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EFFECTS OF VISCOUS DISSIPATION ON FREE CONVECTION BOUNDARY LAYER FLOW TOWARDS A HORIZONTAL CIRCULAR CYLINDER

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ABSTRACT

In this study, the numerical investigation of the viscous dissipation on free convective boundary layer flow towards a horizontal circular cylinder with constant wall temperature is considered. The transformed partial differential equations are solved numerically by using an implicit finite-difference scheme known as the Keller-box method. Numerical solutions are obtained for the reduced Nusselt number and the reduced skin friction coefficient as well as the velocity and temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number and Eckert number are analyzed and discussed. The results in this paper is original and important for the researchers working in the area of boundary layer flow and this can be used as reference and also as complement comparison purpose in future.

Keywords: constant wall temperature, free convection, horizontal circular cylinder, viscous dissipation.

1. INTRODUCTION

Convection boundary layer flow is an important topic to be considered in industrial and engineering activities nowadays. These configurations are applied especially in thermal effects managements, which involves fluid as cooling medium in many industrial outputs for example in electronic devices, computer power supply and also in engine cooling system such as heatsink in car radiator. Air, electrolyte, water, polymer, and nanofluid are the example of the fluid that may be involve in convection process. Because of the large contributions and issues, this topic has attracted many researchers to study and expand the knowledge so that it could be applied in order to handle the thermal problems produced by these industrial outputs [1, 2]. Free convection refers to the convection process occur naturally. Physically, free convection occurs when the fluid motion is generated by gravitational field and changes in the fluid density. It is contrary different with forced convection where the forced convection generated mechanically by the external agents like blower, fan or nozzle.

In considering the convection on a horizontal circular cylinder, Blasius [3] is the first one who solved the momentum equation of forced convection boundary layer flow. [4] then solved the energy equation for this problem by considering the constant wall temperature (CWT). Since then, this topic has attracted many researchers to study the constant wall temperature and constant heat flux. Merkin [5] considered the free convection boundary layer on an isothermal horizontal cylinder and became the first one who got the exact solution for this problem. Merkin and Pop [6] studied free convection boundary layer on a horizontal circular cylinder with constant heat flux. Next, Nazar et al. [7] extended [5] and [6] in micropolar fluid while Molla et al. [8] investigated the heat generation effects on free convection flow on an isothermal horizontal circular cylinder. Recently, Salleh and Nazar [9] and Sarif et al. [10] updated [7] with Newtonian heating and convective boundary conditions, respectively. Both problems are solved numerically by using the Keller-box method.

From literature study, it is found that Gebhart [11] is the first person who studied viscous dissipation in free convection flow. The viscous dissipation effects on unsteady free convective flow over a vertical porous plate was then investigated by Soundalgekar [12]. Vajravelu and Hadjinicolaou [13] then studied the viscous dissipation effects on the flow and heat transfer over a stretching sheet. Chen [14] and Partha et al. [15] observed the mixed and MHD free convection heat transfer from a vertical surface and exponentially stretching surface with Ohmic heating and viscous dissipation, respectively. Recently, Yirga and Shankar [16] considered this topic with thermal radiation and magnetohydrodynamic effects on the stagnation point flow towards a stretching sheet. It is worth to mention that the viscous dissipations effect is important to study in order to understand the behavior of temperature distributions when the internal friction is not negligible.

Therefore, the purpose of the present study is to investigate the effects of viscous dissipation on free convection boundary layer flow towards a horizontal circular cylinder. The governing partial differential equations are solved numerically and the variation of pertinent physical parameters are analyzed and discussed in detail with the aid of tables and figures.

2. MATHEMATICAL FORMULATION

The horizontal circular cylinder of radius, which is heated to a constant temperature, embedded in a

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viscous fluid with ambient temperature as shown in Figure-1. The orthogonal coordinates of and are measured along the cylinder surface, starting with the lower stagnation point and normal to it, respectively. Under the assumptions that the boundary layer approximations is valid, the dimensional governing equations of steady free convection boundary layer flow are [17, 9]:

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+g\beta(T-T_\infty)\sin\frac{\bar{x}}{a}, \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2, \tag{3}$$

subject to the boundary conditions

$$\begin{aligned}&\bar{u}(\bar{x},0)=\bar{v}(\bar{x},0)=0,\ T(\bar{x},0)=T_w,\\&\bar{u}(\bar{x},\infty)\to 0,\ T(\bar{x},\infty)\to T_\infty\end{aligned} \tag{4}$$

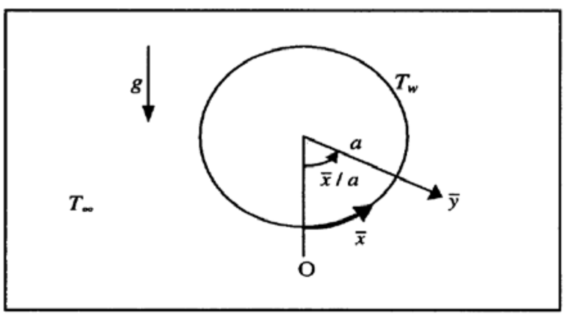

Figure-1. Physical model of the coordinate system.

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively μ is the dynamic viscosity, ν is the kinematic viscosity, g is the gravity acceleration, α is the thermal diffusivity, β is the thermal expansion, T it local temperature, ρ is the fluid density and C_p is the specific heat capacity at a constant pressure. Next, it is introduced the governing non-dimensional variables:

$$\begin{aligned}&x=\frac{\bar{x}}{a},\quad y=Gr^{1/4}\frac{\bar{y}}{a},\quad u=\frac{a}{\nu}Gr^{-1/2}\bar{u},\\&v=\frac{a}{\nu}Gr^{-1/4}\bar{v},\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}.\end{aligned} \tag{5}$$

Using (5), (1)-(3) becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\theta\sin x, \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2, \tag{8}$$

subject to the boundary conditions

$$\begin{aligned}&u(x,0)=0,\ v(x,0)=0,\ \theta(x,0)=1,\\&u(x,\infty)\to 0,\ \theta(x,\infty)\to 0\end{aligned} \tag{9}$$

where $\Pr=\dfrac{\mu}{\alpha\rho}$ is the Prandtl number,

$Ec=\dfrac{\nu^2 Gr}{a^2 C_p\left(T_w-T_\infty\right)}$ is an Eckert number and

$Gr=\dfrac{g\beta(T_w-T_\infty)a^3}{\nu^2}$ is the Grashof number. In order to solve (6)-(8), the following functions is introduce:

$$\psi=xf(x,y),\qquad \theta=\theta(x,y), \tag{10}$$

where ψ is the stream function defined as $u=\dfrac{\partial\psi}{\partial y}$ and $v=-\dfrac{\partial\psi}{\partial x}$ which identically satisfies (6) and θ is the rescaled dimensionless temperature of the fluid. Substitute (10) into (6)-(8), then the following partial differential equations is obtained:

$$\begin{aligned}&\frac{\partial^3 f}{\partial y^3}+f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\sin x}{x}\theta\\&=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right),\end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+f\frac{\partial \theta}{\partial y}\\&=x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}-xEc\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right),\end{aligned} \tag{12}$$

with boundary conditions

$$\begin{aligned}&f(x,0)=0,\quad \frac{\partial f}{\partial y}(x,0)=0,\quad \theta(x,0)=1,\\&\frac{\partial f}{\partial y}(x,\infty)\to 0,\quad \theta(x,\infty)\to 0\end{aligned} \tag{13}$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x are [8]:

$$C_f=\frac{\tau_w}{\rho u_\infty^2},\qquad Nu_x=\frac{aq_w}{k(T_w-T_\infty)}. \tag{14}$$

The surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \qquad q_w = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}, \tag{15}$$

with $\mu = \rho\nu$ and k being the dynamic viscosity and the thermal conductivity, respectively. Using (10) and (15), the reduced skin friction $C_f Gr^{1/4}$ and reduced Nusselt number $Nu_x Gr^{-1/4}$ stated as:

$$C_f Gr^{1/4} = x\frac{\partial^2 f}{\partial y^2}(x,0) \text{ and } Nu_x Gr^{-1/4} = -\frac{\partial \theta}{\partial y}(x,0). \tag{16}$$

Furthermore, the velocity profiles and temperature distributions can be obtained from the following relations:

$$u = f'(x,y), \qquad \theta = \theta(x,y). \tag{17}$$

3. RESULTS AND DISCUSSION

Eqns. (11) and (12) subject to the boundary conditions (13) were solved numerically using the Keller-box method with two parameters considered, namely the Prandtl number Pr and the Eckert number Ec. The step size $\Delta y = 0.02$, $\Delta x = 0.005$ and boundary layer thickness $y_\infty = 8$ and $x_\infty = \pi$ are used in obtaining the numerical results. Tables-1 and 2 show the comparison values of $Nu_x Gr^{-1/4}$ and $C_f Gr^{1/4}$ with previous results for various values of x, respectively. It has been found that they are in good agreement. We can conclude that this method works efficiently for the present problem, and we are also confident that the results presented here are accurate.

Table-3 presents the values of $Nu_x Gr^{-1/4}$ with various values of x and Ec. From table, it is suggest that as Ec and Pr are fix, the $Nu_x Gr^{-1/4}$ decreases as x increases which means the reducing in convective heat transfer capabilities. As tables goes to the right, it is found that the increase of Ec also results to the decrease of $Nu_x Gr^{-1/4}$ as of x and Pr are fixed.

Figures-2 and 3 displayed the temperature and velocity profiles for various values of Pr, respectively. It is found that the increase of Pr have reduce the thermal boundary layer thickness in Figure-2. It is due to decrease in thermal diffusivity which reduced the energy ability and the thermal boundary layer thickness. In Figure-3, the small Pr produced high velocity distribution. It is because of the small Pr usually has low in viscosity which high in momentum diffusivity.

Next, Figures-4 and 5 illustrated the variations of reduced Nusselt number $Nu_x Gr^{-1/4}$ and the reduced skin friction coefficient $C_f Gr^{1/4}$ for various values of Pr against x, respectively. In Figure-4, it is concluded that the $Nu_x Gr^{-1/4}$ decreases as x increases. Furthermore, the effects of Pr are more pronounced at a small value of x. In Figure-5, as expected, the small Pr produced large $C_f Gr^{1/4}$ compared to large Pr. This situation is related with Figure-3. Physically, the high velocity gradient will produced high in skin friction coefficient.

Lastly, in order to understand the effects of viscous dissipation Ec in the convective boundary layer flow, Figures-6 and 7 are plotted. Figure-6 shows the variations of $Nu_x Gr^{-1/4}$ with various values of Ec against x. It is seen that the viscous dissipation Ec are negligible at the lower stagnation region $(x = 0)$. The effects of Ec are significance as x increase to the middle of cylinder then converged back at the end of the cylinder $(x = \pi)$. In Figure-7, at the early stage, it is found that the $C_f Gr^{1/4}$ is unique for all Ec value. From figure, it is understand that the Ec influenced a small effects on $C_f Gr^{1/4}$ Noticed that as Ec increases, the $C_f Gr^{1/4}$ also increases. Furthermore, from numerical calculation, it is concluded that Ec does not affect the temperature and velocity profiles.

Table-1. Comparison values of $Nu_x Gr^{-1/4}$ with previous published results for various values of x when $\Pr = 1$, $Ec = 0$.

x	Merkin [5]	Nazar et al. [7]	Molla et al. [8]	Salleh and Nazar [9]	Present
0	0.4214	0.4214	0.4214	0.4214	0.4214
$\pi/6$	0.4161	0.4161	0.4161	0.4162	0.4163
$\pi/3$	0.4007	0.4005	0.4005	0.4006	0.4008
$\pi/2$	0.3745	0.3741	0.3740	0.3744	0.3744
$2\pi/3$	0.3364	0.3355	0.3355	0.3360	0.3364
$5\pi/6$	0.2825	0.2811	0.2812	0.2817	0.2824
π	0.1945	0.1916	0.1917	0.1939	0.1939

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Table-2. Comparison values of $C_f Gr^{1/4}$ with previous published results for various values of x when $\Pr = 1,\ Ec = 0.$

x	Merkin [5]	Nazar et al. [7]	Molla et al. [8]	Present
0	0.0000	0.0000	0.0000	0.0000
$\pi/6$	0.4151	0.4148	0.4145	0.4121
$\pi/3$	0.7558	0.7542	0.7539	0.7538
$\pi/2$	0.9579	0.9545	0.9541	0.9563
$2\pi/3$	0.9756	0.9698	0.9696	0.9743
$5\pi/6$	0.7822	0.7740	0.7739	0.7813
π	0.3391	0.3265	0.3264	0.3371

Table-3. Values of $Nu_x Gr^{-1/4}$ with various values of x and Ec when $\Pr = 7$.

x / Ec	-0.1	0	0.1	0.2
$\pi/6$	0.7927	0.7821	0.7714	0.7607
$\pi/3$	0.7875	0.7508	0.7135	0.6756
$\pi/2$	0.7605	0.6971	0.6318	0.5647
$2\pi/3$	0.6926	0.6185	0.5412	0.4605
$5\pi/6$	0.5614	0.5024	0.4401	0.3744
π	0.2928	0.2749	0.2556	0.2348

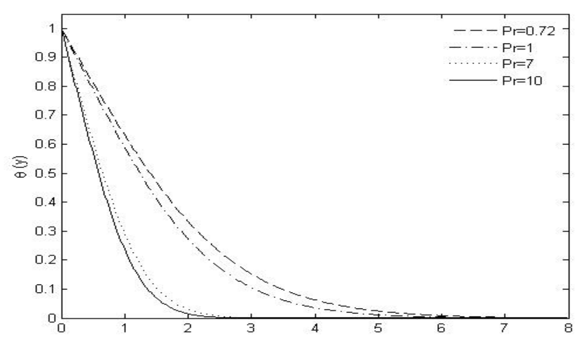

Figure-2. Temperature profiles $\theta(y)$ against y for various values of $\Pr$.

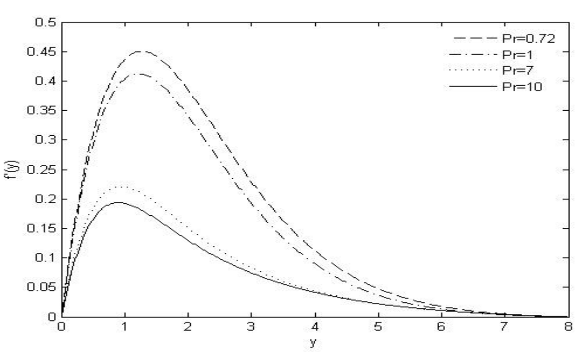

Figure-3. Velocity profiles $f'(y)$ against y for various values of $\Pr$.

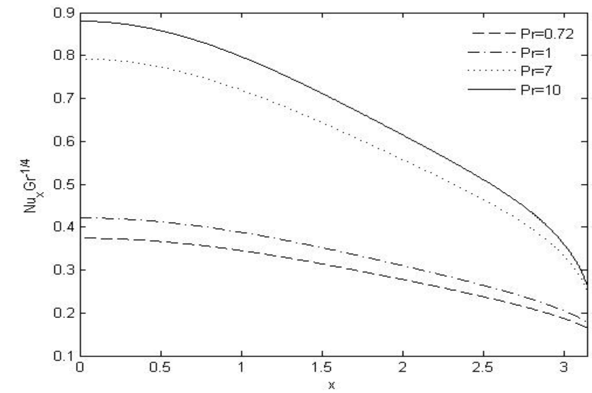

Figure-4. Reduced Nusselt number $Nu_x Gr^{-1/4}$ against x for various values of $\Pr$.

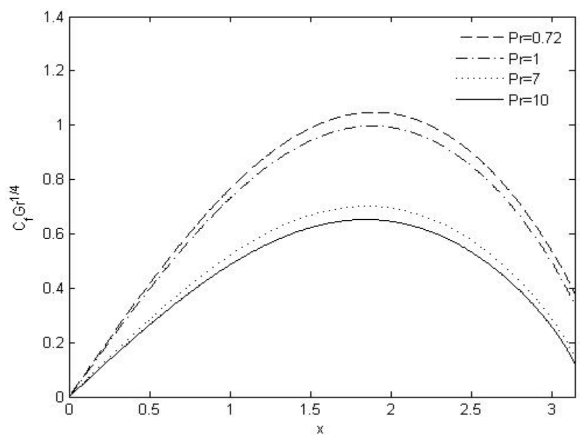

Figure-5. Reduced skin friction coefficient $C_f Gr^{1/4}$ against x for various values of $\Pr$.

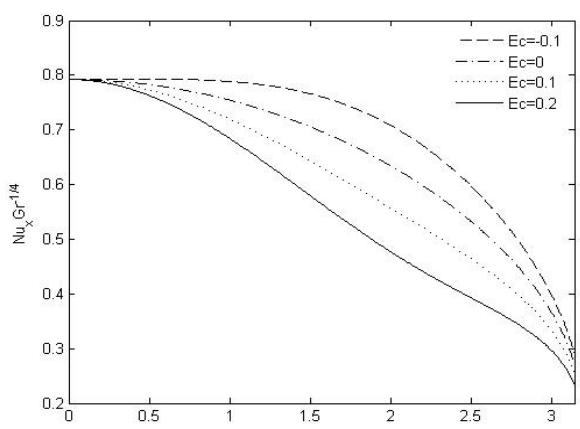

Figure-6. Reduced Nusselt number $Nu_x Gr^{-1/4}$ against x for various values of Ec.

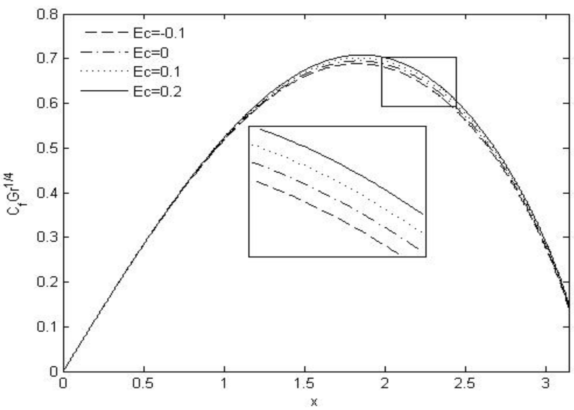

Figure-7. Reduced skin friction coefficient $C_f Gr^{1/4}$ against x for various values of Ec.

4. CONCLUSIONS

In this paper, the viscous dissipation effect on free convection boundary layer flow towards a horizontal circular cylinder is numerically studied. It is shown how the Prandtl number Pr and the Eckert number Ec affect the values of the reduced Nusselt and the reduced skin friction coefficient as well as the velocity and temperature profiles. As a conclusion, the increase of Pr result to the decrease of thermal boundary layer thickness, velocity profiles and the reduced skin friction while the reduced Nusselt number increases. It is because, an increase of Pr means the increase in viscosity but decrease in thermal diffusivity which reduced the energy ability and the thermal boundary layer thickness.

Next, the influenced of Ec on the reduced Nusselt number are negligible at the lower stagnation region $(x=0)$, Ec played the role pronouncedly at the middle of the cylinder. Meanwhile, the Ec effects are small on the reduced skin friction coefficient.

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