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CALCULATED CHARACTERISTICS OF A PROTOTYPE MODEL OF BEAM RECIRCULATION IN A LINEAR ACCELERATOR WITH STANDING WAVE

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ABSTRACT

In this paper we discuss the justification of a beam recirculation scheme in a prototype model of an accelerator with standing wave, the justification of a calculation method for bending magnets and the choice of their geometry, the calculation of the radial and longitudinal dynamics of particles in a system with beam recirculation, and the influence of various parameters of the scheme on the beam characteristics. In selecting the beam recirculation scheme, the fact is determinative that, in the operation of the linear electron accelerator in the standing wave mode, it becomes possible to perform an acceleration of the beam in the opposite direction in the same structure, which does not lead to a significant increase in the size of the entire setup. It is also reasonable to stay with the double passing of the beam along the biperiodic retarding structure, which greatly simplifies the technological implementation of the recirculation process scheme.

Keywords: beam recirculation, linear electron accelerator, standing wave, biperiodic accelerating structure, accelerating section, bending magnet, beam characteristics, median plane.

1. INTRODUCTION

An important issue now is the issue of increasing the efficiency of particle acceleration in linear electron accelerators (LEA) by virtue of repeated passage of particles in the accelerating structure (Ermakov, 2003). The paper (Ayzatsky *et al.*, 2008) discusses a reconstruction project of the linear accelerator of the UHF electrons with the particle energy of 20 *MeV* of the Argonne National Laboratory which aims at increasing the electron energy. It is shown that the proposed modernization scheme for the accelerator based on beam recirculation will provide an electron beam at the accelerator exit with a pulse current of 0.5 A and the particle energy of 45 *MeV*. As shown in the paper (Nechaev, 1978), one of the most effective and promising acceleration schemes is a beam recirculation scheme in the LEA with standing wave.

The multiple passing of the electron beam through the accelerating gap was first suggested by V.I. Veksler (Veksler, 1945). At present, several split microtrons have been launched and are running, the character of acceleration in which has much in common with beam recirculation.

In the currently operating niobium superconducting accelerator for 3.5 *MeV*, the energy of 9.5 *MeV* is achieved by three-time acceleration (Young, 1973). The vertical defocusing, inevitable in the bending magnets, is eliminated by the introduction of additional windings, which create an opposite field on the edge of the magnet.

The microtron, described in the paper (Livingood, 1969), consists of three accelerating cells with the lateral connection resonators and two sector 180^0 magnets. The beam energy varies from 4.5 to 18 *MeV* for the power of a high-frequency generator equal to 3.5 *MW*. The vertical stability is achieved by using electrostatic focusing elements, which also serve to rotate the beam from the gun. In the microtron, the beam passes the accelerating structure each time in one and the same direction.

The same recirculation scheme is considered in (Volodin and Kuzyakov, 1975), where a round diaphragm-type waveguide operating on a traveling wave is used as the accelerating structure. The transport channel, providing the repeated beam entering the accelerator, is calculated using the matrix formalism. The paper (Muntyan *et al.*, 1979) describes an acceleration complex for the energy of 40 *MeV* with the recirculation of beam into accelerator with a traveling wave.

In the works (Kolomensky, 1967; Bulykin, 1979) it is noted that, in the case of LEA operating in the standing wave mode, the second counter passing of the beam of charged particles is possible with the purpose of its further acceleration. The possibility of electron acceleration to 45 *MeV* with the power of high-frequency (HF) power supply of 0.5 *MW* is discussed in the works (Baglin, 1971; Leiss, 1972). There the acceleration is realized almost continuously, whereas the accelerating section consists of two parts of a biperiodic retarding structure (BRS) with external connection cells. The required energy is gained as a result of double passage along the accelerating section, whereas the beam rotation after initial acceleration is achieved by using an achromatic magnetic reflector. The possibility of using such reflectors is considered in the work (Kolomensky and Gapanovich, 1968). The perturbation of the longitudinal movement of the particles is caused by the momentum dispersion at the entrance and is characterized by an increment of the path in the reflector as compared with the equilibrium particle. The phase shift at the second entering the accelerator for the reflector consisting of two consecutive magnetic sectors of 180^0 with oppositely directed magnetic fields will be expressed as follows:

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$$\Delta\varphi = \pm 2\pi \left(1.1\left|\frac{\Delta W}{W}\right| + 2.4|x_0|\right)\frac{r_0}{\lambda},$$

where r_0 is the radius of the equilibrium orbit, x_0 is the beam divergence at the reflector entrance.

For a magnetic sector of 270^0, this value is an order of magnitude greater, but these reflectors are achromatic and are quite suitable for a single rotation. For multiple rotation, the reflection magnets have been proposed (Gapanovich, 1970), in which isochronism is achieved by splitting this magnet into sectors and adding doublets, which certainly complicates the design.

An operating LEA with beam recirculation is described in the paper (Schreiber *et al.*, 1977). The principle diagram of this accelerator includes an accelerating system based on BRS and a magnetic mirror, implementing the beam rotation by 180^0 for the particles with the energy spread $\frac{\Delta W}{W} \leq 16\%$. The injector is situated at an angle of 90^0 to the accelerator axis, while the beam entrance into the accelerating system is provided by a bending magnet. At the power of high-frequency power supply of 1.8 *MeV* and the pulse current of 20 *mA*, the energy of 8 *MeV* is obtained, and at the current of 8 *mA*, 25 *MeV*. The total length of the setup is 2.8 *m*, whereas the beam losses in the magnetic mirror do not exceed 15%.

2. METHODS

2.1 General statements

In a recirculation scheme we will consider a prototype model of a single-section accelerator where the electrons, accelerated in the forward direction, are able to accumulate the energy from 3 to 5 *MeV*. We will discuss the justification of a beam recirculation scheme in a prototype model of an accelerator with standing wave, justification of a method of calculation of the bending magnets and a choice of their geometry, the calculation of radial and longitudinal dynamics of particles in a system with beam recirculation, the influence of various parameters of the scheme on the beam characteristics.

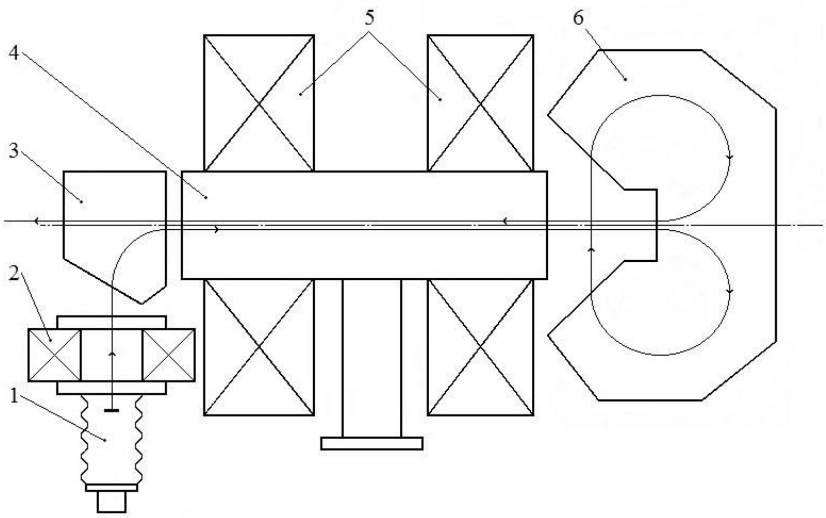

Figure-1. A scheme of LEA with beam recirculation: 1 - injector, 2- magnetic lens, 3 - magnetic 90^0-wedge, 4 - accelerating section, 5 - focusing coils, 6 - recirculation magnet.

Figure-1 shows the scheme under consideration of an accelerator with standing wave and beam recirculation. The electron gun injects an electron beam with the energy of 40 *keV* at an angle of 90^0 to the axis of the accelerating section. The beam rotation is carried out by a bending magnet of the type of magnetic wedge. The beam is accelerated then in the forward direction in a single-section accelerator to an energy of about 3.5 *MeV* and gets into the field of a recirculation magnet, where its turn by 540^0 is carried out. After that the beam again enters the accelerating section, passes it in the opposite direction and is accelerated to the energy of 6 *MeV*. The magnetic field of the 90^0-bending magnet does not significantly affect the movement of the beam in the opposite direction, because the magnet is designed to rotate a beam with the injection energy.

Additional elements of the scheme are magnetic lens of the electron gun, which can be used to change the injection characteristics of the beam, and the focusing coils, which provide the beam guiding through the accelerating section in the opposite direction, since in this case the HF-focusing (Novozhilov *et al.*, 2015) may turns out to be ineffective.

Let us consider in more detail the justification of a choice of the main elements of the scheme. As the source of electrons, an injector is taken with the following parameters: the beam radius is 2 *mm*; the beam divergence is 0.025 *rad*; the injection energy is 40 *keV*. A magnetic wedge, carrying out the beam rotation by 90^0, is placed between the electron gun and the accelerating section. For the calculation of such magnet depicted in Figure-2, one can use the transformation matrices in the direction of the *x*- and *z*-axis (Steffen, 1969).

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$$M_x = \begin{bmatrix} 1 & 0 \\ -1/\rho \tan\gamma & 1 \end{bmatrix} \begin{bmatrix} \cos\varphi & \rho\sin\varphi \\ -1/\rho & \cos\varphi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/\rho \tan\varepsilon & 1 \end{bmatrix}, \quad (1)$$

$$M_z = \begin{bmatrix} 1 & 0 \\ 1/\rho \tan(\gamma+\theta) & 1 \end{bmatrix} \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/\rho \tan(\varepsilon+\psi) & 1 \end{bmatrix}, \quad (2)$$

where $\psi = \frac{1}{\rho}\frac{b}{\cos\varepsilon}, \theta = \frac{1}{6}\frac{b}{\rho\cos\varepsilon}$; $\varphi, \rho, \varepsilon, \gamma, l$ are shown in Figure-2; b is the distance, from which the magnetic field is taken into account. In our case b is defined as shown in Figure-3.

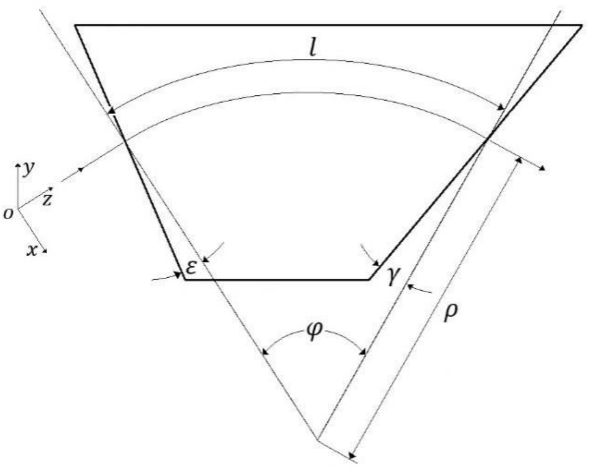

Figure-2. Bending magnetic wedge.

The magnetic field value in the 90^0 magnetic wedge is chosen on the basis of considerations of its compactness and minimum distance of the electron gun from the accelerating section. Calculations show that the most optimal are the following values of parameters: the magnetic field intensity H = 135 *Oe* at the rotation radius ρ = 50 *mm*.

The angles of the magnetic wedge ε and γ, depicted in Figure-2, mainly determine the radial parameters of the beam after the rotation. Since later on the beam should get into the axially symmetric electromagnetic field of the accelerating structure, it is necessary to preserve its axially symmetrical structure after the turn. Furthermore, with respect to radius, the beam not only must not exceed the aperture of the drift channel of the accelerating section, but also be parallel (to the extent possible) to the axis. In order to select the appropriate values of the angles, some variants were calculated with different values of ε and γ according to the formulas (1) and (2).

The field distribution on the edge of the magnet is obtained experimentally and approximated by a linear relationship, as shown in Figure-3.

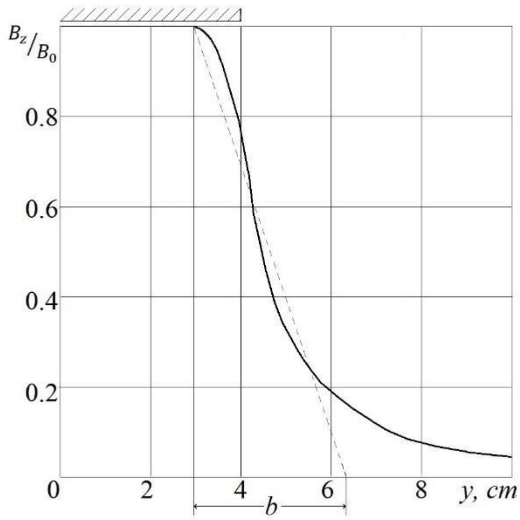

Figure-3. The experimentally measured distribution of the edge magnetic field and its linear approximation (dashed line).

For the selected rotation radius $\rho = 50\ mm$ and the parameter $b = 3.5\ cm$, the values of ε and γ that provide the required characteristics at the exit of the magnet with the purpose of further acceleration are $\varepsilon = -25^0$ and $\gamma = 0$. Thus, after the 90^0 turn, we succeed in keeping the axially symmetric structure of the beam, which gets into the accelerating section and gains the energy of 3.5 *MeV* while passing in the forward direction.

At the exit of the accelerating section, a recirculation magnet is situated, the configuration of the pole-pieces of which is shown in Figure-1. The total rotation angle of the beam in such a magnet is 540^0. In this case, the particles successively pass a 270^0 turn, an interval of drift in the edge field and without a field, and a second 270^0 turn. This magnet has a rather simple design and small sizes; however, it should be noted that, having achromatism which is useful in our case, this magnet is non-isochronous. Providing isochronism would make the design much more complex and increase the dimensions. At the same time, the isochronism requirement in our particular case is not critical; therefore the shape of the magnet shown in Figure-1 is justified. The magnet is designed to turn the electron beam with the energy from 3 to 5 *MeV*; thus at the radius of the turn of the equilibrium orbit 50 *mm*, the field must change from 2200 to 3500 *Oe*.

2.2 Methods of calculation

For detailed calculation of the particle dynamics in the proposed recirculation scheme, one can use two methods: a matrix one and numerical integration of the equations of radial and longitudinal dynamics. The use of the matrix calculation method, as the most simple, is justified for the selection of preliminary parameters of bending magnets. However, the matrix method has fundamental limitations that make it impossible to use this method in the calculation of particle dynamics in such a scheme.

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These limitations include the following ones: the matrix method does not allow taking into account the radial distribution of the fields on the edge and just inside the magnets and is extremely inconvenient in taking into account the time effects during acceleration and particle formation. These circumstances have led to use the method of numerical integration of the equations of motion of the particle in the real field of the magnet, which distribution in the median plane has been obtained experimentally.

This method is based on the equation of motion of a particle in a magnetic field, which can be derived from (Artzimovich and Luk'yanov, 1978)

$$\frac{d}{dt}(m\boldsymbol{v}) = q\boldsymbol{E} + q\mu_0[\boldsymbol{vH}].$$

If the electric field strength is set to be zero, then the change of the particle velocity can be expressed through the parameters of the particle and magnetic field in the following form

$$\frac{d}{dt}(m\boldsymbol{v}) = q\mu_0[\boldsymbol{vH}], \tag{3}$$

where t is time, q and m are the charge and mass of the particle, $\boldsymbol{v}$ is the particle velocity, c is the velocity of light, μ_0 is the magnetic constant, $\boldsymbol{H}$ is the magnetic field intensity of the bending magnet.

We solve the equation (3) in the Cartesian coordinates (x, y, z) and proceed to dimensionless quantities, convenient for numerical integration. As a result, we obtain a system of equations equivalent to the vector equation (3):

$$\begin{aligned} \frac{d^2\left({x}/{\lambda}\right)}{d\tau^2} &= \frac{1}{\gamma}\left(\frac{d\left({y}/{\lambda}\right)}{d\tau}B_z - \frac{d\left({z}/{\lambda}\right)}{d\tau}B_y\right), \\ \frac{d^2\left({y}/{\lambda}\right)}{d\tau^2} &= \frac{1}{\gamma}\left(\frac{d\left({z}/{\lambda}\right)}{d\tau}B_x - \frac{d\left({x}/{\lambda}\right)}{d\tau}B_z\right), \\ \frac{d^2\left({z}/{\lambda}\right)}{d\tau^2} &= \frac{1}{\gamma}\left(\frac{d\left({x}/{\lambda}\right)}{d\tau}B_y - \frac{d\left({y}/{\lambda}\right)}{d\tau}B_x\right); \end{aligned} \tag{4}$$

where γ is the particle energy in the units of the rest energy, $\tau = {ct}/{\lambda}$ is the dimensionless time, λ is the wavelength of the generator, B_x, B_y, B_z are the dimensionless components of the magnetic field.

In solving the system of equations (4), we use the boundary distribution of the magnetic field, measured experimentally for a magnet with the distance of $25\ mm$ between the poles and depicted in Figure 3. The purpose of calculation of the particle dynamics in the proposed recirculation scheme is the final selection of the constructive parameters of the bending magnets and possible focusing elements, as well as the longitudinal and transverse characteristics of the beam. Using the system of equations (4) and taking into account the actual distribution of the magnetic field, we calculated the particle dynamics in the 90^0 magnetic wedge.

3. RESULTS

The conducted calculations have shown that it is necessary to focus the beam of the electron gun before entering the bending magnet, since without it the beam sizes would exceed the aperture of the vacuum chamber. Therefore, it was decided to install a magnetic lens after the electron gun. The calculation results have demonstrated the dependence of the radial characteristics of the beam at the entrance of the accelerating section on the magnetic field value in the lens. At the magnetic field strength of 250 *Oe*, a convergent beam of the diameter of about 4 *mm* can be obtained.

In addition, the calculations have been carried out, when the lens was situated just after the 90^0-magnet; however, in this case the results were much worse. Therefore, the lens position immediately after the electron gun is preferable. The performed calculation of the dynamics in the 90^0 bending magnet indicates that the injection of the electron beam into the accelerating section at an angle of 90^0 with the parameters acceptable for further acceleration is possible.

We turn now to the calculation of particle dynamics in the recirculation magnet, the first stage of which is strict account of the boundary distribution of the magnetic field and its effect on the particle motion. This is connected with the fact that the boundary magnetic field can distort the path of the particles, since the time of the particles being under the action of the magnetic field increases, and therefore, the rotation angle exceeds 540^0. Because of this, we have to adjust the coordinate of the particles entry into the magnetic field region. A regulating parameter Δx is the distance from the bevel edge of the magnet to the rectangular face of the pole-piece, as shown in Figure-4.

To determine the optimal value of Δx, some variants with different values of it were calculated. In addition, zero values of the beam divergence were taken as the initial data, and the energy was assumed to be 3.5 *MeV*, which corresponds to the operating mode of the accelerating section. Figure 4 illustrates the selection of the optimum value of Δx. As seen from the figure, the radial characteristics of the beam in this case are critical with respect to the value of Δx. The optimal variant corresponds to the case of $\Delta x = 23.7\ mm$. If the motion of particles in the opposite direction takes place in the accelerating section with an external magnetic field, then this criticality is lost, since, in the whole range of change of Δx , the beam with the focus located at the entrance (in the opposite direction) into the accelerating section turns out to be convergent. Consequently, the presence of two focusing coils in the accelerating section greatly facilitates guiding of the beam in the opposite direction. In what follows, when calculating the dynamics of particles in the accelerating section, the optimum value will be found of the external magnetic field in terms of forward and reverse motion of the particles.

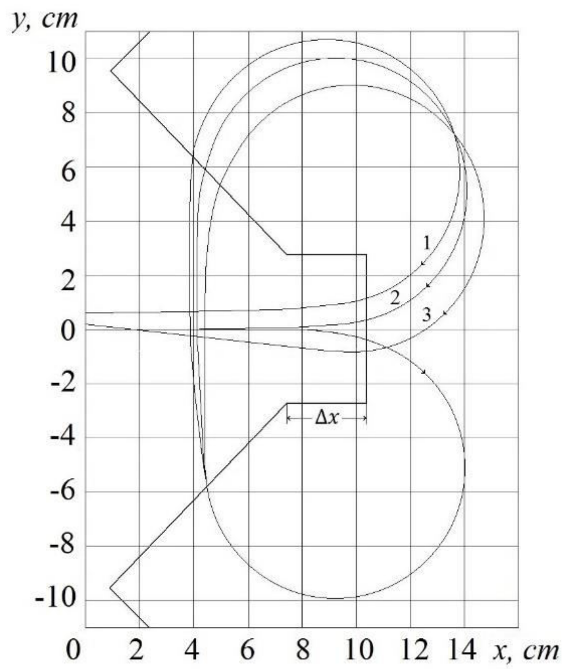

Figure-4. The trajectories of the particles in the recirculation magnet for different values of the parameterΔx: $1 - \Delta x = 25.7$ mm; $2 - \Delta x = 23.7$ mm; $3 - \Delta x = 20.3$ mm.

In order to identify the influence of the initial deflection along the y axis on the beam exit parameters in the median xy-plane, the calculations were conducted that show that the recirculation magnet has unsatisfactory properties with respect to movement in the direction of the y-axis. Already at the initial deflection of the order of one millimeter and the zero divergence at the entrance, the particles have a quite noticeable divergence at the exit equal to 0.035 *rad*. Therefore, as mentioned earlier, for this scheme of the beam recirculation, it becomes necessary to have an external focusing magnetic field, since the HF focusing of the beam effectively operates only in the forward direction.

A necessary condition for an effective acceleration in the opposite direction is to ensure getting of a bunch of particles into a given phase of the HF field. It is possible to fulfil this condition in this case by a change of the flight time of the electrons in the recirculation magnet due to a magnetic field change. This change, as shown by the calculation results, influences the radius of the equilibrium orbit, but does not take out the beam from the magnet pole-pieces.

To study the dynamics of particles in the considered prototype model, the trajectories of particles in the accelerating section were calculated in two variants: for the external focusing field of two coils with the maximum intensity of 900 *Oe* and the HF focusing. The initial radial characteristics were set taken into account the radial beam characteristics obtained after the lens and the 90^0 bending magnet. Figure-5 shows the radial characteristics of the accelerated beam at the entrance to the recirculation magnet, at its exit and after the second acceleration in external focusing field of 900 *Oe*.

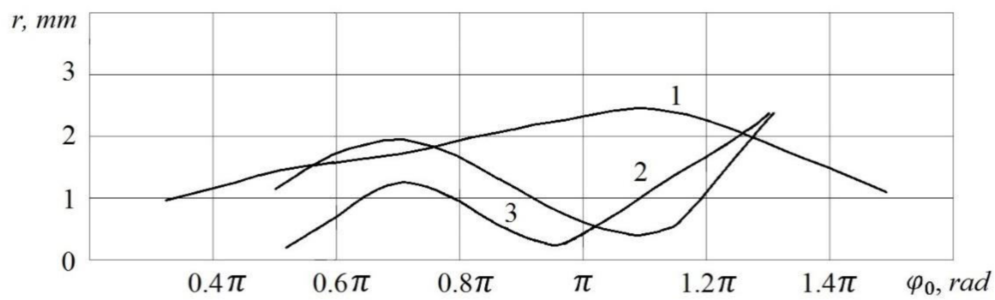

Figure-5. Dependence of the beam radius on the initial phase: 1 - at the exit of the accelerating section; 2 - at the exit of the recirculation magnet; 3 - after the second acceleration.

Calculations show that, as a result of the beam recirculation, the capture efficiency decreases to 40% instead of 60%, that is, a third of the particles are lost during the second acceleration. The data in Figure-5 correspond to the movement of the particles in the xz-plane.

4. DISCUSSIONS

The scheme considered here consists of an accelerating section on the basis of a BRS with internal connection cells and a recirculation 540^0 magnet. The total rotation angle in the recirculation magnet is 540^0; the particles successively pass a 270^0 turn, an interval of drift in the edge field and a second 270^0 turn. This magnet has a fairly simple design and small dimensions; however, having achromatism, this magnet is non-isochronous. In our particular case, the isochronism requirement is not obligatory, so the use of such magnet in the scheme is justified.

The greatest difficulties are encountered when calculating the dynamics of particles in the recirculation magnet. The matrix method does not allow taking into account the real distribution of the magnetic field on the edge and just inside the magnet, therefore this method is extremely inconvenient in taking into account the time effects during acceleration and formation of particles.

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These circumstances impelled us to use a method of numerical integration of the equations of particle motion in a real magnetic field, the distribution of which on the median plane of the magnets was obtained experimentally. Calculations show that some problems can arise in the particle movement in the vertical plane of the recirculation magnet. A violation of the beam symmetry after the turn complicates its guiding via the accelerating section in the opposite direction, so in this case the use of external focusing devices is a necessity.

As a result of the carried out calculations of the parameters of the main elements of the scheme, it is shown that doubling of the electron energy is possible in this scheme, whereas the radial and phase characteristics of the beam will be largely determined by the accelerating section.

5. CONCLUSIONS

The problem of increasing the accelerator efficiency can be solved with the help of the beam recirculation. When selecting the beam recirculation scheme, the fact is determinative that in the LEA operation in the standing wave mode, one can implement the beam acceleration in the opposite direction in the same structure. It is also reasonable to stay with two passes of the beam along the BRS, which greatly simplifies the technological implementation of the recirculation process scheme.

From the obtained calculation results, we can conclude that the movement in the xz-plane in the proposed beam recirculation scheme can be successfully realized. As far as the particle motion in the xy-plane is concerned, there the beam characteristics are less satisfactory.

Indeed, the axial symmetry of the beam after passing through the recirculation magnet is substantially broken. As a result, the beam becomes divergent along the z-axis. Therefore, the transmission efficiency is mainly determined by the movement in the direction of the y-axis. The authors plan to continue the research of the recirculation magnet form to improve the characteristics of the electron beam for the acceleration in the opposite direction without external focusing elements, which allows reducing the radial dimensions of the setup as a whole.

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