



DETERMINATION OF COUPLING FACTORS FOR ADHESIVE-BONDED PLATES

Achuthan C. Pankaj and S. M. Murigendrappa
National Institute of Technology Karnataka, Surathkal, India
E-Mail: acpankaj@nal.res.in

ABSTRACT

Adhesive bonding has gained importance in structural bonding in aircraft industry as an alternative method of joining materials together over the more conventional joining methods. It is gaining interest due to the increasing demand for joining similar or dissimilar structural components, mostly within the framework of designing light weight structures. In this present study, a finite element model of a structure, consisting of two Acrylic/Perspex plates joined by an adhesive has been modeled using ANSYS software. Comparisons have been made for the computed coupling factors and velocity responses for the adhesive bonded plates using finite element method and analytical wave approach of the same plates for a line junction at the joint. The results obtained from the studies signify the importance of modeling of adhesive joints in computation of the coupling factors and its further use in computation of energies and velocity responses using statistical energy approach as compared to the values obtained using analytical wave approach for a continuous line junction. Coupling factors have been computed from the velocity responses for the adhesive bonded plates using finite element method and compared with the values obtained from the analytical wave approach for the same plates with a line junction at the joint.

Keywords: statistical energy analysis, coupling factor, adhesive bonded joints, vibration.

INTRODUCTION

Statistical Energy Analysis (SEA) is one of the widely used energy methods, developed in the early 1960s to predict the vibration response of structures at high frequencies [1]. SEA parameters can be computed by analytical wave approach, power injection method, experimental approach, finite element method or the receptance method. SEA involves predicting the vibration response of a complex structure by dividing it into a number of subsystems, and is characterized by mean energy per mode. The change in energy level between subsystems is characterized by internal and coupling loss factors. Internal loss factor corresponds to damping factor (η_i) in the subsystem itself and CLF (η_{ij}) corresponds to the energy dissipation during flow across the subsystems. Coupling loss and internal loss/damping factors constitute a matrix of energy balance equations, which is used to compute the energies by the power balance approach, once the power inputs are known. The CLFs can be obtained using analytical wave approaches from coefficients of energy propagation, via junctions of subsystems, known for several types of junctions. Alternatively, the values can also be found by the power injection approach after computing the energies (E_i) and power inputs (P_{ij}) through experiments or finite element analysis for a particular frequency (ω) of excitation by using.

$$\omega \begin{bmatrix} \left(\eta_1 + \sum_{i=1}^N \eta_{1i} \right) n_1 & -\eta_{12} n_1 & \cdots & -\eta_{1N} n_1 \\ -\eta_{21} n_2 & \left(\eta_2 + \sum_{i=2}^N \eta_{2i} \right) n_2 & \cdots & -\eta_{2N} n_2 \\ \vdots & \vdots & \ddots & \vdots \\ -\eta_{N1} n_N & \cdots & \cdots & \left(\eta_N + \sum_{i=N}^{N-1} \eta_{Ni} \right) n_N \end{bmatrix} \times \begin{bmatrix} \frac{\langle \bar{E}_1 \rangle}{n_1} \\ \frac{\langle \bar{E}_2 \rangle}{n_2} \\ \vdots \\ \frac{\langle \bar{E}_N \rangle}{n_N} \end{bmatrix} = \begin{bmatrix} \bar{P}_{1,1} \\ \bar{P}_{1,2} \\ \vdots \\ \bar{P}_{1,N} \end{bmatrix} \quad (1)$$

ANALYSIS & PROCEDURE

The coupling factor and velocity responses for two Acrylic/Perspex plates joined by a double sided adhesive tape shown in Figure-1 have been analyzed by using analytical wave approach and finite element method. The structure consists of two Acrylic/Perspex plates with the same dimensions (300mm×300mm×1.1mm) joined together by a double sided adhesive tape. The overlap length of the two sheets is 12.5 mm.

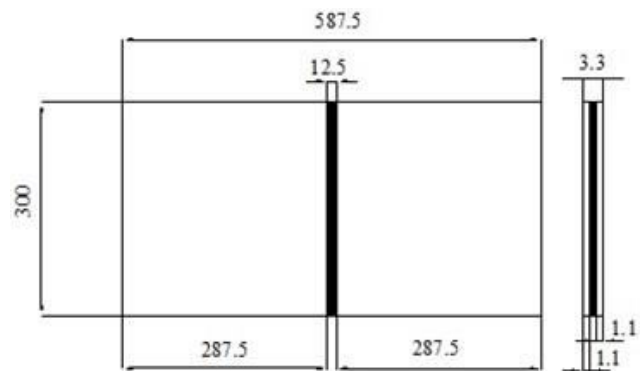


Figure-1. Geometric dimensions of the adhesive bonded plates (in mm).



The material properties assumed for the configurations are given in Table-1. The plates are square shaped having dimensions of 0.3 m with a thickness of 1.1 mm. The internal loss/damping factor of the plates has been assumed to have a value of 0.07.

Table-1. Material and geometrical specifications.

Parameter	Values
Internal Damping (η)	0.07
Width (w)	0.3 m
Length (L)	0.3 m
Thickness (t)	1.1 mm
Density (ρ)	1140 kg/m ³
Poisson's ratio (μ)	0.37
Young's Modulus (E)	2.91 GPa
Force (F)	1 N
Frequency (f)	1000 to 8000 Hz

In addition studies have also been carried out to estimate the coupling factors and velocity responses for frequencies from 1000 to 8000 Hz in steps of 1000 Hz. The analytical computations for SEA of plates are as explained in the next section.

ANALYTICAL WAVE APPROACH FOR PLATES

The subsystems in consideration have been analyzed for flexural waves, which plays an important role for vibrations at high frequencies and sound radiation. The CLF η_{12} between two plates for a line junction is given by [2].

$$\eta_{12} = \frac{2C_B L \tau_{12}}{\pi \omega A}, \quad (2)$$

Where, ω is the angular forcing frequency, A is the surface area, L is the length of the junction of the two plates and C_B is the bending wave speed of the first plate for two connected plates as the function of center frequency, f given by [3].

$$C_B = \sqrt{1.8 C_L t f}, \quad (3)$$

The wave transmission coefficient (τ_{12}) is defined as the ratio of transmitted power to the incident power. The wave transmission coefficient for random incidence vibrational energy of two coupled plates to each other can be calculated by the approximate formula as:

$$\tau_{12} = \tau_{12}(0) \frac{2.754X}{1 + 3.24X}, \quad (4)$$

Where, X is the ratio of plate thicknesses. The normal transmission coefficient $\tau_{12}(0)$ may be calculated as:

$$\tau_{12}(0) = 2 \left(\psi^{1/2} + \psi^{-1/2} \right)^{-2}, \quad (5)$$

$$\psi = \frac{\rho_1 C_{L1}^{3/2} t_1^{5/2}}{\rho_2 C_{L2}^{3/2} t_2^{5/2}} \quad (6)$$

The modal density of flat plate in flexural vibration is given by [3].

$$n(\omega) = \frac{A\sqrt{12}}{2\pi C_L t}, \quad (7)$$

Where, longitudinal wave speed is given by

$$C_L = \sqrt{\frac{E}{\rho(1-\nu^2)}}, \quad (8)$$

E is the Young's modulus, ν is the Poisson's ratio, A is the surface area and t the thickness of the plate under consideration. The time averaged power input for a unit force F is given by

$$P_m = \frac{1}{2} |\tilde{F}|^2 \operatorname{Re} \{ \tilde{Z}_m^{-1} \}, \quad (9)$$

The real part of drive-point mechanical impedance of an infinite plate of thickness t and mass per unit area ρ_a in flexural vibration is given by:

$$\operatorname{Re} \{ \tilde{Z}_m^{-1} \} = 8 \sqrt{\frac{Et^3 \rho_a}{12(1-\nu^2)}}, \quad (10)$$

The forcing frequencies are in the range of 1000-8000 Hz. The energies in each subsystem can be computed by the matrix inversion approach from Equation. (1) after computation of power input and coupling factor. The maximum velocity response V_i of each subsystem can be obtained from the obtained energy E_i under a particular power input by

$$V_i = \sqrt{\frac{2E_i}{M}}, \quad (11)$$

The coupling factors and velocity responses have been computed by in-house program built using the analytical wave approach as discussed above, in MATLAB software.

FINITE ELEMENT ANALYSIS

In numerical methods the behaviour of SEA parameters with change in inputs (geometry, boundary conditions and damping) for the given structure, can be modelled easily and is less time consuming as compared with the experimentation of the real structure. The other advantages of numerical method include cost efficiency and flexibility. A FE model of the structure as shown in Figure-2 is built and its natural frequencies are calculated by using ANSYS software. The upper and lower plates are modelled using SHELL 281 elements. SHELL 281 is suitable for analyzing thin to moderately-thick shell structures. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes,



and rotations about the x, y, and z-axes. SHELL 281 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Shell 281 has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

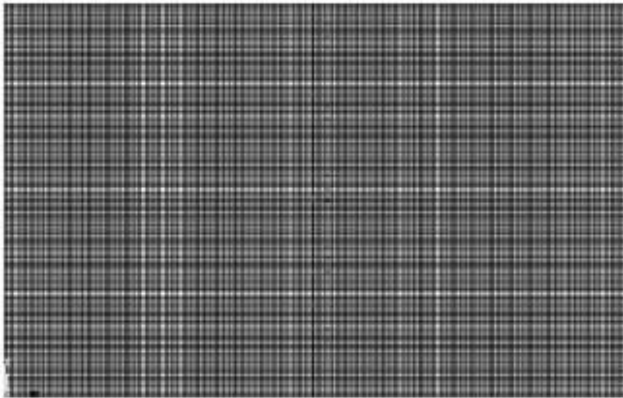


Figure-2. Finite element model.

The adhesive joint patch of 12.5 mm width has been modeled using solid elements (SOLID 186 with 3.125 mm square and 1.1 mm high). The natural frequencies and the mode shapes are calculated under the free-free boundary conditions. The upper and lower plates have been meshed with a mean mesh size of 3.125 mm. SOLID 186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions.

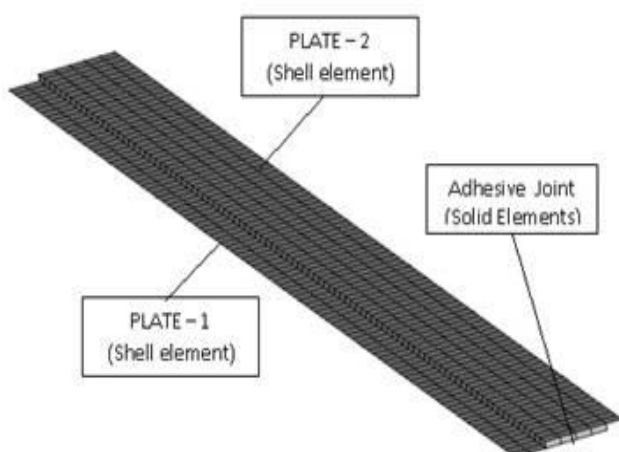


Figure-3. Finite element model (adhesive joint).

The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyper elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly

incompressible elastoplastic materials, and fully incompressible hyper elastic materials [4].

A harmonic force with unit load intensity has been applied in the range of frequencies of 1000-8000 Hz. The load has been applied on one plate and the velocity responses on both the plates have been computed. The velocity responses at all the nodes of the plates including the power input location has been determined. Macros have been developed in ANSYS Parametric Design language (APDL) for automating the computation of energy (E_i) of each subsystem with mass (M_i) and maximum subsystem velocity (V_i) according to

$$E_i = \frac{M_i V_i^2}{2} \quad (12)$$

The coupling factors are computed by the matrix inversion approach from above equation after computation of power inputs and corresponding energies in all the subsystems. The maximum average velocity response (V_i) of each subsystem can be obtained directly from the post-processing of the output results.

RESULTS AND DISCUSSION

The displacement response plots for the adhesive bonded plates with an excitation force of 1N and at frequencies of 2000 and 8000 Hz have been shown in Figure-4 and Figure-5 respectively. It has been observed in Figure-6 that with the increase in excitation frequency the deviation between the coupling factors computed by the analytical wave approach with the assumption of a continuous line junction and that computed through finite element analysis of adhesive joints decreases.

Table-2. Material and geometrical specifications.

Frequency (Hz)	Coupling Loss Factor	
	Analytical	FEM
1000	0.004027	0.0229
2000	0.002848	0.0123
3000	0.002322	0.0096
4000	0.00201	0.0072
5000	0.001799	0.0052
6000	0.001639	0.0044
7000	0.00152	0.0031
8000	0.001422	0.0022
Average	0.002198	0.0083

The velocity responses for plate 1 excited with a force of 1 N at various frequencies between 1000 – 8000 Hz is as shown in Figure-6. The coupling factor computed using the finite element analysis is higher than the analytical ones at lower frequencies. Similar trend can be seen in the plotted velocity response for the second plate Figure-8.

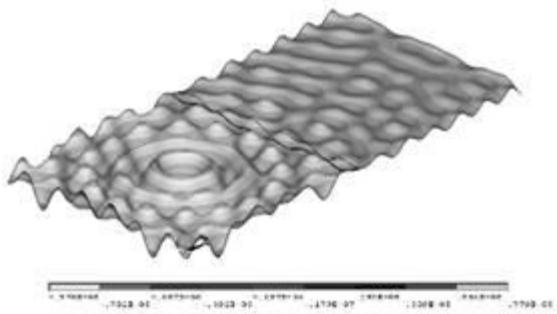


Figure-4. Displacement response at an excitation frequency of 2000 Hz.

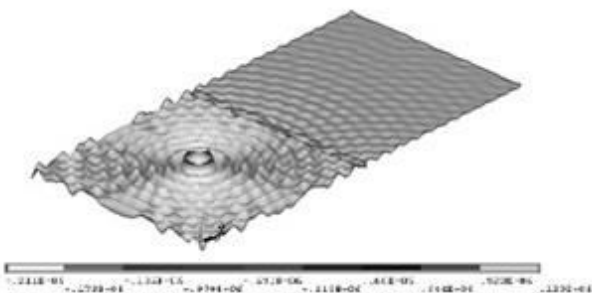


Figure-5. Displacement response at an excitation frequency of 8000 Hz.

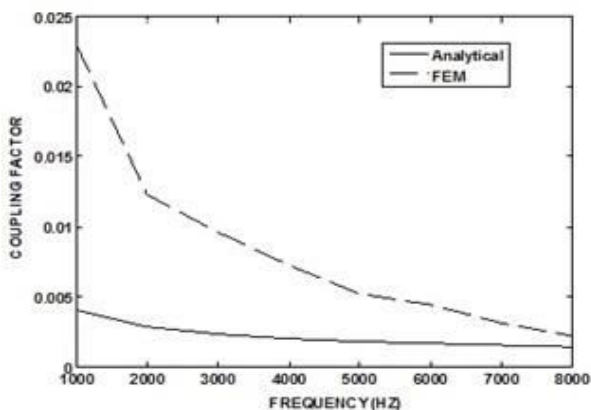


Figure-6. Variation of coupling loss factor with excitation frequencies.

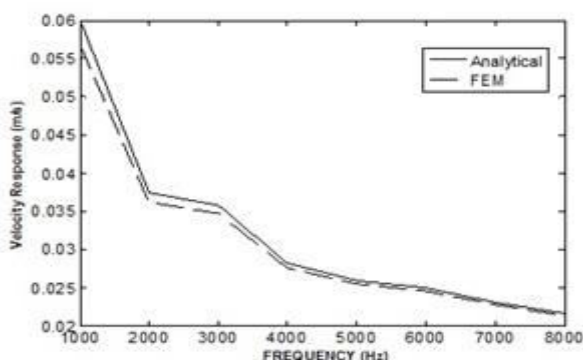


Figure-7. Velocity response for plate 1.

The results obtained from the studies signify the importance of modeling of discrete joints like the spot welded joints in computation of the coupling factors and its further use in computation of energies and velocity responses using statistical energy approach as compared to the values obtained using analytical wave assuming a continuous line junction.

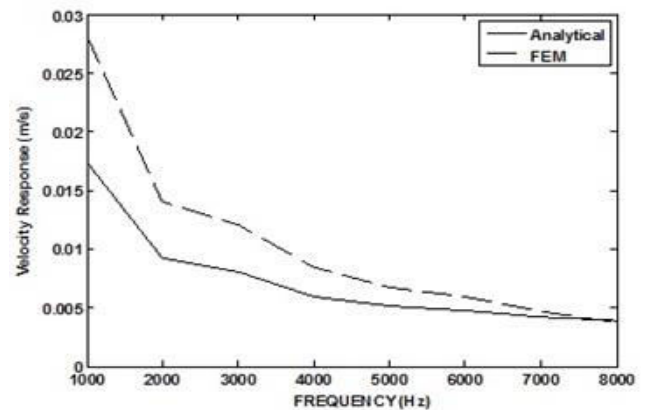


Figure-8. Velocity response for plate 2.

CONCLUSIONS

The coupling factors for two plates joined by a double sided adhesive tape has been determined through the finite element analysis method using the velocity responses obtained under a unit load harmonic excitation in ANSYS software. The finite element model of the adhesive joint has been created using solid elements and the plates by shell elements to study the dynamic behavior of sheet metal structures. Based on the model created, the effect of coupling factors computed using finite element method has been compared with the analytical wave based approach. The results obtained from the analyses have revealed that the deviation between the coupling factors computed by the analytical wave approach with the assumption of a continuous line junction and the values computed through finite element analysis of modeled adhesive joint is reduced with the increase in the excitation frequencies. The results obtained from the studies signify the importance of modeling of adhesive joints in computation of the coupling factors and its further use in computation of energies and velocity responses using statistical energy approach as compared to the values obtained using analytical wave assuming a continuous line junction.

REFERENCES

- [1] Lyon R.H. 1995. Theory and Applications of Statistical Energy Analysis, Second Edition, Butterworth-Heinemann, Boston.
- [2] D.A. Bies and S. Hamid. 1980. Insitu determination of coupling loss factors by the power injection method. Journal of Sound and Vibration.70:187-204.



- [3] Norton M.P. 1989. Fundamentals of Noise and Vibration Analysis for Engineers, Cambridge University Press, Cambridge.
- [4] ANSYS 10.0 Version Manual.
- [5] Cremer L, Heckl M and Ungar E.E. 1973. Structure-Borne Sound, Springer-Verlag, New York.
- [6] Hopkins C. 2007. Sound Insulation, 1st Edition, Butterworth-Heinemann, Boston.
- [7] A. Kaya, S. Mehmet and F.Findik. 2004. Effects of various parameters on dynamic characteristics in adhesively bonded joints. Materials Letters. 58: 3451–3456.
- [8] S.R. Pawar and M.V. Sulakhe. 1998. Finite element analysis of adhesively bonded single lap joint by varying joint parameters. The Shock and Vibration Digest. 30 (2): 91-105.
- [9] H. Vaziri and Nayeb-Hashemi. 2002. Dynamic response of tubular joints with an annular void subjected to a harmonic axial load. International Journal of Adhesion and Adhesives. 22: 367–373.
- [10] Xiaocong He. 2011. A review of finite element analysis of adhesively bonded joints. International Journal of Adhesion & Adhesives. 31: 248–264.