VOL. 11, NO. 10, MAY 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

STRUCTURAL TOPOLOGY OPTIMIZATION SUBJECTED TO RELAXED STRESS AND DESIGN VARIABLES

Hailu Shimels Gebremedhen, Dereje Engida Woldemichael and Fakhruldin M. Hashim
Mechanical Engineering Department, Universiti Teknologi Petronas, Bandar Seri Iskandar, Tronoh, Perak, Malaysia
E-Mail: hailu.sg_g02867@utp.edu.my

ABSTRACT

Sustainability, the ability of humans to live within our means, becomes a major concern for engineers and designers now a day's. Engineering optimization, which uses techniques of selecting best elements from set of alternatives to achieve design goals, is one means for sustainability. Structural topology optimization, which is one type of engineering optimization, deals with finding optimal layout of a structure through optimal material distribution with a given design domain. Topology optimization problems have been formulated and solved by means of compliance minimization. There are some efforts for formulating and solving a topology optimization problem with stress constraints. Though formulating an optimization problems with stress constraints seems acceptable and reliable from engineering point of view it has been facing challenges associated with high nonlinear local stress constraints and design variables. In this paper an optimization problem is formulated to minimize volume based on von mises stress theory subjected to stress constraints for two dimensional problems. A mathematical model which takes into consideration the singularity phenomenon associated with the design variables and stress constraints is developed. The results of the model is compared to the results of the compliance based approach by solving two numerical cases. The numerical results shows that the proposed method has comparable efficiency and accuracy by having less transition elements and securing elements in the design domain free from stress failure.

Keywords: topology optimization, stress constraint, stress relaxation, compliance.

INTRODUCTION

Topology optimization is a mathematical approach which seeks optimal material layout within a given design domain for a given set of boundary and loading conditions. It includes determination of connectivity, geometries of cavities and location of voids in the design domain. Unlike the other types of optimization methods used in structural optimization (Grandhi 1986, Birker 1992), in topology optimization problem, the number of holes, shapes, and bodies are not decided prior to the optimization process.

Most of the researches in this area are focused on the formulation and solution of an optimization problem by compliance minimization (Rozvany 2009, Bruggi and Duysinx 2012). Though, the method get popular in structural topology optimization, it has some drawbacks including unable to consider multiple loading cases, variation of results with the amount of material to be distributed and unfeasibility of the final result(París 2010). For designing structures with ductile materials, stress is often used as the solely concerned objective or constraint function in the studies of optimal topology design of continuum structures. A research on formulation and solution of topology optimization to include stress concentration have been going on since the last three decades (Guo, Zhang *et al.* 2014, Cai and Zhang 2015).

Different approaches and algorithms have been suggested for problem formulation and updating the design variables in the optimization process. Optimality criteria method, convex linearization, method of moving asymptotes, successive linearization method are among the algorithms used for updating design variable. Homogenization method (Kikuchi 1991),Evolutionary Structural Optimization (ESO) (Zhou and Rozvany 2001, Xie and Huang 2010, Zuo and Xie 2014) and Bi-directional Evolutionary Optimization (BESO) (Querin, Steven *et al.* 1998, Huang and Huang 2011, Deaton and Grandhi 2014, Zhao 2014), Level- Set Method (Wang, Xiaoming *et al.* 2003, Guo, Zhang *et al.* 2011, James, Lee *et al.* 2012) and Solid Isotropic Material with Penalization (SIMP) (Bendsoe 1989, Zhou and Rozvany 1991, Xie and Steven 1993, Holmberg, Torstenfelt *et al.* 2013) are among the approaches used for problem formulation. Among the approaches for problem formulation the SIMP approach is the common one due to its conceptual simplicity and high computational efficiency (Deaton and Grandhi 2014).

Most of the researches develped cosiders the realxation of the design variables only which will let the opmization to loose a controll over the singularity phenomenon associated with the stress values of the void elements(París 2010, Luo, Wang *et al.* 2013). The main aim of this paper is to formulate a stress based topology optimization subjeted to relaxed stress constraint and design varialble based on SIMP approach for ductile material based designs.

PROBLEM FORMULATION

The optimization problem is stated to minimize the volume of the structure subjected to stress constraints using a power law approach as shown in Equation 1. The problem is formulated using von mises stress failure theory, where failure in a material occurs when the von mises stress induced in the material exceeds yield strength of a material.

$$\min V = \sum_{e}^{N} \rho^{p} v_e$$

$$subejectedto$$

$$\frac{\sigma_{vm}}{\sigma_{yield}} < 1$$

$$KU = F$$

$$0 < \rho_{\min} \le \rho \le 1 \tag{1}$$

V = The volume to be minimized,
p = Penalization factor where the value is taken to be 3,
v_e = Volume of each element in the design domain,
σ_{vm} = Von mises stress,
σ_{yield} = Maximum (yield stress).
K = Global stiffness matrix,
U = Global displacement vector,
F = Global force vector,
ρ = Design variable,
ρ_{min} = The minimum density to control the singularity phenomenon associated with the design variable,

The stress constraint is relaxed by an approach suggested by (Duysinx .P 1998) to avoid the singularity phenomenon associated with the discontinuity of the constraint function due to the removal of elements as shown in Equation. 2

$$\frac{\rho \sigma_{vm}^{e}}{\rho^{p} \sigma_{yield}} = \xi(1-\rho) \tag{2}$$

ξ = Relaxation parameter having a value [0.001 – 0.1]

For relating the macro stress levels to the micro stress levels, a local stress interpolation proposed by (Duysinx and Bendsøe 1998) is used as shown in Equation 3.

$$\sigma(x) = \frac{D_e(x)\overline{\varepsilon(x)}}{\rho(x)^{q}} \tag{3}$$

$\sigma(x)$ = Local stress of a material point
$\varepsilon(x)$ = Macroscopic average strain of a material point
$D_e(x)$ = Macroscopic elastic tensor

Macroscopic elasticity tensor is related to the constitutive elasticity tensor using a power law approach as shown in Equation 4.

$$D_e = \rho^{p} Do \tag{4}$$

The design domain is assumed to be rectangular and discretized by square finite elements. An average strain of an element at the centroid of the element ($\overline{\varepsilon_e}$) can be expressed as

$$\overline{\varepsilon_e} = B_e^{c} u_e \tag{5}$$

B_e^{c} = Strain displacement matrix at the centroid of an element in the design domain
u_e = The elemental displacement vector

Substituting Equation (4) and Equation (5) into Equation (6) the stress state at any arbitrary point in the design domain becomes

$$\sigma(x) = \frac{\rho^{p} D_O B_e^{c} u_e}{\rho(x)^{q}} \tag{6}$$

The exponent $q > 1$ is used for preserving physical consistency in the modeling of a porous SIMP material (Bruggi and Duysinx 2012). The von mises stress of an element can be calculated as

$$\begin{aligned}\sigma_{vm}^{e} &= \sqrt{(\sigma_e)^{T} m(\sigma_e)} \\ &= \sqrt{(\rho_e^{p-q} D_O B_e^{c} u_e)^{T} m(\rho_e^{p-q} (D_O B_e^{c} u_e))}\end{aligned} \tag{7}$$

For the plane stress state, the constant matrix m is given by (Luo, Wang *et al.* 2013).

$$m = \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

From the above derivations and relations the stress based topology optimization problem in Equation 1 can be expressed as shown in Equation 8.

The design variable is relaxed from the lower boundary to avoid the discontinuity of the stress constrains and stiffness matrix

$$\min\ V = \sum_{e=1}^{N} \rho^{p} v_e \tag{8}$$

Subjected to:

$$KU = F$$

$$\rho\left(\frac{\sqrt{(\rho_e^{p-q} D_o B_e^{c} u_e)^{T} m(\rho_e^{p-q} (Do B_e^{c} u_e))}}{\rho^{p} * \sigma_{yield}} - 1\right) \le \xi(1-\rho)$$

$$0 < \rho_{\min} \le \rho_e < 1$$

VOL. 11, NO. 10, MAY 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

METHODOLOGY

A Matlab code was written for testing the developed mathematical model for different cases based on (O.sigmund 2001) with a flow chart as shown in Figure-1. The developed code was tested within a range of the design variable to select the appropriate value of the design variable for initializing the optimization process.

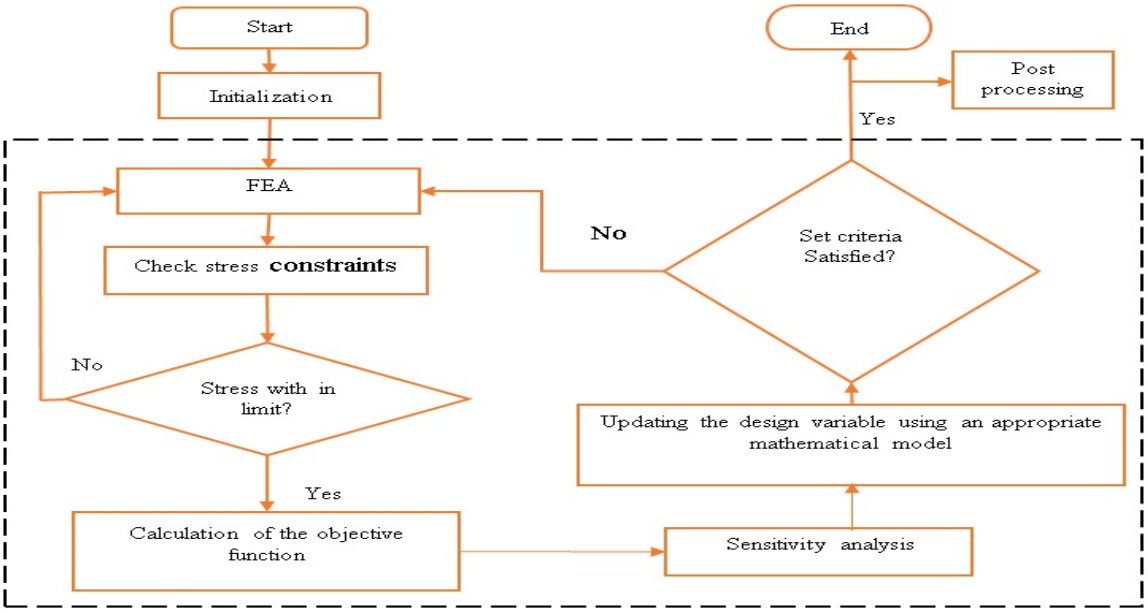

Figure-1. Flow chart for the developed code.

NUMERICAL RESULTS

Maintaining initial requirement, number of transition elements and number of solid elements are used as a selection criteria among the simulation results. From the simulation result, it can be seen those values of the design variables in the range of 0.1- 0.4 failed to maintain the initial requirement and the number of transition materials were more even if the number of solid elements is less as show in Figure-2(a-d) and Figure-3 (a-d). The simulation results within the range of 0.6 - 1 have a good output for maintaining the initial requirement but the number of solid elements was much more as shown Figure-2 (f-j) and Figure-3 (f-j). From the simulation result shown in Figure-2 and Figure-3, a value of design variable is selected for the analysis of the two cases described in Figure-4 and Figure-6 based on number of transition materials, for maintaining initial requirements and less number of solid elements.

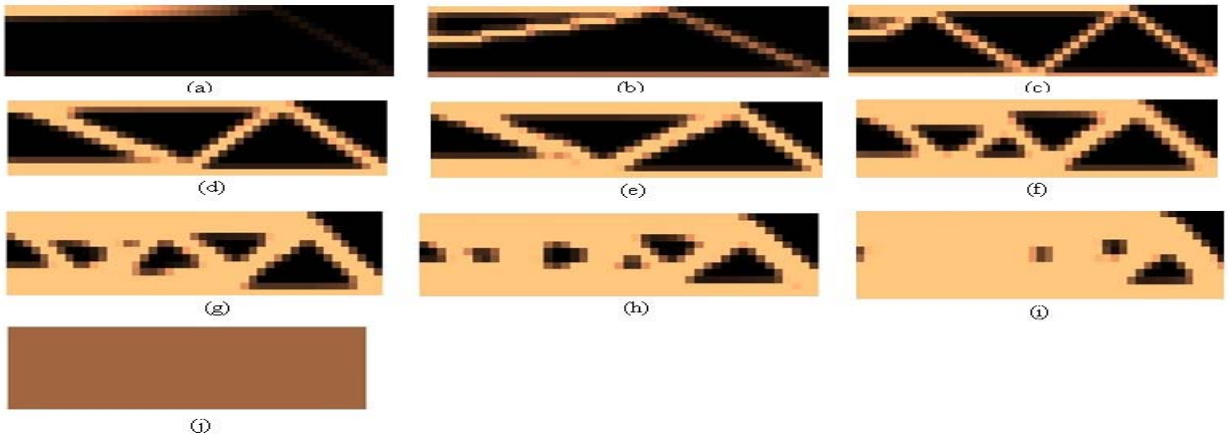

Figure-2. Selection of appropriate initial value for design variable for the MBB (a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4 (e) 0.5 (f) 0.6 (g) 0.7 (h) 0.8 (i) 0.9 (j)1.

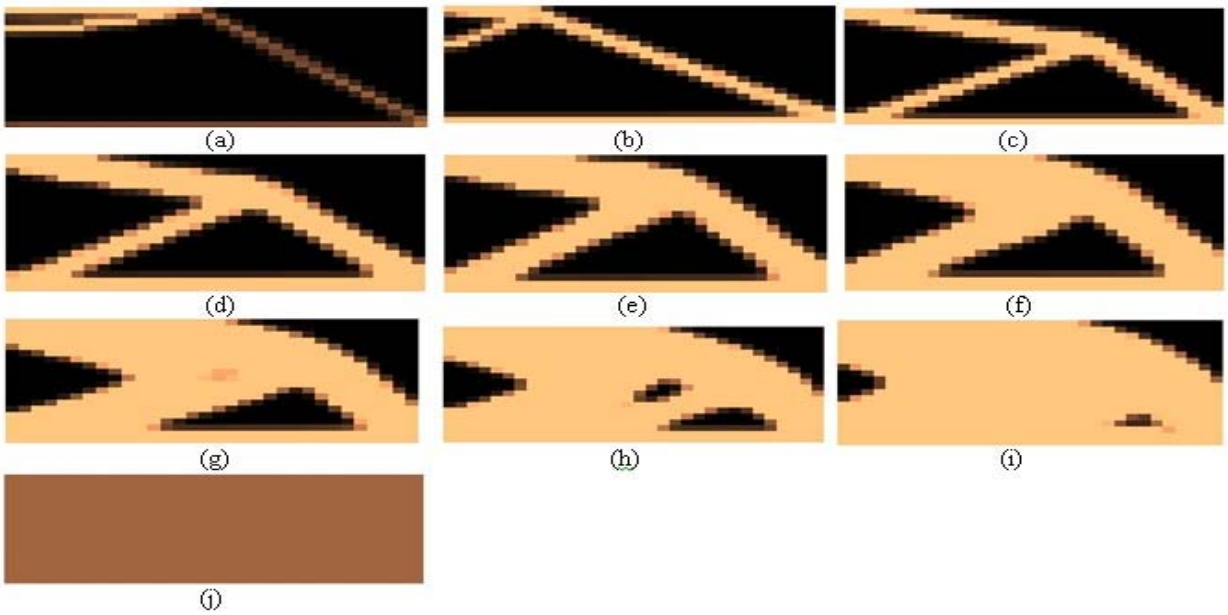

Figure-3. Selection of appropriate initial value for design variable for the cantiliver beam (a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4 (e) 0.5 (f) 0.6 (g) 0.7 (h) 0.8 (i) 0.9 (j) 1.

Case 1

The first case corresponds to an optimization of a support beam from a civil aircraft produced by Messerschmit-Bolkow-Blohm (MBB- type beam), which is a classical problem in topology optimization. Only half of the beam is considered for analysis because of symmetry as shown in Figure-4. The design domain is discretized into 45 x 12 by 4 node square finite elements having unit thickness.

Figure-5 shows the solutions obtained by means of the proposed model and (O.sigmund 2001). As it can be seen from the results executed, the solutions obtained are similar for keeping the general requirements, keeping solid materials at the supports and the point where the external load is applied. The compliance obtained with the developed code was higher than that of the results from (O.sigmund 2001). The stress distribution was much more uniform and all the elements were safe from stress failure as per the failure theory considered for developing the model. The number of transition and solid elements is much less than that of the (O.sigmund 2001) results which will make the outputs from the simulation to be easily manufactured.

Figure-4. MBB beam domain definition and external load.

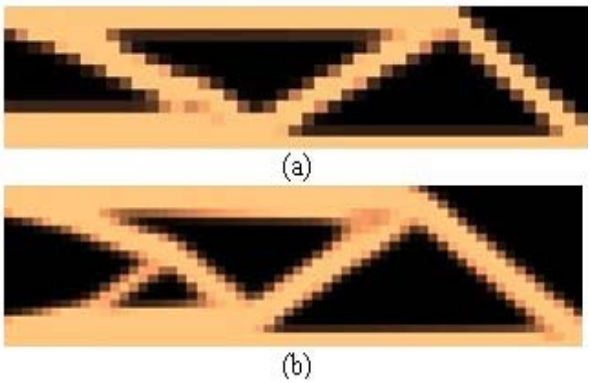

Figure-5. Material distribution of the MBB beam (a) developed model and (b) (O.sigmund 2001).

Case II

The second example corresponds to a cantilever beam subjected to a unit load at the bottom right corner refer with Figure-6. The design domain is discretized in 32 x 20 by 4 node square finite elements having unit thickness. Figure-7 shows the solution obtained by means of the proposed model and (O.sigmund 2001), As it can be seen from the simulation result, the solutions obtained are similar for keeping solid materials at the support and the right bottom corner where the load is applied. Like the simply supported beam the number of transition and solid elements is much less that of (O.sigmund 2001), which will decrease the time for post processing (Sigmund and Petersson 1998).

Figure-6. Cantilever beam domain definition and loads applied.

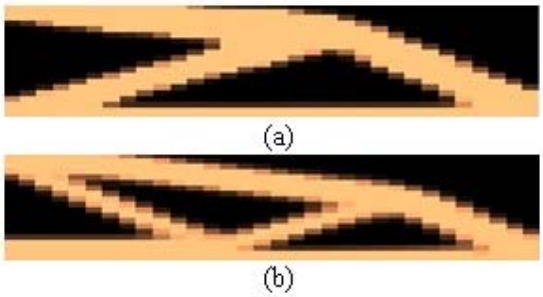

Figure-7. Material distribution of the cantilver beam (a) developed model and (b) (O.sigmund 2001).

From the simulation results of the two cases presented in the paper, the compliance of the stress based topology optimization was higher than that of the compliance based approach. For those designs where stress is a big concern, results from stress based topology optimization approaches can be used directly at the conceptual design stage. In the compliance based approach, there is no means that the designer can check the stress level of elements in the design domain. Therefore, it will be difficult to use those results from compliance based approach for those designs where stress is the major design concern. Though the compliance approach is good for those designs which are based on stiffness, considering the compliance approach for stress based designs will have some doubt unless the designer or end user incorporates some safety factors like the iterative design approach which needs more computational time and cost.

CONCLUSIONS

Topology optimization is a mathematical approach which seeks optimal material layout within a given design domain for a given set of boundary and loading conditions. It has been formulated and solved based for compliance minimization. Some efforts has been done to formulate and solve including stress constraints. Though considering stress in the formulation and solution of optimization problem is more acceptable from engineering point of view it has been facing challenges associated with the stress constraints and design variables.

In this paper, stressed based topology optimization problem has been modeled based on a von Mises stress theory. The developed model considers relaxation of the stress constraints in addition to relaxation of the design variables, which helps to control the singularity phenomenon associated with the stress values associated with the void materials. The developed model is transferred to Matlab code and simulated for different values of design variables to determine the appropriate value of the design variable for initializing the optimization process. The Matlab code developed is tested for simply supported MBB and cantilever beam. The simulation results shows

- The stress based topology optimization will let the designer to have an optimized structure which has elements free from stress failure.
- The proposed method results a final structure with less transition materials than that of the compliance based approach.
- The compliance obtained from stress based approach was higher than that of the compliance based approach due to the consideration of the stress constraints.
- There is no means which can assure all the elements in the design domain are free from stress failure.
- It is difficult to assure that all the elements in the design domain are free from stress failure unless a stress constraint is induced for further analysis for those designs where stress is a big concern.

Further research can be done on testing the developed model for different type of beams as well complicated problems.

ACKNOWLEDGEMENTS

The authors acknowledge Universiti Teknologi Petronas for the financial support to produce this paper.

REFERENCES

[1] Bendsoe, M. P. (1989). "Optimal shape design as a material distribution problem." Structural Optimization 1: 193-202.

[2] Birker, G. I. N. R. M. Z. a. (1992). "Generalized shape optimization without homogenization" struct.Multidisc Optm 4: 250-252.

[3] Bruggi, M. and P. Duysinx (2012). "Topology optimization for minimum weight with compliance and stress constraints." Structural and Multidisciplinary Optimization 46: 369-384.

[4] Cai, S. and W. Zhang (2015). "Stress constrained topology optimization with free-form design domains." Computer Methods in Applied Mechanics and Engineering 289: 267-290.

[5] Deaton, J. and R. Grandhi (2014). "A survey of structural and multidisciplinary continuum topology optimization: post 2000." Structural and Multidisciplinary Optimization 49(1): 1-38.

[6] Duysinx .P, S. O. (1998). "New development in handling stress constraints in optimmam material distribution" 7th AIAA/USAF/NASA/ISSMO

symposium on multidisplanry analysis and optimization. A collection of technical papers St. Louis, Missouri 3: 1501-1509.

[7] Duysinx, P. and M. P. Bendsøe (1998). "Topology optimization of continuum structures with local stress constraints." International Journal for Numerical Methods in Engineering 43(8): 1453-1478.

[8] Grandhi, R. T. H. R. V. (1986). "Structural shape optimization—A survey." computer methods in applied mechanics and engineering 57: 91-106.

[9] Guo, X., W. Zhang and W. Zhong (2014). "Stress-related topology optimization of continuum structures involving multi-phase materials." Computer Methods in Applied Mechanics and Engineering 268: 632-655.

[10] Guo, X., W. S. Zhang, M. Y. Wang and P. Wei (2011). "Stress-related topology optimization via level set approach." Computer Methods in Applied Mechanics and Engineering 200: 3439-3452.

[11] Holmberg, E., B. Torstenfelt and A. Klarbring (2013). "Stress constrained topology optimization." Structural and Multidisciplinary Optimization 48(1): 33-47.

[12] Huang, R. and X. Huang (2011). Matlab implementation of 3D topology optimization using BESO, Melbourne, VIC.

[13] James, K. A., E. Lee and J. R. R. A. Martins (2012). "Stress-based topology optimization using an isoparametric level set method." Finite Elements in Analysis and Design 58: 20-30.

[14] Kikuchi, K. S. a. N. (1991). "Homogenization method for shape and topology optimization." compter methods in Applied Mechanics and Engineering 93: 291-318.

[15] Luo, Y., M. Y. Wang and Z. Kang (2013). "An enhanced aggregation method for topology optimization with local stress constraints." Computer Methods in Applied Mechanics and Engineering 254: 31-41.

[16] O.sigmund (2001). "A 99 line topology optimization code written in Maltlab." Structural and Multidisciplinary Optimization 21: 120-127.

[17] París, J. N., F,Colominas, I,Casteleiro, M. (2010). "Block aggregation of stress constraints in topology optimization of structures." Advances in Engineering Software 41(3): 433-441.

[18] Querin, O. M., G. P. Steven and Y. M. Xie (1998). "Evolutionary structural optimisation (ESO) using a bidirectional algorithm." Engineering Computations 15(8): 1031-1048.

[19] Rozvany, G. I. N. (2009). "A critical review of established methods of structural topology optimization." Structural and Multidisciplinary Optimization 37(3): 217-237.

[20] Sigmund, O. and J. Petersson (1998). "Numerical instability in topology optimization: a survey on procedures dealing with checkerboards, mesh-dependencies and local minima." Structural Optimization 16: 68-75.

[21] Wang, M. Y., W. Xiaoming and D. Guo (2003). "A level set method for structural topology optimization" Compter methods in Applied Mechanics and Engineering 192(1-2): 227-246.

[22] Xie, Y. M. and X. Huang (2010). "Recent developments in evolutionary structural optimization (ESO) for continuum structures." IOP Conference Series: Materials Science and Engineering 10: 012196.

[23] Xie, Y. M. and G. P. Steven (1993). "A simple evolutionary procedure for structural optimization." Computers & Structures 49(5): 885-896.

[24] Zhao, F. (2014). "Topology optimization with meshless density variable approximations and BESO method." Computer-Aided Design 56: 1-10.

[25] Zhou, M. and G. I. N. Rozvany (1991). "The COC algorithm, Part II: Topological, geometrical and generalized shape optimization." Computer Methods in Applied Mechanics and Engineering 89: 309-336.

[26] Zhou, M. and G. I. N. Rozvany (2001). "On the validity of ESO type methods in topology optimization." Structural and Multidisciplinary Optimization 21: 80-83.

[27] Zuo, Z. H. and Y. M. Xie (2014). "Evolutionary topology optimization of continuum structures with a global displacement control." Computer-Aided Design 56: 58-67.