2);@GIN(H3);
@GIN(H4);@GIN(H5);@GIN(H6);
@GIN(L1);@GIN(L2);@GIN(L3);
@GIN(L4);@GIN(L5);@GIN(L6);
@GIN(I1);@GIN(I2);@GIN(I3);
@GIN(I4);@GIN(I5);@GIN(I6);
@GIN(S1);@GIN(S2);@GIN(S3);
@GIN(S4);@GIN(S5);@GIN(S6);
@GIN(P1);@GIN(P2);@GIN(P3);
@GIN(P4);@GIN(P5);@GIN(P6);
@GIN(C1);@GIN(C2);@GIN(C3);
@GIN(C4);@GIN(C5);@GIN(C6);
End

```



## 7. COMPUTATIONAL EFFICIENCY

An intel CORE i5 processor (Second Generation) with 4GB RAM was used to process the model. Branch and Bound solver was used.

### A. Numerical Problem size

Total variables: 53  
Nonlinear variables: 0  
Integer variables: 40  
Total constraints: 80  
Nonlinear constraints: 0  
Total nonzeros: 191

### B. Run Time

The problem was solved in less than 1 second.

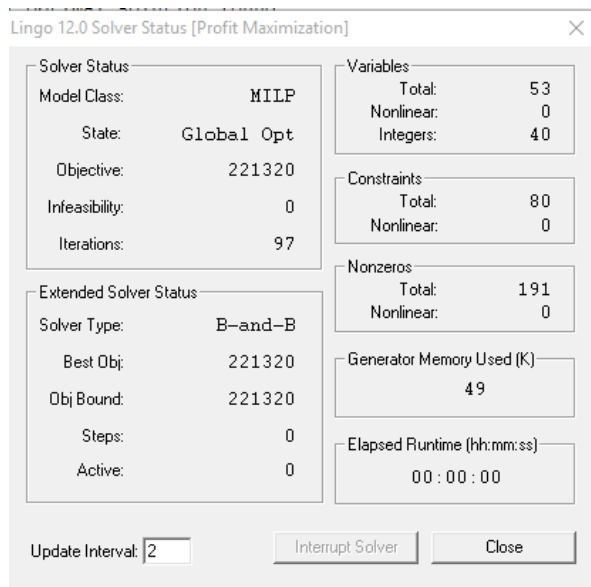


Figure-1. Result.

## 8. RESULT

By optimizing the objective function subjected to listed constraints the aggregate plan is shown in table 4 and 5.

Table-4. Aggregate plan for red tomato part I.

t	H <sub>t</sub>	L <sub>t</sub>	W <sub>t</sub>	O <sub>t</sub>
0	0	0	80	0
1	0	15	65	0
2	0	0	65	0
3	0	0	65	0
4	0	0	65	0
5	0	0	65	0
6	0	0	65	0

Table-5. Aggregate plan for red tomato part II.

t	I <sub>t</sub>	S <sub>t</sub>	C <sub>t</sub>	P <sub>t</sub>
0	1000	0	0	-
1	600	0	0	2600
2	800	0	0	2600
3	840	0	0	2600
4	0	300	60	2600
5	100	0	0	2600
6	500	0	0	2600

## 9. CONCLUSION

Thus the global optimal solution was found. Thus in this paper we have used LINGO program to solve the SCND problem. The answer is the same as obtained using excel solver by Sunil Chopra *et al* (7).

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