MCDM-AHP METHOD IN DECISION MAKINGS

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ABSTRACT
Analytic Hierarchy Process (AHP) is among the most popular methods of decision-makings. This article contains the tutorial document composed of a short discussion on AHP presented at the International conference on “Proceedings of Engineering Technology International Conference”. The conference would be held at Bali, Indonesia during August 10-11, 2015. The paper starts with the short review of Multi-criteria Decision Making and AHP. Then it discusses about the techniques and formulae that are used in the AHP decision-making method. Lastly, this paper recommends AHP to future researchers and professionals with highlights of the reasons to use the methodology process when engaged in complex decision-making problems.

Keywords: multi-criteria decision-making (MCDM), multi-alternative decision making (MADM), analytic hierarchy process (AHP), and decision techniques.

1. INTRODUCTION
Multi-Criteria Decision Making (MCDM) models are suitable for evaluating and making decision for the best alternatives options in order to choose the perfect criteria [1]. This involves a general class of operations research models, which considers problems in decision making in the presence of many decision criteria. There are two types of the MCDM, which are: Multi-Objective Decision Methods (MODM) and Multi-Attribute Decision Methods (MADM) [1]. The Multi-Objective Decision Methods (MODM) is an approach, which uses mathematical optimization technique and mostly involves analysis procedure related to calculation design process. MODM usually involves maximization of mathematical problems involving function that is more objective which need to be simultaneously optimized. Especially MODM was created to solve problems in mathematical programming and design via the best alternatives. However, Multi-Attribute Decision Methods (MADM) is an approach developed for selecting the best criteria or alternative(s). This is used in decision-making problems involving a number of decision-making alternatives. This model is based on the list of criteria chosen, its parameters, variables that one wishes to monitor in decision-making process [2]. The category of MCDM has been used for selecting a minimum number of alternatives. According to Ermatita, et al., [3] two levels are relevant with MADM and these are: (a) Aggregation implementation: The decision that reflects the result equivalent for all areas on each alternative is developed. (b) Alternatives implementation: The alternatives ranking for the aggregation of the result makers. There are many other methods which are used under MADM such as; Analytical Hierarchy Process (AHP); Simple Additive Weighting (SAW); Ordered Weighted Averages (OWA); Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS); Elimination et Choice Translating Reality (ELECTRE); Decision trial and Evaluation Laboratory; Decision Trial and Evaluation Laboratory (DEMATEL); The Simple Multi Attribute Rating Technique (SMART); and others, for example: [4, 5, 6]. Notwithstanding all these methods AHP method is regarded as the most famous MCDM tool for decision making problems based on literature studies.

2. AHP METHOD
According to Triantaphyllou and Mann [7], the nice mathematical properties of AHP have attracted many researchers’ interest and AHP input data are easy to obtain. Analytic Hierarchy Process (AHP) that is created under Multi- Criteria Decision Making (MCDM) is composed of techniques are suitable for ranking of critical management problems [8]. The Analytic Hierarchy Process (AHP) was introduced by Saaty [9] since 1970’s. The AHP method is ranking process that is used in making group decision and is widely used around the world in a variety of fields such as business, government, industry, education, health, and others. The method also allows for consistency test in judgment making room to check and reduce inconsistencies in opinions or judgments. The scales of ratio and consistency index are derived from the principal Eigen vectors and Eigen value respectively. The method focuses on prioritizing the selection criteria, and distinguishing the more important criteria from the less important ones. Although some researchers argue some disadvantage of AHP [6, 10], AHP is simple method with focus placed on peer to peer comparisons that are suitable to evaluate both qualitative and quantitative design [11]. More also, AHP method uses judgment to analyze the data.

3. AHP CONCEPTUAL
Although AHP is a very popular decision making method, authors only find few articles to algorithmically review AHP applications (i.e. [12]). This article tries to fill the need of application tutorials. This paper discusses four main steps of AHP that could be used in decision-making problems and these include; problem modeling, weight valuation, weight aggregation, and sensitivity analysis. The steps start with hierarchy construction where objective
are highlighted and criteria and alternatives identified. Then comparisons of the criteria and alternatives and their relative rankings are calculated [11]. Firstly, the criteria in the hierarchy construction must be determined and this can be done through different stages from the top level, middle level, and to the bottom level. Top level: This set the goal or the objective and is on upper stage in the hierarchy; Middle level: made up of both criteria and sub-criteria in the second stage; and Bottom level: made up of down stage including alternatives. In addition, the weighting of criteria, sub-criteria and alternatives must be evaluated. This process or function is vital to group decision making as its presents important top elements for target achievement. AHP is simple peer-to-peer differences and to determine the best criteria and/or alternatives there should be focus on weight of the factors. More also, comparison matrix will have to be expanded by calculating the weights of the criteria and local weight of the alternatives to obtain the matrix weight. Table-1, shows the important evaluation scales of pair-wise comparison. It is related to AHP process for making a decision such as ranking size to evaluate the scales. In this table, we have five related important values that will be transforming to matrix. In this regard, this article recommends Expert Choice Software for the easier AHP weighting and all of the criteria and sub-criteria that are created should be analysis and evaluate using computer software.

**Table-1.** Evaluation scales of pair-wise comparison [8].

<table>
<thead>
<tr>
<th>Important Value</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal direct influence</td>
<td>Both of elements are same with objective</td>
</tr>
<tr>
<td>3</td>
<td>Weak direct influence</td>
<td>Experience and judgment minimum evaluate process between another factor</td>
</tr>
<tr>
<td>5</td>
<td>Moderate direct influence</td>
<td>Experience and judgment medium evaluate process between another factor</td>
</tr>
<tr>
<td>7</td>
<td>Strong direct influence</td>
<td>Very useful process between another factor</td>
</tr>
<tr>
<td>9</td>
<td>Very Strong direct influence</td>
<td>Extreme processes factor compare with all of the factors</td>
</tr>
</tbody>
</table>

AHP also need to listed some criteria and alternative conceptual. It is because to easier AHP ranking and evaluate [13]. There are three steps to illustrate AHP data hierarchy [14]: goal/objective, alternative, and criteria/sub-criteria will be seeing Table-2 below.

**Table-2.** AHP reference conceptual of criteria for selecting alternative.

<table>
<thead>
<tr>
<th>Step</th>
<th>Dimension of Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal</td>
<td>The vision/mission to do and develop. Example: To buy a perfect car.</td>
</tr>
<tr>
<td>Alternative</td>
<td>The types of vision/mission Example: Proton, Honda, Toyota, Perodua, Ford, Nissan, and Mercedes</td>
</tr>
<tr>
<td>Criteria</td>
<td>The explanations of reason to choose that alternative. Example: Price, Type, Size, Color, Design, Insurance, and Maintenance,</td>
</tr>
</tbody>
</table>

4. **AHP PROCESS**

Although, the technique of AHP processes have been explained. There are forth mathematical calculations in Analytic Hierarchy Process that needs to be followed as stated by Richard et al. [12] and Alam, et al. [14]: Firstly, the process of hierarchy ranking were be constructed starting from goal or objective to achieve and identify and/or determine both of criteria and alternatives. Secondly, the pair-wise comparison matrices of alternatives and criteria must be constructed as illustrated in the formulae below.

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & ... & a_{1n} \\
    a_{21} & a_{22} & ... & a_{2n} \\
    ... & ... & ... & ... \\
    a_{n1} & a_{n2} & ... & a_{nn}
\end{bmatrix}
\]

\[ [aij], \text{where, } i, j = 1, 2, ..., n, \]

\[ A_{ij} = 1 \text{ for } i = j, \]

\[ A_{ij} = \frac{1}{A_{ji}} \text{ for } i \neq j \]

The above formula deals with the matrix of dimension (n x n), where n = comparison number of factors. The elements value that are related to the diagonal of the matrix is equal to 1 such as Aij = 1. Based on AHP, it is possible for preference that is reciprocal and this is expressed by Aij=1/Aji for i ≠ j. For example if factor of i-th is, x times more important than the j-th factors (e.g: Aij
\[ x \), meanwhile the automatically assume that j-th factors is \( 1/x \) as important between i-th element \( (a_{ij} = 1/x) \) and/or \( A_{ij} = 1/a_{ij} \).

Thirdly, the calculations of criteria weight and alternative local weight that are selected from existing matrices are calculated through the use normalization procedure. Equations of criteria weight and alternatives local weight are explained as below.

Calculation of total data of each row.

\[
W_i = \sum_{j=1}^{n} a_{ij}, i = 1, 2, \ldots, n
\]

Normalization of local weight.

\[
W_i = \frac{\sum_{j=1}^{n} a_{ij}}{\sum_{j=1}^{n} a_{kj}}, i = 1, 2, \ldots, n
\]

Eigen Vector.

\[
w_i = \frac{1}{n} + (A1 + A2 + A3 + \cdots + An)
\]

Weight Vector.

\[
W = [W^1, W^2, \ldots, W^n]^T
\]

Lastly, obtained the alternatives global weight that are synthesize with the local weight

\[
B \times V = \begin{bmatrix}
    b^{11} & b^{12} & \cdots & b^{1n} \\
    b^{21} & b^{22} & \cdots & b^{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    b^{n1} & b^{n2} & \cdots & b^{nn}
\end{bmatrix} \times
\begin{bmatrix}
    v^1 \\
    v^2 \\
    \vdots \\
    v^n
\end{bmatrix}
\]

From above equation, \( B \) explains alternatives of local weight (row) and the criteria of local weight (column). Also, \( V \) explained transpose of criteria local weight and the Global weight is derived by multiplying the matrices of \( B \) and \( V \). For easier AHP weighting, all of alternatives and criteria that are created should be analyzed and evaluated using computer software. More also, consistency is an important aspect of AHP and must be checked. The AHP consistency test has been represented in the formulae below. However, the consistency of data can also be tested using Expert Choice Software.

Maximal Eigenvalue.

\[
\lambda_{max} = \frac{1}{n} = \sum_{i=1}^{n} \frac{(Aw)i}{wi}
\]

Consistency Ratio.

\[
CR = \frac{Cl}{RandomAverageCI} = \frac{\lambda_{max} - n}{r(n-1)} \times 100%
\]

Where [15];

\[ CR \leq 10\% = \text{Inconsistent is acceptable} \]

\[ CR > 10\% = \text{Revise process of subjective judgment.} \]

The random consistency index is an important value to calculate the CR. Random Index (RI) is value to calculate the CR. Table-3 show the value index of RI from \( n = 1 \) until \( n = 15 \) with RI value such as table shown.

<table>
<thead>
<tr>
<th>n</th>
<th>Random Index (RI)</th>
<th>Cut-off CI (10%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.58</td>
<td>0.058</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
<td>0.090</td>
</tr>
<tr>
<td>5</td>
<td>1.21</td>
<td>0.121</td>
</tr>
<tr>
<td>6</td>
<td>1.24</td>
<td>0.124</td>
</tr>
<tr>
<td>7</td>
<td>1.32</td>
<td>0.132</td>
</tr>
<tr>
<td>8</td>
<td>1.41</td>
<td>0.141</td>
</tr>
<tr>
<td>9</td>
<td>1.45</td>
<td>0.145</td>
</tr>
<tr>
<td>10</td>
<td>1.49</td>
<td>0.149</td>
</tr>
<tr>
<td>11</td>
<td>1.51</td>
<td>0.151</td>
</tr>
<tr>
<td>12</td>
<td>1.48</td>
<td>0.148</td>
</tr>
<tr>
<td>13</td>
<td>1.56</td>
<td>0.156</td>
</tr>
<tr>
<td>14</td>
<td>1.57</td>
<td>0.157</td>
</tr>
<tr>
<td>15</td>
<td>1.59</td>
<td>0.159</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

In this paper, the authors presented a short review of Multi-criteria Decision Making (MCDM), Multi-Attribute Decision Methods (MADM), and Analytic Hierarchy Process (AHP). They then discussed about the techniques and formulae that are used in the AHP decision-making method. It was observed that, Analytic Hierarchy Process (AHP) method is suitable for ranking and analyzing complex decision-making problems. AHP is also regarded as one of perfect and easiest method under MCDM because it is easy to use and makes room for checking and reducing inconsistencies in opinion(s). There is therefore sufficient evidence based on the formulae, along with comprehensive literature review by the authors to strongly recommend AHP to future researchers and
professionals to use the methodology process when engaged in complex decision-making problems involving many criteria, sub-criteria, and alternatives. Also, this paper recommends Expert Choice Software and other software that may be useful and will make weighting, all of alternatives and criteria easier.

REFERENCES


