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A SHORTEST PATH LENGTH ON A FUZZY NETWORK WITH TRIANGULAR INTUITIONISTIC FUZZY NUMBER

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ABSTRACT

In this paper, a fuzzy network method is proposed for finding the Shortest Path length (SPL) with Triangular Intuitionistic Fuzzy Numbers (TIFN). Furthermore, to find the smallest path of the edge by the intuitionistic fuzzy distance using graded mean integration. We discussed the SPL from a specified vertex to all other edges in a fuzzy network. An illustrative example is given to express our proposed work.

Keywords: triangular intuitionistic fuzzy number, fuzzy network, shortest path length, bellman's equation.

AMS classifications: 03B52, 03E72, 05C72, 90C70.

1. INTRODUCTION

In this paper, we proposed a new network approach that can obtain the triangular intuitionistic fuzzy number. The SPL requires determining the shortest route for fuzzy network. The SPL is one of the most fundamental and well-known combinatorial optimization problems that appear in many applications as a sub-problem.

In recently, many researchers have developed much concentration to the fuzzy SPL since it is essential to many of applications [1-3, 6-10]. In some applications, the numbers associated with the edges of networks may represent characteristics other than lengths, and we may want the optimum paths, where optimum can be defined by different criteria. In the fuzzy SPL, the fuzzy network shortest length and the corresponding shortest path problems are the useful information for the decision makers. The length of arcs in the network represents travelling time, cost, distance or other variables.

The SPL in a fuzzy network has discussed many previous researchers since it is important to many applications such as communications, routing and transportation [4, 12, 13]. In a fuzzy network problem, the arcs length is assumed to represent transportation cost. In the real life, the transportation cost may be known only approximately due to pensiveness of information.

The fuzzy SPL was first developed by Dubois and Prade [5]. Although the shortest path length can be obtained, a corresponding shortest path cannot be identified [11]. Liu and Kao's algorithm for finding the shortest path can obtain a non-dominated shortest path [14]. In1980 Dubois and Prade [5] first introduce fuzzy SPP. Okada and Soper [11] developed an algorithm based on multiple labeling approaches which is useful to generate number of non-dominated paths. Applying minimum concept they have introduced an order relation between fuzzy numbers. Applying extension principle Klein [4], has given an algorithm which results in a dominated path on a fuzzy network.

In this paper is organized as follows. In section 2, preliminary ideas and definitions are given. The procedure for finding SPL using TIFN derived in section 3 and 5. An example is provided in section 4 to find the shortest path length and discussed the conclusion in section 5.

2. PRELIMINARIES

Definition 2.1

Let $T=\{x,\mu_T(x),\gamma_T(x)/x\in X\}$ be an IFS, then we call $(\mu_T(x),\gamma_T(x))$ an Intuitionistic Fuzzy Number (IFN). We denote it by $(\langle x,y,z\rangle,\langle l,m,n\rangle)$ where $\langle x,y,z\rangle$ and $\langle l,m,n\rangle\in F(I), I=[0,1]$, $0\le z+n\le 1$.

Definition 2.2

An Intutitionistic Fuzzy Set (IFS) T in X is given by $T=\{x,\mu_T(x),\gamma_T(x)/x\in X\}$ where $\mu_T(x):X\to[0,1]$ and $\gamma_T(x):X\to[0,1]$ and for every $x\in X, 0\le\mu_T(x)+\gamma_T(x)\le 1$.

Definition 2.3

A Triangular Intuitionistic Fuzzy Number (TIFN) 'A' is given by $A=(\langle x,y,z\rangle,\langle l,m,n\rangle)$ with $\langle l,m,n\rangle\le\langle x,y,z\rangle^c$ that is either $1\ge y, m\ge z$ (or) $m\le x, n\le y$ are membership and non-membership fuzzy numbers of A.

The additions of two TIFN are as follows:

For two triangular intuitionistic fuzzy numbers

$T_1=(\langle x_1,y_1,z_1\rangle:\mu_{T_1},\langle l_1,m_1,n_1\rangle:\gamma_{T_1})$ and

$T_2=(\langle x_2,y_2,z_2\rangle:\mu_{T_2},\langle l_2,m_2,n_2\rangle:\gamma_{T_2})$

with $\mu_{T_1}\ne\mu_{T_2}$ and $\gamma_{T_1}\ne\gamma_{T_2}$, define

$$T_1+T_2=\begin{pmatrix}\langle x_1+x_2,y_1+y_2,z_1+z_2\rangle:Min(\mu_{T_1},\mu_{T_2}),\\ \langle l_1+l_2,m_1+m_2,n_1+n_2\rangle:Max(\mu_{T_1},\mu_{T_2})\end{pmatrix}.$$

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Definition 2.4

For each $S_d = \left(\langle a,b,c\rangle, \langle e,f,g\rangle \right) \in F$, the signed distance of S_d measured from 0 and is defined by $d\left(S_d,0\right)=\frac{1}{4}\left[(a+e),2(b+f),(c+g)\right]$. from this definition for each $S_d=\left[\left(\langle a_1,a_2,a_3\rangle,\langle a_4,a_5,a_6\rangle\right),\left(\langle b_1,b_2,b_3\rangle,\langle b_4,b_5,b_6\rangle\right),\left(\langle c_1,c_2,c_3\rangle,\langle c_4,c_5,c_6\rangle\right)\right]$, we obtain $s_d=\frac{1}{4}\left[A+2B+C\right]$, where

$A=\left(a_1+a_4,a_2+a_5,a_3+a_6\right)$, $B=\left(b_1+b_4,b_2+b_5,b_3+b_6\right)$ and $C=\left(c_1+c_4,c_2+c_5,c_3+c_6\right)$.

3. SHORTEST PATH LENGTH (SPL) PROCEDURE IN BELLMAN'S EQUATION

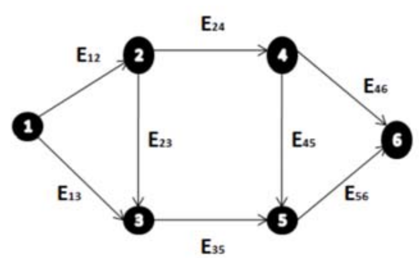
According to Bellman's equation, a Dynamic Programming formulation for the SPL can be given as follows:

Given a network with an acyclic directed graph $G=(V,E)$ with n vertices numbered from 1 to n such that 1 is the source and n is the destination. Then we have,

$$g(n)=0,\ g(i)=\min_{i<j}\left\{ e_{ij}+g(j)/\langle i,j\rangle\in E\right\}\cdots\cdots\cdots(1)$$

Here e_{ij} is the weight of the directed edge $\langle i,j\rangle$ and $g(i)$ is the length of the shortest path from vertex i to vertex n. From the following figure, the solution of Dynamic Programming can be derived as follows:

4. AN EXAMPLE



Consider a network with the triangular intuitionistic fuzzy arc lengths as shown below. The arc lengths are assumed to be

$E_{12}=\left(\langle 17,30,42\rangle,\langle 37,50,56\rangle\right)$;

$E_{13}=\left(\langle 25,35,49\rangle,\langle 37,51,56\rangle\right)$;

$E_{23}=\left(\langle 33,49,56\rangle,\langle 52,61,69\rangle\right)$;

$E_{24}=\left(\langle 33,41,53\rangle,\langle 42,55,64\rangle\right)$;

$E_{35}=\left(\langle 14,21,29\rangle,\langle 24,31,39\rangle\right)$;

$E_{45}=\left(\langle 14,38,52\rangle,\langle 44,52,54\rangle\right)$;

$E_{46}=\left(\langle 20,28,36\rangle,\langle 30,40,49\rangle\right)$;

$E_{56}=\left(\langle 23,35,52\rangle,\langle 43,55,65\rangle\right)$.

The possible paths are as follows:

$W_1: 1-2-4-5-6;\ L_1=\left(\langle 87,144,199\rangle,\langle 166,212,239\rangle\right)$

$W_2: 1-2-4-6;\ L_2=\left(\langle 70,99,131\rangle,\langle 109,145,169\rangle\right)$

$W_3: 1-2-3-5-6;\ L_3=\left(\langle 87,135,179\rangle,\langle 156,197,229\rangle\right)$

$W_4: 1-3-5-6;\ L_4=\left(\langle 62,91,130\rangle,\langle 104,137,160\rangle\right)$.

The solution of Dynamic Programming can be derived as follows:

$$g(6)=0,\ g(5)=E_{56}=\left(\langle 23,35,52\rangle,\langle 43,55,65\rangle\right)$$

$$\begin{aligned} g(4)&=\min_{4<j}\left\{e_{4j}+g(j)/\langle 4,j\rangle\in E\right\}\\ &=\min\left\{E_{46},E_{45}+E_{56}\right\}\\ &=E_{46}=\left(\langle 20,28,36\rangle,\langle 30,40,49\rangle\right)\end{aligned}$$

$$\begin{aligned} g(3)&=\min_{3<j}\left\{e_{3j}+g(j)/\langle 3,j\rangle\in E\right\}\\ &=E_{35}+g(5)\\ &=E_{35}+E_{56}=\left(\langle 37,56,81\rangle,\langle 67,86,104\rangle\right)\end{aligned}$$

Similarly, we obtain,

$$\begin{aligned} g(2)&=\min\left\{E_{24}+g(4),E_{24}+E_{25}+g(5)\right\}\\ &=E_{24}+E_{46}=\left(\langle 53,69,89\rangle,\langle 72,95,113\rangle\right)\end{aligned}$$

and

$$\begin{aligned} g(1)&=\min\left\{E_{12}+g(2),E_{13}+g(3),E_{12}+E_{23}+g(3),E_{12}+E_{24}+g(4),E_{12}+E_{24}+E_{45}+g(5)\right\}\\ &=\left(\langle 62,91,130\rangle,\langle 104,137,160\rangle\right).\end{aligned}$$

5. TO COMPUTE THE SHORTEST PATH LENGTH (SPL)

In this problem we consider is that the edge weight in the network denoted by e_{ij} and the edge weight should be expressed using fuzzy linguistics, and also this used in TIFN.

$$\tilde{e}_{ij}=\left(\alpha,e_{ij},\beta\right)\cdots\cdots(2)$$

where $\alpha=e_{ij}-\delta_{ij}$, $\beta=e_{ij}+\delta_{ij}$ and $0<\delta_{ij}<e_{ij},\delta_{ij}>0$ since δ_{ij} should be determined by the Decision Maker. Now consider the fuzzy case. We look for inequalities that satisfy $e_{ii_1}+e_{i_1i_2}+\cdots+e_{i_{m(i)^n}}\le e_{ij_1}+e_{j_1j_2}+\cdots+e_{j_{m(i)^n}}$, $\forall i<j,\ \langle i,j\rangle\in E$. when $i=1$, $e_{13}+g(3)<e_{12}+g(2)$, that is $e_{13}+e_{35}+e_{56}<e_{12}+e_{23}+g(3)<e_{12}+e_{23}+e_{35}+e_{56}$ (or) $e_{13}+g(3)<e_{12}+e_{24}+e_{45}+e_{56}$.

when $i=2$, $e_{24}+g(4)<e_{23}+g(3)$, that is $e_{24}+e_{46}<e_{23}+e_{35}+e_{56}$.

when $i=4$, $e_{46}+g(6)<e_{45}+g(5)$, that is $e_{46}<e_{45}+e_{56}$.

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Then the parameters of $\delta_{i i_1} + \delta_{i_1 i_2} + \cdots + \delta_{i_m(i)^n} \le \delta_{i j_1} + \delta_{j_1 j_2} + \cdots + \delta_{j_m(i)^n}$, $\forall i < j, \langle i, j \rangle \in E$ based on the above inequalities are derived as

$$\delta_{13} + \delta_{35} + \delta_{56} < \delta_{12} + \delta_{24} + \delta_{46} < \delta_{12} + \delta_{24} + \delta_{45} + \delta_{56} \cdots\cdots (3)$$
$$\delta_{24} + \delta_{46} < \delta_{23} + \delta_{35} + \delta_{56}$$
$$\delta_{46} < \delta_{45} + \delta_{56}$$

If the Decision Maker chooses the values of parameters:
$\delta_{12} = (\langle 3,4,6\rangle, \langle 5,7,8\rangle)$; $\delta_{13} = (\langle 5,6,7\rangle, \langle 8,9,10\rangle)$;
$\delta_{23} = (\langle 4,8,10\rangle, \langle 9,11,12\rangle)$; $\delta_{24} = (\langle 3,6,7\rangle, \langle 7,9,10\rangle)$;
$\delta_{35} = (\langle 2,4,6\rangle, \langle 5,7,8\rangle)$; $\delta_{45} = (\langle 4,6,7\rangle, \langle 7,9,11\rangle)$;
$\delta_{46} = (\langle 6,8,9\rangle, \langle 10,11,13\rangle)$; $\delta_{56} = (\langle 4,6,7\rangle, \langle 9,11,12\rangle)$ to satisfy the conditions in (3), then the fuzzy numbers can be determined as

$\tilde{e}_{ij}$	$\alpha = e_{ij} - \delta_{ij}$	e_{ij}	$\beta = e_{ij} + \delta_{ij}$
$\tilde{e}_{12}$	$(\langle 14,26,36\rangle, \langle 32,43,48\rangle)$	$(\langle 17,30,42\rangle, \langle 37,50,56\rangle)$	$(\langle 20,34,48\rangle, \langle 42,57,64\rangle)$
$\tilde{e}_{13}$	$(\langle 20,29,42\rangle, \langle 29,42,46\rangle)$	$(\langle 25,35,49\rangle, \langle 37,51,56\rangle)$	$(\langle 30,41,56\rangle, \langle 45,60,66\rangle)$
$\tilde{e}_{23}$	$(\langle 29,41,46\rangle, \langle 43,50,57\rangle)$	$(\langle 33,49,56\rangle, \langle 52,61,69\rangle)$	$(\langle 37,57,66\rangle, \langle 61,72,81\rangle)$
$\tilde{e}_{24}$	$(\langle 30,35,46\rangle, \langle 35,46,54\rangle)$	$(\langle 33,41,53\rangle, \langle 42,55,64\rangle)$	$(\langle 36,47,60\rangle, \langle 49,64,74\rangle)$
$\tilde{e}_{35}$	$(\langle 12,17,23\rangle, \langle 19,24,31\rangle)$	$(\langle 14,21,29\rangle, \langle 24,31,39\rangle)$	$(\langle 16,25,35\rangle, \langle 29,38,47\rangle)$
$\tilde{e}_{45}$	$(\langle 10,32,45\rangle, \langle 37,43,43\rangle)$	$(\langle 14,38,52\rangle, \langle 44,52,54\rangle)$	$(\langle 18,44,59\rangle, \langle 51,61,65\rangle)$
$\tilde{e}_{46}$	$(\langle 14,20,27\rangle, \langle 20,29,36\rangle)$	$(\langle 20,28,36\rangle, \langle 30,40,49\rangle)$	$(\langle 26,36,45\rangle, \langle 40,51,62\rangle)$
$\tilde{e}_{56}$	$(\langle 19,29,45\rangle, \langle 34,44,53\rangle)$	$(\langle 23,35,52\rangle, \langle 43,55,65\rangle)$	$(\langle 27,41,59\rangle, \langle 52,66,77\rangle)$

From (2.4), we obtain the following estimate of the edge weights in the fuzzy sense:

$E_{12}^0 = \langle 54,80,98\rangle$	$E_{13}^0 = \langle 62,86,105\rangle$	$E_{23}^0 = \langle 85,110,125\rangle$
$E_{24}^0 = \langle 75,96,117\rangle$	$E_{35}^0 = \langle 38,52,68\rangle$	$E_{45}^0 = \langle 58,90,106\rangle$
$E_{46}^0 = \langle 50,68,85\rangle$	$E_{56}^0 = \langle 66,90,117\rangle$	

The fuzzy network $G = (V, E)$ with $\{e_{ij}^0 / \langle i, j\rangle \in E\}$.
where
$g^0(1) = E_{13}^0 + g(3) = E_{13}^0 + E_{35}^0 + g(5) = E_{13}^0 + E_{35}^0 + E_{56}^0$.

The fuzzy shortest path is $1-3-5-6$ with length $\langle 166, 228, 290\rangle$.

6. CONCLUSIONS

In this paper, a shortest path length is obtained using a procedure based on Bellman's equations in a fuzzy network. The SPL arc lengths are considered as uncertain and are characterized by triangular intuitionistic fuzzy numbers. The fuzzy shortest length is the useful information for the decision makers in an intuitionistic fuzzy SPL. This algorithm can be computed using the TIFN by the decision maker. It provides the good output for different types of fuzzy network. The effectiveness of the method is tested by an example.

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