



DEVELOPMENT OF MATHEMATICAL MODELS AND OPTIMIZATION OF THE LASER WELDING PROCESS PARAMETERS USING RESPONSE SURFACE METHODOLOGY

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ABSTRACT

One of the benefits of fiber laser welding technology is that the amount of heat generated at the weld position is very less. This method is said to offer a great advantage for today's modern manufacturing needs. The input parameters involved in the laser welding process play an important role in deciding the quality of the weld joint. The various properties that can define the quality of the weld are mechanical aspects, the geometry of the weld bead and distortion. In this research work, the geometry of the weld bead such as ultimate tensile strength, weld bead width, depth of penetration of the laser welded butt joints of mild steel 2062 sheets are examined. With the help of design expert software, the Response Surface Methodology [RSM] was used in developing the empirical relationships relating the process parameters such as laser power, travelling speed and focal position with the output responses such as ultimate tensile strength, depth of penetration and weld bead width. The acceptability of the developed mathematical models is validated with the help of analysis of variance using design expert software. The investigation was further carried out using the desirability approach in achieving an optimal welding combination, such that, it would maximize the ultimate tensile strength, depth of penetration, and minimize the weld bead width.

Keywords: laser welding, response surface methodology, desirability approach, and optimization.

INTRODUCTION

The mathematical models for controlling the quality of weld joint, weld properties and productivity in arc welding processes have been studied [1]. The study showed various practical situations where the mathematical models can be developed and the relationship and influences between the process parameters and output responses can be found. These relationships can only be developed based on the experimental results, as the relationship between the process parameters and the weld bead geometry in the process are non-linear. It is rather difficult to develop a mathematical model that can predict the response of the welding process and determine the optimum welding condition expressed in terms of typical constraints. In general, all the welding processes are intended to obtain a welded joint with the optimal weld bead parameters with good mechanical properties and a low level of distortion. In order to achieve such a result, people nowadays use the application of design of experiment (DoE) to develop a model which will lead to the optimal weld quality. The use of Response Surface Methodology has a very high optimization accuracy level and the computational time is shorter. Also, the understanding of the technique is said to be easier than the other techniques [2]. The weld bead geometry is considered as an important aspect in finding the mechanical properties of the welded joints. This shows that the selection of proper welding process parameters is more important for obtaining optimal weld bead geometry [3-5]. The combination of the laser power, travelling speed, focal position is more important for a correct transverse cross-section shape [6]. These parameters must

be selected in a way that the deeper penetration is achieved with smaller widths for fused zone [7]. Alexandra P Costaa *et al.* [8] worked on the laser beam welding hard metals to steel and examined laser beam weldability of hard metals to steel with high power CO₂ laser and Nd: YAG laser. Balasubramian K R *et al.* [9] worked on the mathematical and ANN modelling of Nd: YAG laser welding of thin SS sheets and compared the neural network model and multiple linear regression model. Padmanabhan G *et al.* [10] worked on the optimization of laser beam welding process parameters in achieving maximum tensile strength of AZ31B magnesium alloy and concluded that the welding speed has the greatest influence on tensile strength, followed by laser power and focal position. Dhavalkumar K Soni *et al.* [11] conducted an experimental investigation and prediction of the laser welding process for mild steel 2062 sheets of 1 mm using a fiber laser. They conducted the experiment and designed the experimental work using design expert software. They predicted the output responses by Artificial Neural Network (ANN) using MATLAB.

In the current research, an attempt has been made to establish mathematical relationships by relating the process parameters to the output responses and thereby optimizing the process parameters using the desirability approach in achieving an optimal welding combination with the help of Response Surface Methodology. The objective of the optimization is to maximize the ultimate tensile strength, depth of penetration, and minimize the weld bead width.



RESPONSE SURFACE METHODOLOGY

The mathematical models are developed with the help of this technique. This methodology is preferred as it has a collection of statistical and mathematical techniques which is mainly used in building a significant model. The desirability approach is mainly used for its simplicity and flexibility in giving preference level for individual responses.

ANALYSIS OF VARIANCE USING RSM

In the Table-1, the model F-value of 378.86 shows that the model developed for the output response ultimate tensile strength is significant. If the values of "Prob>F" are less than 0.05, it is said to indicate that the model terms are significant. From Table-1, it is clear that the terms A, B, C, AB, AC, BC, A², C² are all significant. The "Pred R-Squared" of 0.9876 is found to be in reasonable agreement with the "Adj R-Squared" of 0.9924. "Adeq Precision" measures the signal to noise ratio, greater than 4 is desirable. The ratio of 68.494 indicates an adequate signal and thus the model can be used to navigate the design space. In the Table-2, the model F-value of 64.01 shows that the model developed for the output response depth of penetration is significant. From Table-2, it is clear that the terms A, B, C, AC are all significant. The "Pred R-Squared" of 0.9164 is in reasonable agreement with the "Adj R-Squared" of 0.9562. The ratio of 29.010 indicates an adequate signal and thus the model can be used to navigate the design space. In the Table-3, the model F-value of 11.56 shows that the model developed for the output response weld bead width is significant. From Table-3, it is clear that the terms A, B are all significant. The "Pred R-Squared" of 0.6553 is in reasonable agreement with the "Adj R-Squared" of 0.7852. The ratio of 11.595 indicates an adequate signal and thus the model can be used to navigate the design space.

DEVELOPMENT OF EMPIRICAL RELATIONSHIPS

The mathematical relationships expressed in relating the output responses and the process parameters are given in the Equations. (1) – (3).

Ultimate tensile strength = $97.85185 + 277.11111 * \text{Laser power} + 0.022389 * \text{Travelling speed} - 84.84127 * \text{Focal position} - 0.010000 * \text{Laser power} * \text{Travelling speed} + 44.76190 * \text{Laser power} * \text{Focal position} + 7.61905 * 10^{-3} * \text{Travelling speed} * \text{Focal position} - 59.55556 * \text{Laser power}^2 + 1.77778 * 10^{-6} * \text{Travelling speed}^2 + 32.19955 * \text{Focal position}^2$ (1)

Depth of penetration = $0.84108 + 0.12333 * \text{Laser power} + 1.72222 * 10^{-6} * \text{Travelling speed} + 0.021349 * \text{Focal position} + 6.66667 * 10^{-6} * \text{Laser power} * \text{Travelling speed} - 0.026667 * \text{Laser power} * \text{Focal position} + 7.14286 * 10^{-6} * \text{Travelling speed} * \text{Focal position} - 0.026667 * \text{Laser power}^2 - 1.33333 * 10^{-9} * \text{Travelling speed}^2 + 0.016327 * \text{Focal position}^2$ (2)

Weld bead width = $0.94611 + 0.082556 * \text{Laser power} - 1.13333 * 10^{-5} * \text{Travelling speed} - 6.42857 * 10^{-3} * \text{Focal position} + 3.33333 * 10^{-6} * \text{Laser power} * \text{Travelling speed} + 3.80952 * 10^{-3} * \text{Laser power} * \text{Focal position} + 3.33333 * 10^{-6} * \text{Travelling speed} * \text{Focal position} - 0.021333 * \text{Laser power}^2 - 5.42657 * 10^{-22} * \text{Travelling speed}^2 + 6.80272 * 10^{-3} * \text{Focal position}^2$ (3)

OPTIMIZATION

The need of relating the ultimate tensile strength, depth of penetration and weld bead width must be addressed, in order to establish a model of optimized values. The optimal welding conditions at which the desirable responses can be achieved is noted based on the optimization study carried out. Once we assign the criteria for which the models are developed, the optimum welding conditions can be obtained. The criteria implemented is shown in Table-4. The criteria was set to reach the maximum ultimate tensile strength, depth of penetration and minimum weld bead width by using the input parameters in the range. The optimal solutions obtained through desirability approach are given in the Table-5 and Table-6. The solutions obtained through the desirability approach show that for an optimization criteria of maximum ultimate tensile strength and depth of penetration, and minimum weld bead width, the travelling speed has to be around the limit of 1900 mm/min. The optimal conditions provide an ultimate tensile strength of 428.12 MPa, depth of penetration of 1.023 mm and weld bead width of 1.014 mm with a desirability of 0.820. The optimized values and their responses are shown for each parameter in the Figure-1.

Table-1. ANOVA results for the response 1 (Ultimate tensile strength).

Source	Sum of Squares	Mean Square	F Value	p-value Prob > F
Model	2967.7	329.746	378.8	< 0.0001
Laser power	1624.5	1624.5	1866.4	< 0.0001
Travelling speed	256.88	256.888	295.1	< 0.0001
Focal position	684.5	684.5	786.4	< 0.0001
AB	18.75	18.75	21.54	0.0002
AC	184.08	184.083	211.5	< 0.0001
BC	21.333	21.3333	24.51	0.0001
A ²	83.129	83.1296	95.51	< 0.0001
B ²	1.1851	1.18518	1.361	0.2594
C ²	93.351	93.3518	107.25	< 0.0001
R-Squared	0.995	Pred R-Squared	0.987	
Adj R-Squared	0.992	Adeq Precision	68.49	

**Table-2.** ANOVA results for the response 2 (Depth of penetration).

Source	Sum of Squares	Mean Square	F Value	p-value Prob > F
Model	0.00457	0.0005	64.009	< 0.0001
Laser power	0.00273	0.0027	344.43	< 0.0001
Travelling speed	0.000213	0.0002	26.864	< 0.0001
Focal position	0.001494	0.0014	187.96	< 0.0001
AB	8.3×10^{-6}	8.3×10^{-6}	1.048	0.3202
AC	6.5×10^{-5}	6.5×10^{-5}	8.218	0.0107
BC	1.8×10^{-5}	1.8×10^{-5}	2.358	0.1430
A ²	1.6×10^{-5}	1.6×10^{-5}	2.096	0.1658
B ²	6.6×10^{-7}	6.6×10^{-7}	0.0838	0.7756
C ²	2.4×10^{-5}	2.4×10^{-5}	3.0191	0.1004
R-Squared	0.9713	Pred R-Squared	0.9164	
Adj R-Squared	0.9561	Adeq Precision	29.010	

Table-3. ANOVA results for the response 3 (Weld bead width).

Source	Sum of Squares	Mean Square	F Value	p-value Prob > F
Model	0.00037	4.145×10^{-5}	11.562	< 0.0001
Laser power	0.00015	0.00015	41.903	< 0.0001
Travelling speed	0.0002	0.0002	55.788	< 0.0001
Focal position	5×10^{-7}	5×10^{-7}	0.1394	0.7134
AB	2.08×10^{-6}	2.08×10^{-6}	0.5811	0.4563
AC	1.33×10^{-6}	1.33×10^{-6}	0.3719	0.5500
BC	4.08×10^{-6}	4.08×10^{-6}	1.1390	0.3008
A ²	1.06×10^{-5}	1.06×10^{-5}	2.9753	0.1027
B ²	0	0	0	1.0000
C ²	4.16×10^{-6}	4.166×10^{-6}	1.1622	0.2961
R-Squared	0.85957	Pred R-Squared	0.6552	
Adj R-Squared	0.78523	Adeq Precision	11.595	

Table-4. Optimization criteria for laser welding process.

Name	Goal	Lower Limit	Upper Limit
Laser power	is in range	1.5	2
Travelling speed	is in range	1000	2000
Focal position	is in range	-0.7	0
Ultimate tensile strength	maximize	389	427
Depth of penetration	maximize	0.979	1.023
Weld bead width	minimize	1.005	1.021

Table-5. Optimized solutions showing the input parameters.

Laser power	Travelling speed	Focal position
1.935519	1999.9999	-0.69998
1.938384	1999.9997	-0.69614
1.941178	1999.9824	-0.69241
1.943892	1999.9978	-0.6888
1.947846	1999.9978	-0.68357
1.951555	1999.9919	-0.6787
1.955482	1999.9989	-0.67353
1.960577	1999.9995	-0.66691

Table-6. Optimized solutions showing the output responses.

Ultimate tensile strength	Depth of penetration	Weld bead width	Desirability
428.12824	1.0230001	1.014221	0.82021681
428.00737	1.0230000	1.014225	0.82010474
427.89081	1.0229999	1.014229	0.81999268
427.77905	1.0230001	1.014233	0.81989176
427.61805	1.0230001	1.014238	0.81974206
427.46974	1.0230009	1.014243	0.81960191
427.31377	1.0229997	1.014248	0.81945944
427.11636	1.0230004	1.014254	0.81927844

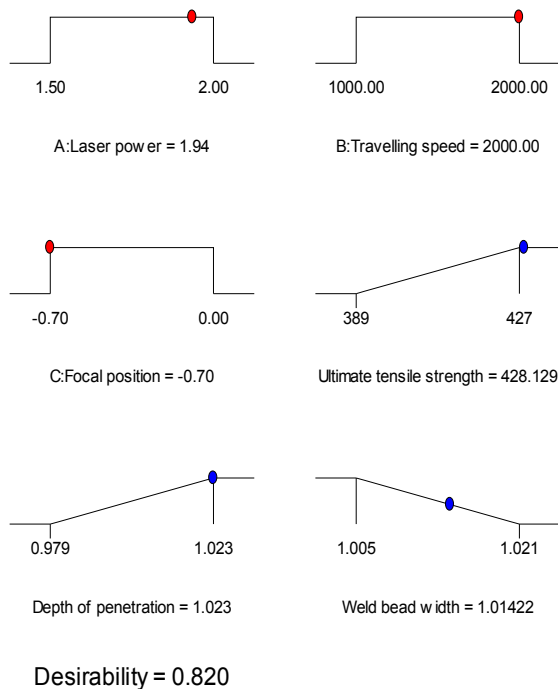


Figure-1. Optimized result for each parameter.

DESIRABILITY

The values of the desirable level of all the input parameters and output responses are shown individually in the Figure-2. The combined desirability of the optimized model is said to be 0.820.

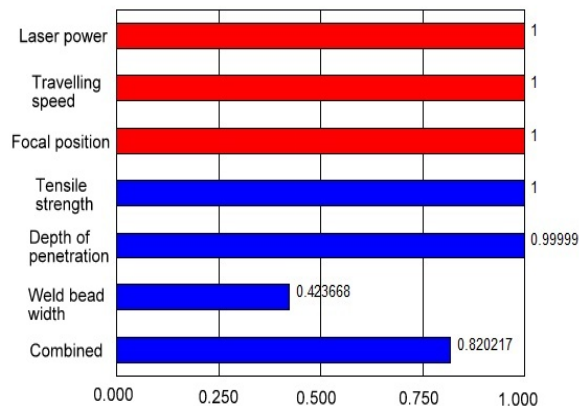
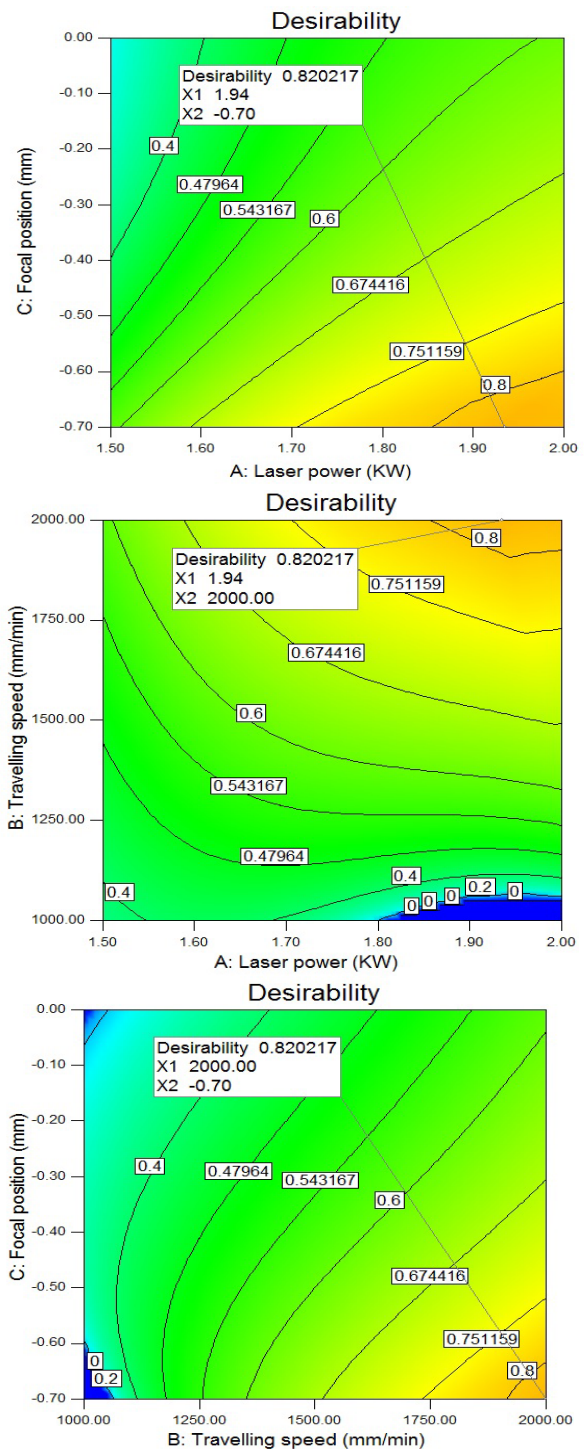


Figure-2. Desirability aspect for individual parameters and combined level.

CONTOUR AND OVERLAY PLOTS

The graphical plots of the optimal solution show the influence of each parameter level with the desirability of the optimization process. The desirability and the optimal values of the input parameters are shown in Figure-3(a), Figure-3(b) and Figure-3(c). The overlay plot shown in Figure-3(d) is the result of the graphical

optimization of the welding process in which the yellow shaded regions are the portions that come under the desired response criteria. Figure-4(a), Figure-4(b), Figure-4(c) shows the cube plot that has the ability to show the influence of all the three input parameters for a particular output response and predict the optimal conditions for obtaining the desired responses.



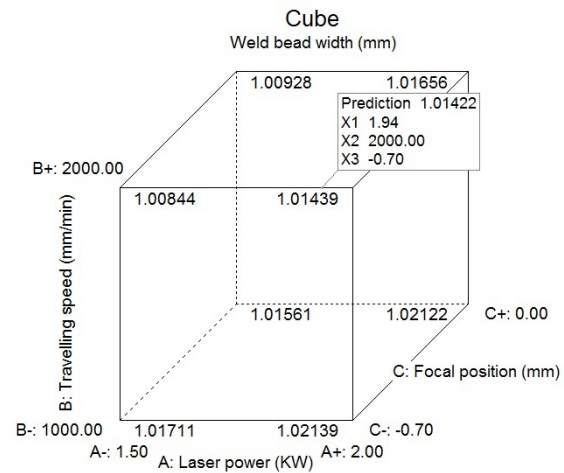
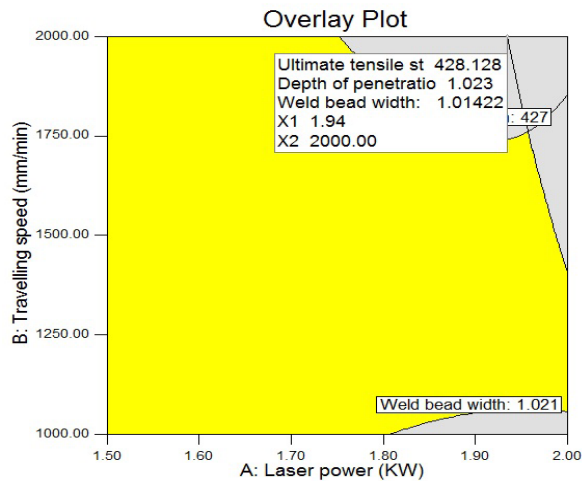


Figure-3. Desirability plots (a) Contour plot showing laser power and focal position (b) Contour plot showing laser power and travelling speed (c) Contour plot showing travelling speed and focal position (d) Overlay plot showing the optimized weld zone.

CUBE PLOTS

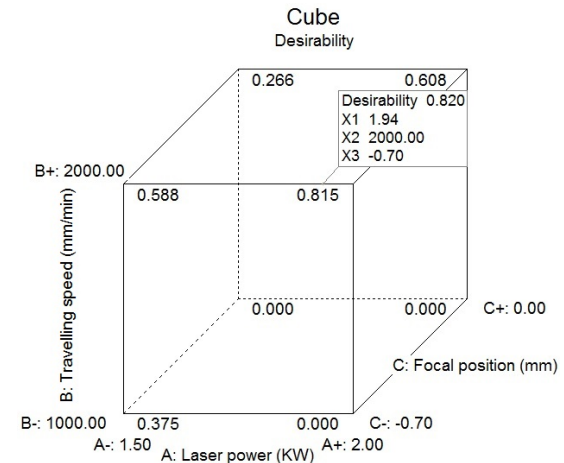
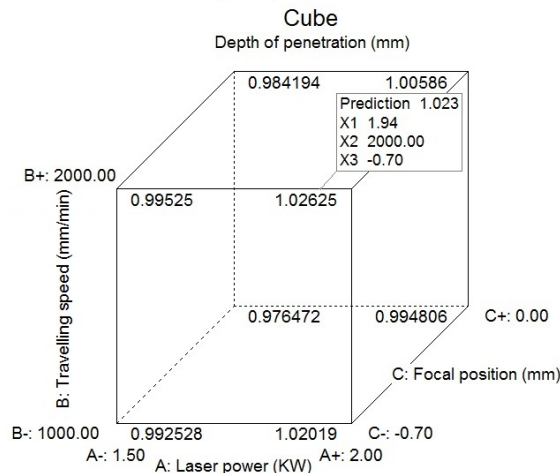
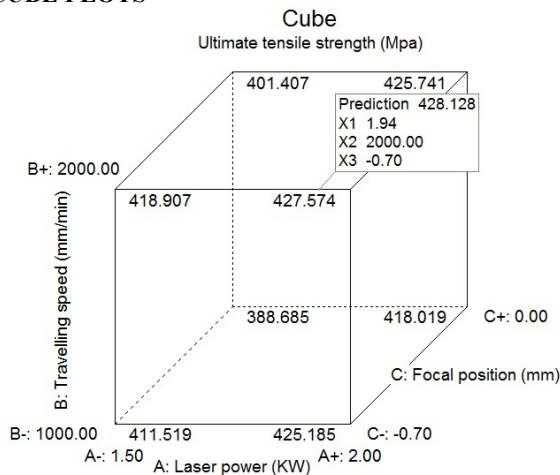


Figure-4. Cube plots (a) Ultimate tensile strength (b) Depth of penetration (c) Weld bead width (d) Desirability.

CONCLUSIONS

From the results obtained, the following conclusions were listed.

Investigation on the laser welding process is carried out and the relationship between the input parameters such as laser power, travelling speed and focal position with the output responses such as ultimate tensile strength, depth of penetration and weld bead width is modelled through RSM. The developed RSM model is used to optimize the welding parameters with the help of desirability approach using design expert software.

A travelling speed between 1980 and 2000 rpm is an optimum input for obtaining an excellent laser welded result. The travelling speed is the most influencing parameter of the output response, weld bead width.

During the laser welding process, the optimized laser power between 1.93 and 1.94 is said to have a higher influence on the output response, depth of penetration. In this case, the travelling speed does not influence the depth of penetration much.

The travelling speed has less influence on the output response, ultimate tensile strength, which means the



input parameters laser power and focal position are the most influencing aspects of the output response considered.

Thus, it is clear that the optimized results show that the maximum tensile strength of 428.12 MPa, depth of penetration of 1.023 mm/min and weld bead width of 1.014 mm for the input parameters (Laser power = 1.94 KW, Travelling speed = 1999.99 mm/min & Focal position = -0.699 mm) obtained through desirability approach.

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