



EFFECTIVE MATRIX BLOCK SIZES IN PERCOLATION MODEL AND FILTRATIONAL PARAMETERS OF FRACTURED ENVIRONMENTS

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ABSTRACT

The relationship of matrix block sizes and fluid filtration parameters in fractured porous media is considered. It is shown that the Barenblatt hypothesis operates quite well starting from the times smaller than the characteristic times of pressure redistribution in saturated porous media and relaxation times in fractures media with two type of porosity. By using of percolation model of a naturally fractured reservoir with uniform and normal distributions of matrix block sizes it is calculated linear lengths of the blocks. It is occurred that the relatively large linear sizes of the blocks in fractured porous media (10^{-1} - 10^1 meters) correspondsto the relaxation times in interval 10^2 - 10^5 seconds.

Keywords: fractured porous media, filtration, blocks sizes, relaxation times.

1. INTRODUCTION

To describe the filtration of fluids infractured porous media is commonly used Barenblatt [1] and Warren and Root models [17] including the concept of a dual porosity medium: fractured space and porous blocks separated by these fractures [16].

Following Barenblatt the system of unsteady filtration equations in fractured porous media can be written as (Molokovich *et al.*, [9, 10]):

$$\begin{aligned} \vec{w}_f &= -\frac{k_f}{\mu} \text{grad}P_f, \vec{w}_m = 0; \\ \rho_0 \text{div} \vec{w}_f + \frac{\partial(m_f \rho)}{\partial t} - q &= 0, \quad \frac{\partial(m_m \rho)}{\partial t} + q = 0; \\ m_f \rho &= m_{f0} \rho_0 + \rho_0 \beta_f P_f, \quad m_m \rho = m_{m0} \rho_0 + \rho_0 \beta_m P_m; \\ q &= \alpha \frac{\rho_0 k_m}{\mu a^2} (P_m - P_f) = \rho_0 A (P_m - P_f). \end{aligned} \quad (1)$$

In the equations (1) are used the pressure deviation functions in fractures (P_f)and blocks (P_m).This functions satisfy the equation

$$\chi^* \Delta (P_{f,m} + \tau_m \frac{\partial P_{f,m}}{\partial t}) = \frac{\partial}{\partial t} (P_{f,m} + \tau \frac{\partial P_{f,m}}{\partial t}). \quad (2)$$

Here and below indices f –refers to fractures, m –corresponds to blocks, 0 –corresponds to the initial parameter values, k –permeability, m –porosity, w –filtration velocity, β –total compressibility, β_f and β_m –fractures and porous compressibility, ρ –density, $\chi = k/\mu\beta$ –diffusivity coefficient, $\chi_f = k_f / \mu\beta_f$ – fracture diffusivity coefficient, $\chi^* = \chi_f \tau_f / (\tau_m + \tau_f) = \chi_f \beta_f / (\beta_m + \beta_f)$ –diffusivity coefficient, taking into account the cumulative effect of the blocks and fractures, a – linear block size, A – dimensionless parameter order unit, $A = \alpha k_m / \mu a^2$, $\tau_{f,m} = \beta_{f,m}/A$ –constant of the time dimensions - relaxation times, $\tau = \tau_f \tau_m / (\tau_f + \tau_m)$ -relaxation factor.

An important assumption in the filtrational model under consideration is a description of the interporosity flow function between blocks and fractures (q) as a linear function versus the pressure difference (Δp) between the

fractures and the blocks($q \sim \Delta p$).It is an interesting to know the minimal time when this assumption is correct.

The presence of the relaxation time parameters τ_f and τ_m in the filtrational model enable to adequately interpret the pressure build-up test results in fractured porous media. For instance, in figure 1is demonstrated the accordance between the model simulation and experiment data for the well 4788 using time parameterst_m=25000 s and $\tau_f = 6250$ s. The experimental curve is shown 1 by points, model – solid line. We see the relaxation times (non-local time parameters in (1) equals 10^4 - 10^5 s. This corresponds to an average linear block size $L = \sqrt{\pi \chi \tau} = 13.4m$ [4].

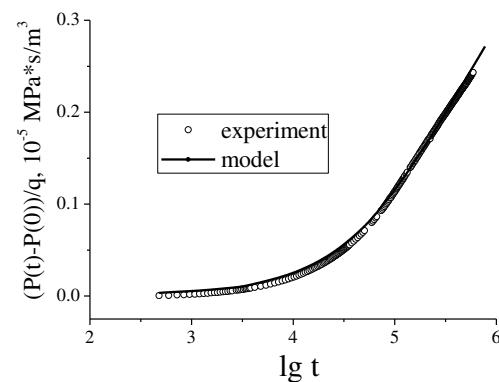


Figure-1. Build-up pressure change in well 4788. Model and experimental curves.

Currently, there are a large number of works dealing with block size distribution in fractured medium Cinco-Ley *et al.*, [3]; Kazemi *et al.*, [6]; Moench *et al.*, [8]. The typical values of the blocksize distributions are relevant for the interpretation of the well pressure built-up test results (Hassansadeh *et al.*, [5]; Montazeri *et al.*, [11]; Rodriguez *et al.*, [14], [15]). Much less works devoted to the block space modeling and research (Liu *et al.*, [7]).

In this paper is discussed the Barenblatt hypothesis applicability to the interpretation of the well



testing data in fractures media, the non-stationary hydrodynamics wave approach to the intensification of fluid flows between the blocks and the fractured spaces in context of blocksize distributions using percolation model to imitate the block space.

2. MODEL 1. FILTRATIONAL WAVES AND BLOCK SIZES

Let us estimate the time limits of the Barenblatt hypothesis ($q \sim \Delta p$) applicability. We will consider a sphere of radius R , which simulates a block (matrix) of the dual porosity medium and assume that the filtration in the sphere occurs on Darcy's law. So the process of pressure redistribution will be described by the standard diffusion equation in spherical coordinates

$$\frac{\partial p}{\partial t} = \chi \left(\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} \frac{\partial p}{\partial r} \right). \quad (3)$$

Here pressure depends only on the distance r from the center of the sphere. We will solve (3) with the initial and boundary conditions $p_{t=0} = p_0$, into sphere $0 \leq r \leq R$ and in fracture space $p(R) = p_1$ when $t > 0$. In other words on the sphere surface of the sphere-in the fractured space the pressure jump from p_0 to p_1 at the time $t=0$. Then, solving (3) by the separation of variables method we obtain

$$p(r, t) = \frac{2R}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} [p_1 - (p_0 - p_1) \exp(-\lambda_n t)] \sin\left(\frac{\pi n r}{R}\right). \quad (4)$$

where the notation $\lambda_n = \pi^2 n^2 \chi / R^2$. Determining the difference between the average pressure in the sphere and in the fractured space as $\Delta p = \langle p(r, t) \rangle - p_1$, where V - volume of the sphere, $G = \chi t / R^2$ and average pressure $\langle p \rangle = \int p(r, t) dV / \int dV$ we found

$$q = \frac{16}{3} \pi^4 \frac{k}{\mu} R \Delta p F(t), \quad (5)$$

$$\beta = \frac{16}{3} \pi^4 \frac{k}{\mu} R,$$

where

$$F(t) = \sum_{n=1}^{\infty} \exp(-\pi^2 n^2 G) / \sum_{n=1}^{\infty} (\exp(-\pi^2 n^2 G) / n^2). \quad (6)$$

The relative declination from the Barenblatt hypothesis can be written as

$$F(t) - 1 = (q - \beta \Delta p) / \beta \Delta p. \quad (7)$$

Figure-2 illustrate the function $F(t)-1$ dependence from dimensionless parameter $G = \chi t / R^2$.

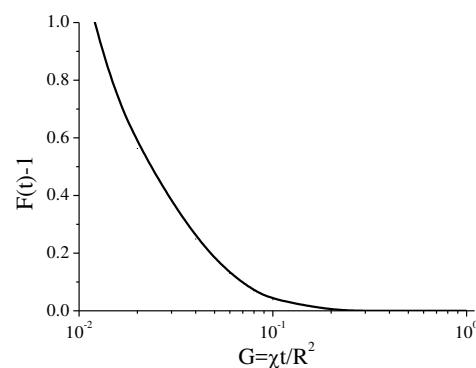


Figure-2. Function $F(t)-1$ vs time dependence.

The relative deviation function becomes less than 10^{-2} , so we can conclude that Barenblatt hypothesis operates quite well beginning from time $t^* \sim 0.2 R^2 / \chi$. This time less than the characteristic time of pressure redistribution $t_{char} \sim R^2 / \chi$.

Let us consider the periodic pressure stimulation in vertical well

$$p_c = p_{c0} + p_{c1} \cos(\omega t). \quad (8)$$

In this case the interporosity flow will be equal to [4]

$$q(r, t) \sim p_{c1} \operatorname{Re} \left\{ \frac{i\omega\tau_2}{1+i\omega\tau_2} \frac{\operatorname{BesselK}_0(r/R(\omega))}{\operatorname{BesselK}_0(r_c/R(\omega))} \exp(i\omega t) \right\}. \quad (9)$$

Here $R(\omega) = \sqrt{\chi_1 \tau (1 + i\omega\tau_2) / i\omega\tau_2 (1 + i\omega\tau)}$, p_{c1} - the amplitude of the pressure oscillations in the well, p_{c0} - average pressure.

For $n = 1$ in (4) and $\lambda_n = \pi^2 n^2 \chi / R^2$ we can estimate the value of period when interporosity flow will have a maximum amplitude (Davletshin et al., [4]; Molokovich et al., [9]; Molokovich [10]),

$$T_{max} = \frac{2}{(\sqrt{2}-1)\pi} \frac{R^2}{\chi} \approx \frac{3}{2} \frac{R^2}{\chi}. \quad (10)$$

It is essential this time value (T_{max}) is more than t^* and depends from linear block size.

3. PERCOLATION MODEL

Bradbury and Hammersley [2] reviewed the overall situation arising from accidental spreading of the liquid through a medium. Following this approach consider 3 D fractured medium with uniform fractures. Let the cube filled by random fractures. The fractures will be presented in the form of flat squares. The size of the fracture is the length of the four square. Since fractures in real rocks tend to have preferential propagation direction, we choose two perpendicular planes of fracture orientation. The location of the fractures will be distributed randomly in a given plane. Connected volume of the solid matrix bounded by fractures are the blocks.

The main focus during the modeling studies will be drawn to determine the specific surface of fractures



filled at the moment of the connected path. Here the specific surface is the area of fractures per unit volume.

With an increasing the linked fracture network, the area occupied by them exceeds some threshold value - percolation threshold and the flow effect occurs. Limited space fissure structures will play the role of the blocks. In rocks, the volume of medium, bounded by fractures, saturated by injected fluid, are the blocks, participating in the transient flow.

The specific surface of fractures can be turnbind to the average linear block size. Assume the the medium consists of a cubic blocks with a sides separated by fissures. In this case, the volume of each unit block (a^3) decreases with an increasing number of blocks in domain ($\sim 3 a^2$) fractures surface area. The inverse to the specific surface area S/V is equal to $V/S = a/3$. Hence, $a \approx 3V/S$ and the average linear block size will be in three times larger than the inverse of the specific surface. Below we take into account that in real medium exists as a hierarchy of fractures and blocks.

4. RESULTS

First of all, consider the case of constant size fractures. Several simulations were run for fractures sizes of $l=5, 10, 20, 30, 40, 50$ and 100 , in model medium with sizes $100 \times 100 \times 100 = 10^6$, $200 \times 200 \times 200 = 8 \times 10^6$, $300 \times 300 \times 300 = 27 \times 10^6$, $400 \times 400 \times 400 = 64 \times 10^6$ conventional units. We use dimensionless quantities so important is only the ratio of fracture length and the length of the sample, of the fracture length and block length.

The dependence of the linear block size vs medium volumes under the various size of fractures shown in the Figures 3 and 4. For small fractures the connected path is formed when a smaller area of fractures is full of injected fluid, and hence a larger transient block size, which by passed liquid. For each size of the fractures according to the amount field changes the result, but tends to a certain limit. When interpreting the results should also be take into account when the size of fractures growth and it is commensurate with the medium length, the results volatility increases. So, we see the more the fracture length the less the linear block size when becomes percolation threshold for a fixed volume.

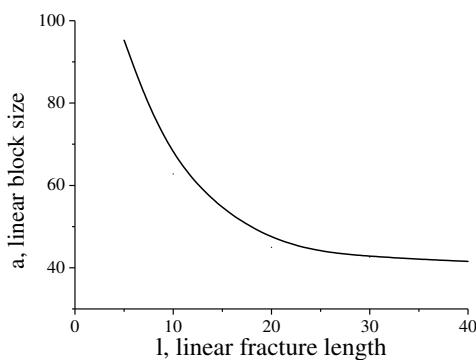


Figure-3. Linear block size vs. fractures length for a fixed media volume.

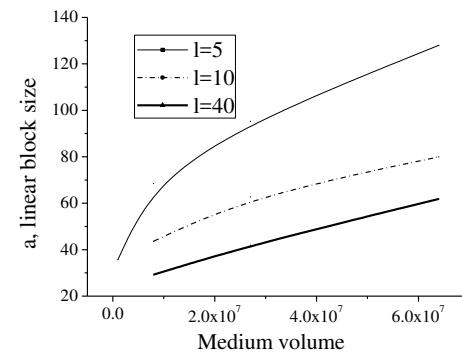


Figure-4. Linear block size (a) vs. volume of medium, for different values of the fractures length (l).

Now consider model with normal fracture size distribution. A significant number of physical parameters is subject to this allocation. In particular, the pore size distribution obeys close to normal as evidenced by data from a number of authors. Number of fractures size is determined according to the formula

$$n(r) = \frac{2\pi}{\sigma} \exp\left(-\left(\frac{l-\xi}{\sigma}\right)^2\right), \quad (11)$$

where n – number of fractures, l – linear dimension of the fractures, σ, ξ - distribution parameters. For a large values ξ distribution is more symmetrical representation, while it means a more uniform representation and increase in the size of the fractures. So when $\xi > 20$ virtually no small fractures. And for small σ sharply reduced the share of smaller and larger fractures ξ . Calculating results shown in Figure 5. The growth parameter σ leads to increasing of the part the shorter and longer fractures compared with the average value of the fracture in the distribution (11) and, in general, reduces the effective linear size of the blocks sufficient to produce a flow in the system.

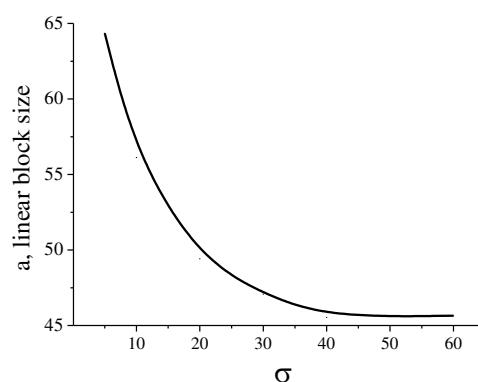


Figure-5. Linear block size vs. parameter σ ($\xi=10$).

The graph shows then apparently the average blocks sizes reaches a certain limit. This is done by larger size of fractures. With decreasing σ the average block size approaches close to that at constant fracture size. With the separarameters, the average block size is changed by 1.5



times. The increase in the dispersion of the distribution function of fracture size reduces the percolation threshold, which is consistent with Wilkinson *et al.*, [18].

For comparison with real media we note, if unit length $\Delta l = 10^{-2}$ - 10^{-1} m, the characteristic linear dimension of the block will be $a = 10^{-1}$ - 10^1 m.

5. DISCUSSIONS AND CONCLUSIONS

We see from a constructed percolation model that linear block sizes may be 10^0 - 10^1 and exceed linear fracture sizes more than 100 times. In all analysed cases observed increase of the specific surface or in other words decrease of the average block size vs. The fractures size regardless of whether any fractures are equal or not. In all cases there is a limit specific surface area, above which there is a connected path from the inlet to the outlet. It is observed that with the increasing the fracture size the average size of the blocks, surrounded by filling fractures decreases.

The pressure drop in the fracture network that ensured the flow, lead to the beginning of mass transfer with blocks. Transients pressure changes will correspond to the blocks. Thus for med timing lag filter relaxational models, such as in equation (1).

In real rock media there are blocks and fracture with various lengths. Most large fractures and are generally and most permeable. But even small-sized fractures can provide network, sufficient to flow by configuring the blocks "delineated" by fractures through which flow is achieved at the time of the passage of fluid throughout the sample. Average block size is several times greater than the fracture size. The process of mass transfer between them will have a considerable characteristic time. The equation describing this process will require a great relaxation time.

The parameters with the time dimension present usually in the models of fluid filtration in fractures media. The values of this parameters often exceeds 10^4 - 10^5 seconds. Such a large times correspond to linear block size equal to 10^0 - 10^1 meters. These values are essentially more than linear fracture sizes.

We see also that the Barenblatt hypothesis works quite well starting from the times more than $0.2 R^2/\chi$ and this magnitude smaller than the characteristic time of pressure redistribution in saturated porous media.

In this way the fracture lengths, linear block sizes, relaxation times in filtrational models and times of Barenblatt hypothesis application are agreed connected each other.

In a practical sense, knowledge of the block size distribution may be helpful for the evaluation of the time-optimal regimes in well exploration in order to intensify the fluid flow between block and fracture areas.

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