



## RECENT RESEARCH IN COOPERATIVE PATH PLANNING ALGORITHMS FOR MULTI-AGENT USING MIXED- INTEGER LINEAR PROGRAMMING

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### ABSTRACT

Path planning is one of the issues to be handled in the development of autonomous systems. For a group of agents, cooperative path planning is crucial to ensure that a given mission is accomplished in the shortest time possible with optimal solution. Optimal means that the resulting path has minimal length hence the total consumed energy by the agents is the least. Cooperative path planning fuses information from all agents to plan an optimal path. There are a number of cooperative path planning methods available in the literature for multi-agent including Cell Decomposition, Roadmap and Potential Field to name but three. This paper will review and compare the performances of those existing methods that can find solution without graph search algorithm such as Mixed-Integer Linear Programming (MILP) techniques which exactly solves the problem and then propose four alternative MILP formulations which are computationally less intensive and suited for real-time purposes, but yield a theoretically guaranteed suboptimal solution.

**Keywords:** path planning, mixed-Integer linear programming.

### Nomenclature

$s_p$  = vehicle state

$u_p$  = control input

$T$  = number of time-steps

$t_{pcik}$  = binary variable (0 or 1)

$M$  = large number for logical constraints

$b_{pqik}$  = binary variable (0 or 1)

$d_x, d_y$  = safety distances

### INTRODUCTION

Multi-agent working cooperatively has main advantages over single-agent in various applications. The advantages of multi-agent include increased speed of task completion as well as improved robustness in the system. While parallelism is achieved by assigning different tasks or abilities to different agents, robustness is a benefit of multi-agent systems that have redundant agents. If control and responsibilities are sufficiently shared among different agents, the system can tolerate failures by one or more of the agents (Peter *et al.*, 2000).

Although a multi-agent system need not be implemented on multiple processors, to provide full robustness against failure, its agents should be distributed across several machines. Another benefit of multi-agent systems is their scalability. Since they are inherently modular, it should be easier to add new agents to a multi-agent system than it is to add new capabilities to a monolithic system which means systems capabilities and parameters are likely need to change over time or across agents can also benefit from this advantage of multi-agent system.

A path planning algorithm is one of the main factors to have a successful mission for multi-agent. It is a process that looks ahead to the future and uses information about the world it has been given or accumulated over time to provide a safe path for the agent, prevent it from dangerous zones and reach the goal in the shortest amount

of time possible (Giesbrecht, 2004). Path planning algorithm related problems have been extensively investigated and solved by many researchers (Nilson, 1969), (Thompson, 1977), (Lozano-Perez *et al.*, 1979), (Tokuta, 1998) mostly focusing on ground robotics and manipulators.

Among important criteria of path planning that are commonly taken into account are the computational time, path length and completeness. A path planning algorithm with less computational time is crucial in real time application, which is desirable in dynamic environments. The generated optimal path in terms of path length by a path planning technique will minimize an agent traversal time, hence extends its endurance and life cycle, minimizes fuel or energy consumption and reduces exposure to possible risks. On the other hand, a path planning approach holds the completeness criteria if it is able to find a path if one exists.

Nevertheless, occasionally there are trade-offs between path planning criteria. For example, a path planning method has to neglect the path optimality in order to increase the computational efficiency. It means that finding a slightly longer path with less computational time may be preferable. On the other hand, higher computational complexity is necessary if an optimal path is required for some reasons (Omar, 2012).

Path planning for multi-agent has been used for area coverage navigation in obstacle environments and for task assignment (Kotari, 2011). Path planning for a team of agents to cover an area of interest while avoiding obstacles and reaching a particular location at specified time and orientation has been proposed by (Tsourdos, 2005). The algorithm was then extended to produce safe and flyable paths using Dubin's path. A number of path planning techniques considering multi-agent have been developed in (Mot *et al.*, 2002), (Schouwenaars *et al.*, 2006) and (Bertucceki *et al.*, 2006).



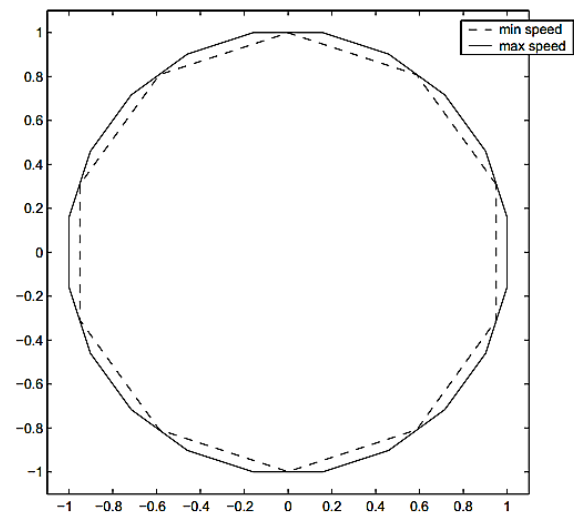
The purpose of this paper is to review recent researches on cooperative path planning algorithms for multi-agent particularly using Mixed-Integer Linear Programming (MILP). MILP is one of the path planning approaches that has been proven to be intensely suitable for obstacle avoidance subject to dynamic constraints. The advantage of MILP is its capability to solve global optimization problems efficiently in obstacle avoidance while incorporating linear vehicle dynamic constraints that may lead to be exceptionally complex problem in other methods, particularly in combinatorial approaches to path planning based on graph formulations and dynamic programming (Hwang *et al.*, 1992).

MILP problem differs from a linear programming (LP) problem only in that some of the variables are restricted to be integers and thus the solution space for a MILP problem is a subset of that of a similar LP problem which is the MILP problem without the integer restrictions (Cedric *et al.*, 2006). The MILP form of the trajectory optimization problems is linear by definition, so the method is immune to issues of local minima and globally optimal solutions can be found.

This approach is that highly optimized can be readily solved using commercial software (Richard *et al.*, 2002). The AMPL/CPLEX optimizer software is used to solve the MILP formulation (IlogAmpl, 2000) that implements the branch-and-bound algorithm in conjunction with many adjustable heuristics, allowing quite large problems to be solved in practical computation times. However, one major drawback of MILP approach is its computational complexity. There have been relatively a few recent researches on the development in MILP approach which are concerned on problem formulation including how the integer constraints can be added to linear programming to account for obstacle avoidance and collision avoidance among group of autonomous systems.

## PROBLEM FORMULATION

In the MILP formulation presented by (Schouwenaars *et al.*, 2001), the combinational of the linear program with the binary constraints for collision avoidance, yielded a large non-convex MILP. To make the problem more realistic, linear or piecewise linear absolute value constraints on maximum and minimum input and state can also be added (Andrew *et al.*, 1999). Example of two polygons that constrain the length of the velocity vector as illustrated in Figure-1. The dashed line represents a polygon associated with minimum speed constraints and the solidline is for the maximum speed (Kuwata, 2001).



**Figure-1.** Convex and non-convex constraints on a normalized velocity vector (Kuwata, 2001).

The corresponding state and input vectors are:

$$s_p = [x_p y_p \dot{x}_p \dot{y}_p]^T \quad (1)$$

$$u_p = [u_{xp} u_{yp}]^T \quad (2)$$

where

$s_p$  = vehicle state

$u_p$  = control input

$T$  = number of time-steps

These equations are discretized resulting in the form

$$s_{(i+1)p} = A s_{(ip)} + B u_{(ip)} \quad (3)$$

Where  $i$  is time step, the states at intermediate points in time must be consistent with the system dynamics where  $A$  and  $B$  are discretized form of the continuous system dynamics.

Assumed that the Vis number of vehicles and  $L$  is number of stationary obstacles and the complete time range be divided into  $N$  time steps. The complete MILP then becomes

$$\forall i \in [0, \dots, N-1] :$$

$$u_{pi} \geq u_{p,min} \quad (4)$$

$$u_{pi} \leq u_{p,max} \quad (5)$$

$$\forall i \in [0, \dots, N] :$$

$$s_{pi} \geq s_{p,min} \quad (6)$$

$$s_{pi} \leq s_{p,max} \quad (7)$$



$\forall i \in [1, \dots, L], \in [1, \dots, N] :$

$$x_{pi} \leq x_{c,min} + Mt_{pci1}$$

$$-x_{pi} \leq -x_{c,max} + Mt_{pci2}$$

$$y_{pi} \leq y_{c,min} + Mt_{pci3}$$

$$-y_{pi} \leq -y_{c,max} + Mt_{pci4}$$

$$\sum_{k=1}^4 t_{pcik} \leq 3$$

$$t_{pcik} = 0 \text{ or } 1$$

where

$t_{pcik}$  = binary variable (0 or 1)

$M$  = large number for logical constraints

$\forall i \in [1, \dots, N], q \in [p+1, \dots, V] :$

$$x_{pi} - x_{qi} \geq d_x - Mb_{pqi1}$$

$$x_{qi} - x_{pi} \geq d_x - Mb_{pqi2}$$

$$y_{pi} - x_{qi} \geq d_y - Mb_{pqi3}$$

$$y_{qi} - x_{pi} \geq d_y - Mb_{pqi4}$$

$$\sum_{k=1}^4 b_{pqik} \leq 3$$

$$b_{pqik} = 0 \text{ or } 1$$

where

$b_{pqik}$  = binary variable (0 or 1)

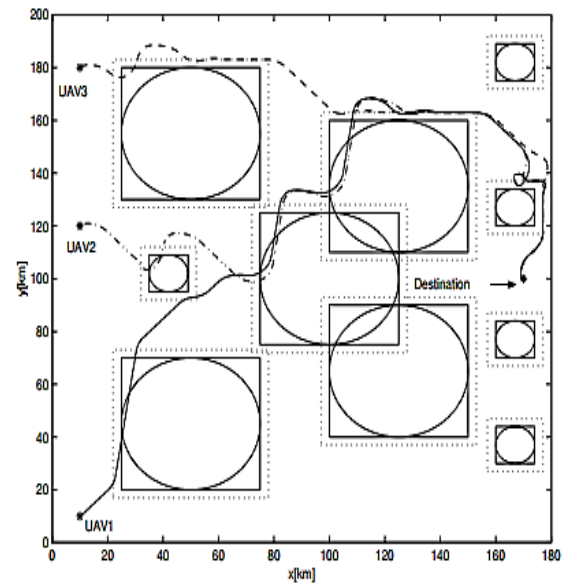
$d_x, d_y$  = safety distances

## TECHNOLOGY OVERVIEW

In this section, several surveys of recent researches have been described briefly on reducing the computational requirements of MILP approach so that it can be more readily used in real-time applications.

A finite receding horizon approach was proposed in (Kamal *et al.*, 2005) which was based on the work of (Richard *et al.*, 2002), (Schouwenaars *et al.*, 2001), (Richard *et al.*, 2001), (Tillerson *et al.*, 2002). Various constraints problem were formulated to avoid radar zones and collisions. These constraints were expanded to be both hard and soft so as to overcome the infeasibility problem. The finite receding horizon approach was numerically stable and could be applied to the path planning of a fleet of UAVs. The authors had proposed the problem formulation with "soft" constraints which accommodate infeasibility and find the least risk (most optimal) flight path in that situation.

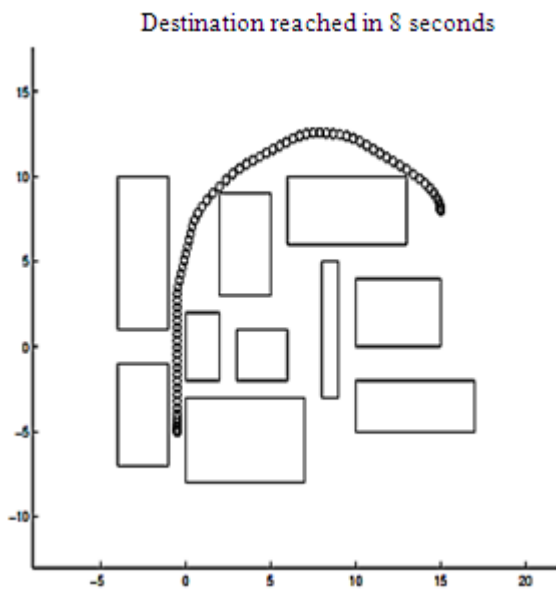
The proposed approach had shown that flight path planning for UAVs could be solved by a linear, constrained optimization formulation with real and integer variables. A finite receding horizon method had been proposed which made use of soft constraints. However, the MILP procedure required a high computational demand. That made it very difficult to perform in real time, though the introduction of finite receding horizon greatly helps the reduction of computation time. Figure-2 shows an example of the operation region is 180 km by 200 km and has 10 defence units (radar and Surface-to-Air- Missile (SAM)) shown as circle in the Figure.



**Figure-2.** Example of a scenario and the obtained flight trajectories (Kamal *et al.*, 2005).

The work of (Schouwenaars *et al.*, 2001) raised several concern about a new approach to fuel-optimal path planning of multiple vehicles using a combination of linear and integer programming. The basic problem formulation was to have the vehicles moving from an initial dynamic state to a final state without colliding with each other, while at the same time avoiding other stationary and moving obstacles.

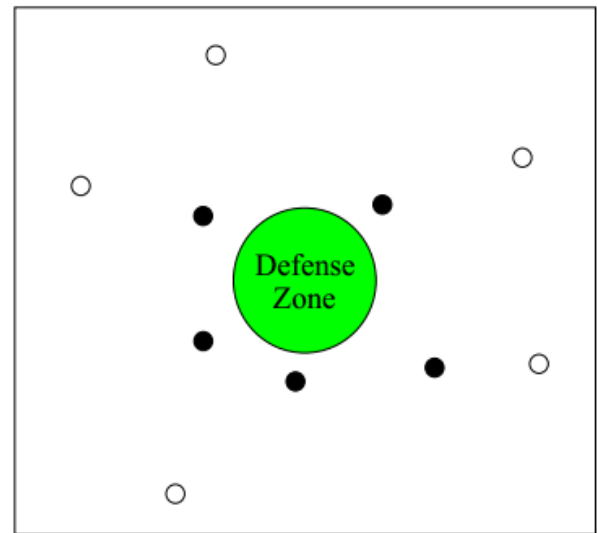
Comparison of receding horizon strategies with fixed arrival time approaches that have been carried out to show that the framework of MILP is well suited for path planning and collision avoidance problems. It was shown that receding horizon strategies, while computationally more attractive than strategies aimed at computing complete trajectories a priority, could lead the system to unsafe conditions. An example of the trajectory for fixed arrival time  $T_{arr} = 8s$  is shown in Figure-3.



**Figure-3.** Fuel-optimal path of unmanned vehicle for a fixed arrival time  $T_{arr}=8s$  (Schouwenaars *et al.*, 2001).

On the other hand, (Matthew *et al.*, 2005) applied methods based on problems derived from an adversarial game between two teams of robots called RoboFlag. In the study, a hybrid systems approach for modeling and cooperative control of multi-vehicle systems were carried out called mixed logical dynamical systems (Bemporad *et al.*, 1999), which were governed by difference equations and logical rules and were subject to linear inequality constraints.

As an alternative approach for multi-vehicle control, first presented in (Earl *et al.*, 2002), (D'andrea *et al.*, 2002), was developed independently from a similar approach developed by (Richards *et al.*, 2002). The first step was to focus on modelling the intelligence of the adversaries with state machines. Second step allowed multiple, possibly non-uniform and time discretizations. The performances of the resulting trajectories were feasible, which was advantageous because they could be applied directly to the multi-vehicle system. Figure-4 shows the drills involved two teams of robots, the defenders and the attackers, on a playing field with a circular region of radius  $R_{dz}$  at its center called the Defense Zone.



**Figure-4.** The drill takes place on a playing field with a Defense Zone at its center. (Matthew *et al.*, 2005).

In (Cedric *et al.*, 2006), the authors introduced a terrain flight formulation and presented the motivations and techniques that helped making the path planning approach suitable for a real-time applications environment. In their approach, the terrain was constructed as a series of obstacles which acted as “floor tiles” in three-dimensional that traveled over by the vehicle. On top of that, an altitude cost to keep the trajectory close to the terrain was introduced. By choosing the right parameters, Nap-of-the-Earth (NOE) with varying aggressiveness could be achieved. For terrain modeling, the authors chose triangulated irregular networks (TIN) to represent the terrain due to its straightforwardness. In terms of computational techniques, the authors used the concept of abstraction to naturally decompose the problem into subproblems of largest timescales, medium timescales and smallest timescales. These techniques managed to reduce the computational intensity of a very large path planning problem by exploiting the structures of the path planning problem and breaking it down into multiple layers of subproblems that were far simpler to solve. These subproblems solutions were then combined to form a composite trajectory. However, the authors had not addressed how the MILP planned trajectory will be implemented on an actual flight trajectory.

## CONCLUDING REMARKS AND FUTURE WORK

In summary, this survey has described some of approaches based on MILP that have been developed for path planning and they provided some possible direction for future study. It was found that most of path planning methods based on MILP produced solutions with high computation time due to its complexity. It also suffers from scalability issues, in which the number of integer variables increases with the number of obstacles. Several methods have been used to address this issue including implementing a MILP trajectory planning problem in a receding horizon framework and breaking the problem



down into many subproblems of smaller size. In any cases, there are a widevariety of effective algorithms available to be used for path planning and much researchis still being undertaken in refining and improving them.The other issue is how implement the planned trajectory on an actual flight trajectory. For future works, the direction of research with regards to MILP can be focused on how to make it less computationally expensive hence it can be applied in real time path planning for multi-agent. Also, the implementation issue is a great deal and must be addressed to ensure that the benefit of MILP, which can produce an optimal path can be enjoyed.

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