ABSTRACT

Image compression based on fractal coding is a lossy compression technique and normally utilize for gray level images in range and domain blocks of rectangular shape. Fractal based digital image compression techniques’ provides a large compression ratio and it is proposed using nearest neighbourhood interpolation using isosceles triangle segmentation. In the present study a partitioned iterative function searching based on isosceles triangle shape dimension of image plane has been considered. During the searching the intensity of each pixel needs be transformed and this is done with intensity interpolation based on nearest neighborhood intensity interpolation. The proposed method has yielded higher PNSR with marginally lesser processing time but with very high compression ratio. It is established that the proposed method is suitable for face image as well as natural object images.

Keywords: fractal compression, iterated function system (IFS), Partition IFS, contractive affine transformation, Isosceles triangle segmentation, transformation of colour image.

1. INTRODUCTION

Fractal is basically based on the concept of fractional geometry which is used to describe irregular and fragmented objects or patterns. There are various objects like cloud, fire flame, snow fall, mountain, waves, and trees etc which are difficult to describe or deal with the help of other geometry. Fractal compression [1] came into being to encode the images which posses self similarity [2]. A lot of work has been carried out by various scientists to compress either gray scale [3]-[5] or colour images. Present day digital images normally are of high resolution with high level quantization of intensity and it requires huge memory space in storage devices and it invites larger processing time by the processor and requires wide band width for data transmission. Thus compressions of high data signals have become necessary and prevalent also. Fractal compression is a lossy compression technique and it generally does not harm image quality in drastic way because all data sets like image data have got redundant portions also which may be deleted or discarded during the compression process colour images have occupied important place due to proliferation of high quality television and internetworking systems [6], and biomedical applications [7]-[9].

The fractal is generated by continuous iteration method starting from an initial data set or matrix indicating the image function. The fractal object can be thought of to be comprised of several copies of itself, each of which is scaled down with proper and feasible translations and rotations. Self similarity is an important word which means the unchangability of appearance if the image is either zoomed or shrinked by any scale. Although Mandelbrot first introduced the idea of fractal geometry, subsequently it was developed by Barnsley [1] and Jacquin [2]. The self similarity properly of the object makes fractal independent of scaling. Barnsley proposed about iterated function system (IFS) which is a model collection of contractive transformation in any metric space, and not necessarily encoding, as a dynamical system. This is used to code an image. Jacquin proposed partition IFS which possesses’ easier searching process compared to the earlier ones. Still a great deal of time is taken during the similarity search process between range and domain blocks.

There have been reported various research works [4],[10],[11] with a view to improvement of coding efficiency, to have greater processing speed as well as preserve the image quality when decoding is done for the restoration of the image. Attempts were made to vary the sizes of range and domain block to minimize the error between them [3]. Apart from the rectangular blocks, other types of blocks like lozenge, triangle [12], [13] block matching technique were also reported for colour images in RGB space and was coded by fractals by separating it into three channels [4].

Fractal coding may be considered as a combination of affine transformation and luminance (intensity) transformation. The latter requires some pre-defined knowledge about the intensity of the pixels in the coded image and this would ultimately decide the contrast of the transformed image.

In the present study, the concepts of nearest neighborhood interpolation method [12] have been taken into consideration for intensely transformation. The fractal coding part is based on the partitioning of image utilizing isosceles triangles geometry on RGB colour image [13]. The quality of the reconstructed image quality has been found to be better in terms of PSNR as well as human perception.
2. MATHEMATICAL BACKGROUND AND PROPOSED METHOD

A. Fractal based mathematical expression

A fractal may be represented by the set of equations, [14]

\[
W \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}
\]  

(1)

A fractal is generated when equation (1) is recursively iterated starting with an initial value of pixel location (x, y) on the image plane. The, a, b, c, d matrix indicates the rotation and skewing operations and e, f determine the translation of the image.

The set \{w\} is termed an affine transformation or contractive affine transformation. A fractal may be represented by a set of contractive affine transformation which are given by

\[
W_0, W_1, W_2, W_3, W_4, \ldots, W_n
\]  

(2)

Where \(W_n\) are the affine transformations and \(P_n\) denotes their individual importance. This representation is generally called IFS coding.

On self similarity concept in fractal based image compression, image blares are seen as rescale and intensely transformed approximate couples of block found elsewhere in the image plane. The compression ratio is proportional to the degree of self similarity.

The fractal coding based image compression can be expressed by following steps--

Step 1:

The image under test is partitioned into sub-images or range blocks. There may be different schemes to find such blocks, viz. quad tree partitioning, triangular partitioning, horizontal vertical partitioning, polygonal partitioning etc. The main aim in fractal compression is to partition an image into a smaller number of range blocks which should be similar to other portion of the image after undergoing different transformations.

Step 2:

The searching of domain block should be done in a statistical manner. The purpose is to search for the best domain which will be mapped to a range. A domain is a region where the transformation mapping starts and a range is the region where it is mapped. If the best matched area between the largest range blocks and the transformed domain blocks has got an error which is found to be greater than the similarity threshold. The range block is similar with each of these blocks. Range blocks which cannot be matched within the similarity threshold continue to partition into smaller range cells until the maximum number of partition is obtained. If this limit is touched and the nearest domain cell does not match a range cell within the said threshold, this area can be termed as anomalous area.

Step 3:

When the match with location is found various transformations like combination of affine and luminance transformation are to be computed. The same can be represented as follows:

\[
W \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ c & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} e \\ f \\ q \end{bmatrix}
\]  

(3)

Here z indicates the pixel intensity (luminance) at the location (x, y), e and f indicate the shift in the position of the range with respect to the domain p and q are the contrast and luminance adjustment to accomplish the transformation the affine transformation.

Step 4:

The foundation of fractal image compression lies in the construction of the IFS that approximate the original image. An IFS in terms of set theory and distance function, is a union of contractive transformation each of which maps into itself. To realize contractive transformation, the following condition should hold good:

\[
d(W(P_i), W(P_j)) < d(P_i, P_j)
\]  

(4)

The above equation is true for any metric space.

B. Fractal based colour image compression using Isosceles triangle segmentation

In [13], the domain and range blocks are divided into isosceles triangle instead of quad tree approach. In this case the length of hypotenuse of non-overlapping range blocks are calculated over the whole image. Compare domain block and range block with the hypotenuse of both blocks. Matching for range block and domain block are depends on certain pre-assigned threshold value. During this process of finding best match domain block, the same is required to undergo shrink, isometric and brightness transformation. In their study two kinds of isometric transformation includes themselves and its vertical mapping along right angle vertex was adopted. In the present paper the transformation have been avoided and replace by simple interpolation method which are stated below:

C. Nearest neighbourhood interpolation

In this method, unchanged intensity of any newly created range blocks are matched by nearest neighborhood intensity. It implies any new range block is assigned with the intensity of the one of the \(D_0\) neighborhood pixels. It is the one of the easiest methods of gray level assignment.

D. Peak Signal to Noise Ratio (PSNR)

Image compression is based on fractal coding is lossy compression method. The quality of image is calculated by Peak Signal to Noise Ratio (PSNR).

\[
\text{PSNR}=10 \log_{10} \frac{255^2}{(\frac{1}{M \times N}) \sum \sum (x_{ij} - \hat{x}_{ij})^2}
\]  

(5)
Where, $M \times N$ is the size of the composite image, and $x_{i,j}$ and $\hat{x}_{i,j}$ are values of the original image and reconstructed image at position $(i, j)$. Naturally higher PSNR value indicates much better image quality.

**E. The PSNR is compared between the images before and after compression**

Two types of images viz. Lena and Pine have been taken in the experiment. The aim of the present study is to segment them with high compression ratio. Fractal based geometry is applied in order to accomplish this. The images are partitioned successively through isosceles triangular shape. The range blocks thus found in each step have been searched to find the self similarity [15]. The RGB colour images have been converted to gray scale images using matlab software. During process, each range blocks are needed to be assigned a new intensity values and this has been done through nearest neighborhoods interpolations which are used for image zooming or shrinking. The process was continued keeping other properties of affine transformation.

*Figure-1(a). Range blocks segmentation based on isosceles triangles segmentation.*

Figures 1 (a), (b) and (c) show that range blocks and domain blocks are divided into isosceles triangles rather than squares. This method is approaching the diagonal edge and using the self-similarity relationship of the image which helps to reconstruct the edge information of original image.

*Figure-1(b). Domain blocks segmentation based on isosceles triangles segmentation.*

In Figure-1 (a) show the non-overlapped range blocks $R_1, R_2, R_3, R_4$ …… $R_n$ for whole image. In Figures 1(b) and 1(c) are the overlapped domain block segmentation.

*Figure-1(c). Range blocks and domain blocks segmentation based on isosceles triangles segmentation.*
Figure-2. Domain block and range block matching process.

In Figure-2 show the matching process of range and domain block. In this process finding the best matching range and domain block based on nearest neighbourhood interpolation using self-similarity parameter. In Figure-3 show the flow chart for proposed method.

3. EXPERIMENTAL RESULT AND DISCUSSIONS

The proposed technique is experimented and observed the simulation result. The simulation results are carried out Core I3 processor, 512GB RAM and Windows XP operating system. Here two colour images are namely Lena and Pine of size 256×256×3 are taken. Here Figures 4(a) show the original images of Lena, (b) show the gray-level images of Lena, (c) show the retrieve images of Lena according to our proposed method (d) show the original images of Pine (e) show the gray-level images of Pine and (f) show the retrieve images of Pine according to our proposed method.

Proposed method can save about 70 percent processing time without degrading image quality and compression ratio compare to RGB coding separately.

Using isosceles triangle segmentation range blocks and domain blocks are need less time for self-
similarity matching compare to square blocks segmentation.

Method [4] has better image quality but compression ratio is not good. Method [13] has the better image quality and improved the time efficiency compared to method [2] and [4] but proposed method has the better image quality and improved the time efficiency compared to other methods. In propose method PSNR will increase compare to their method nominal [13].

Table-1. Comparative table to study PSNR, processing time and compression ratio.

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<tbody>
<tr>
<td></td>
<td>RGB</td>
<td>PSNR (dB)</td>
<td>TIME (S)</td>
<td>Compression ratio</td>
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<tr>
<td>Lena</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>R</td>
<td>34.58</td>
<td>53</td>
<td>4.129</td>
<td>33.68</td>
</tr>
<tr>
<td>G</td>
<td>34.10</td>
<td>53</td>
<td>31.28</td>
<td>31.85</td>
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<tr>
<td>B</td>
<td>33.46</td>
<td>54</td>
<td>32.90</td>
<td>34.04</td>
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<tr>
<td>Pine</td>
<td></td>
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<td></td>
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<tr>
<td>R</td>
<td>31.26</td>
<td>58</td>
<td>4.129</td>
<td>30.19</td>
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<tr>
<td>G</td>
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<tr>
<td>B</td>
<td>30.74</td>
<td>54</td>
<td>30.66</td>
<td>32.04</td>
</tr>
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</table>

Here Table-1 show the comparative data with respect to PSNR, time of execution of the compression process done by the processor and compression ratio using proposed method and RGB coding separately, A.E. Jacquin [2], method proposed H.L. Zhu, Y.L. Zhao [4] and Method proposed Yuli Zhao, Zhiliang Zhu, Hai Yu [13]. From the proposed method [2], images are dividing RGB components into the size of 4×4 separately.
Figure-4(a). The original images of Lena.

Figures-4(b). The gray-level images of Lena.

Figures-4(c). The retrieve images of Lena according to our proposed method.

Figures-4(d). The original images of Pine.

Figure-4(e). The gray-level images of Pine.

Figures-4(f). The retrieve images of Pine according to our proposed method.
In Figure-6 (a) show the comparison chart of PSNR for Pine for all mentioned methods, and value of PSNR is higher (Red = 36.15, Green = 37.63 and Blue = 35.21) in proposed method compare to others methods. Figure-6 (b) show the comparison chart for processing time for Pine for all mentioned methods. Figure-6 (c) show the comparison chart of Compression ratio for Pine for all mentioned methods.
time for Pine for all mentioned methods and processing time for Pine is less (Red = 41, Green = 41 and Blue = 41) in proposed method compare to others methods, Figure-6 (c) show the comparison graph for compression ratio for Pine for all mentioned methods and compression ratio for Pine is higher (Red = 24.98, Green = 25.11 and Blue = 26.19) in proposed method compare to others methods. From the experimental result it has been observed that proposed method is better compare to others.

4. CONCLUSIONS

Partitioned iterative process has been carried out to find the self-similarity regions in the image plane based on the isosceles triangle segmentation. The contractive affine transformation is maintained during the iterative searches. Both Lena and Pine images have been subjected to same procedure. The compression ratio although may be theoretically infinite, in practice, is finite in spite of it passes through a lossy compression process. The finite value of the compression ratio may arise due to the presence of same anomalous region for which no self-similar matched region is found. The basic strength of the current study lies in the interpolation method in pixel intensity values required during iterative search. The study may be considered as a ground work for image zooming and shrinking on the basis of fractal geometry.

Fractal geometry based image segmentation has been carried out over two images. The rescaling of the imaging is done by taking isosceles triangles and intensity transformation is carried out by well known nearest neighbour interpolation. The study has yielded at least eight times more compression ratio with appreciable increase in PNSR in the segmented images. In future works better algorithms can be introduce for enhancing the coding efficiency and compression ratio without reducing the image quality.

REFERENCES

[12] Rafael C. Gonzalez and woods, Digital image processing, 3rd edition, PHI.